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Spectra of Electromagnetic Surface Waves on Plasma–Vacuum Interface Inside a Metallic Cylindrical Pipe

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Abstract: A general nonlinear dispersion relation, for the spectra of electromagnetic surface waves on the interface separating an axially symmetric plasma and vacuum both enclosed in a resistive cylindrical pipe, have been derived and various limiting cases were discussed. The spectra were found to be similar to those of electrostatic surface wave modes existing at plasma–vacuum interface for cases such as thin plasma or a conducting wall far away from the plasma-vacuum interface. For moderate values of ka, where k is the wave number and a is the width of plasma, appreciable modifications of spectral curves have been observed. By including the surface impedance Z_m of the pipe–wall, the dispersion relation of the electromagnetic surface waves becomes a function of the dispersive properties of the wall. Investigating the surface impedance shows no observable effect on the real part of the surface wave spectra, but it introduces an imaginary part into the frequency ω. The corresponding peak values have been obtained for various representative situations. When coupled to an external driver such as particle beams, surface wave fields can be excited, causing beam energy losses, and thus affecting the beam dynamics

Key Words: Electromagnetic Surface Waves, Surface Waves In Plasmas, Resistive Cylindrical Pipe.

Introduction

Surface waves (SW's) are proper modes propagating along the interface between two media, and are characterized by having fields concentrated near the boundary interfaces. In order for a surface wave to exist, the corresponding wave fields must be evanescent in both regions. The presence of the bounding walls can alter the plasma behavior and then the spectra of waves and oscillations existing in such plasmas. Surface waves are of importance in laser –produced–plasmas fusion research [1, 2] and in all industrial applications of guided–wave–produced plasmas [3]. These plasmas can be sharply bounded, inhomogeneous, and anisotropic so that they are capable of supporting surface waves [4].

Being periodic surface charges at boundary interfaces, amplitudes of the fields associated with SW's reach their maximum values at the bounding surfaces and decay in both media by moving far away from the interface [5, 3, 6]. Study of electrostatic surface waves (ESW) on cylindrical cold plasmas was first carried out by Trivelpiece and Gould [7]. For semi–infinite plasma with planar vacuum boundary, surface waves exist with spectra ranging from $\omega = \omega_P / \sqrt{2}$ for plasma-vacuum interface, and $\omega = \omega_P / \sqrt{1+\varepsilon_d}$ for plasma–dielectric interface, down to $\omega = 0$, where ω_P is the bulk plasma frequency, $\varepsilon_P = 1 - \omega_P^2 / \omega^2$ is the cold plasma dielectric constant, $\varepsilon_d = \epsilon_d / \epsilon_0$ is the relative dielectric constant of bounding dielectric, and ϵ_0 is the permittivity of free space.

By adopting a plasma kinetic approach,

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Guernsey investigated the effects of thermal motion on the ESW spectra [8]. It has been found that ESW are more strongly Landau damped than the corresponding bulk mode. For sufficiently large k values, thermal effects may be modeled by replacing the cold plasma dielectric constant by $\varepsilon_P = 1-\left(\omega_P^2 + \gamma_e v_{th}^2 k^2\right)/\omega^2$, where γ_e is the ratio of specific heats for electrons and v_{th} is their average thermal speed. Exact treatment of surface waves shows that $\gamma_e = 3$ corresponds to the wave spectra correction when fluid theory is used for semi–infinite plasma [8]. It has been shown in previous studies that the effect of electron thermal motion becomes important only in the quasi–static limit of small phase velocities of surface waves compared to the speed of light [9].

Electromagnetic treatment of surface waves in plasmas showed that SW's are neither pure longitudinal nor pure transverse [10, 11, 12]. Since perturbations in wave fields are both longitudinal and transverse, surface modes are hybrid modes. Generally, they are mixture of both longitudinal space charge and transverse electromagnetic waves, and only in the frequency domain such that $\varepsilon(\omega)\,\omega/c << k$ where *c* is the speed of light, the magnetic field component of the wave field in a given medium can be neglected. This corresponds to the electrostatic limit in which SW's may be treated as potential waves [7, 13, 14].

In the absence of a steady magnetic field there is no interior space charge bunching and the waves are surface waves. For a homogeneous plasma filling a conducting tube and in the presence of an axial dc magnetic field, the cases of strong and weak magnetic fields $\omega_c > \omega_P$, where ω_c is the gyro-frequency, and $\omega_P > \omega_c$ result in two propagating and two non–propagating (evanescent) bands corresponding to real and imaginary propagation constants, respectively [7, 15]. For a cold plasma partially filling the conducting tube, and as the magnetic field is reduced to zero, only one circularly symmetric mode propagating at low frequency will survive with asymptotic frequency $\omega = \omega_P \big/ \sqrt{1+\varepsilon_d}$ for short wavelengths.

Theoretical studies of surface waves in the presence of plasmas with sharp boundaries, neglecting transition layers with smooth density increase existing near the boundary, are valid to first approximation and can explain most properties of surface waves, as long as wavelengths of interest are much larger than the width of the transition layer [16]. Effects of gradients in plasma density, contrary to thermal and collisional effects which broaden the frequency domain of the surface waves, can lead to a significant modification of the dispersion curve of SW's by forming a maximum in the dispersion curves and shifting it down to lower values [3, 10, 16].

In the present article, we investigate the spectra of electromagnetic surface waves (EMSW's) at the surface of cold, unmagnetized and uniform plasma of finite width in the presence of a finite resistive wall of a cylindrical waveguide. In Sec.2, we derive the EMSW dispersion relation as a function of the dispersive properties of the pipe-wall and discuss some limiting cases such as the quasi–static limit of slow velocity surface waves and the long wavelength limit. In Sec.3, some representative numerical examples of the EMSW's dispersion relation will be given for different plasma– waveguide parameters. Finally, the main conclusions will be presented in Sec.4.

General Dispersion Relation of EMSW

Consider a plasma column of width a surrounded by vacuum in a conducting cylindrical pipe of radius b. The wall of the pipe has a large, but finite conductivity σ_ω. In the presence of a plasma, modes can no longer be separated into pure transverse magnetic (TM) and transverse electric (TE), except for the lowest azimuthal symmetric mode [16, 17, 18, 19]. For uniform plasma with azimuthal symmetry, we only consider transverse magnetic modes such that $H_z = 0$ since transverse electric modes with $E_z = 0$ do not exist for the azimuthal symmetric mode. All other field components will be obtained from E_z using the Maxwell's field equations. In frequency domain, we have the following Maxwell's curl equations,

$$\nabla\times \mathrm{E} = i\omega\mu_0 \mathrm{H}, \quad \nabla\times \mathrm{H} = -i\omega \in \mathrm{E}, \qquad (2.1)$$

where $\in = \in_0 \varepsilon$ and ε is the dielectric constant of the medium under consideration. Due to the azimuthal symmetry, the only nonvanishing field components are E_z, E_r and H_θ. For modes propagating along the positive *z*-axis such as $e^{i(kz-\omega t)}$, and upon using circular cylindrical coordinates, we obtain the following equations,

$$r^2 E_z'' + rE_z' - r^2\left(k^2 - \omega^2 \mu_0 \in\right)E_z = 0 \qquad (2.2)$$

$$H_\theta = -i\frac{\omega \in}{k^2 - \omega^2 \mu_0 \in} E_z' \, , E_r = -i\frac{k}{k^2 - \omega^2 \mu_0 \in} E_z' \, , \quad (2.3)$$

where the prime and double prime stand for differentiation with respect to the radial coordinate r. The wave equations for E_{pz} in the plasma region from $r = 0$ to $r = a$ and for E_{vz} in the vacuum between a and b become

$$r^2 E_{pz}'' + rE_{pz}' - \tau^2 r^2 E_{pz} = 0, \qquad (2.4)$$

$$r^2 E_{vz}'' + rE_{vz}' - \tau_0^2 r^2 E_{vz} = 0, \qquad (2.5)$$

$$\tau^2 = k^2 - \omega^2 \mu_0 \in_0 \varepsilon_P, \; \tau_0^2 = k^2 - \omega^2 \mu_0 \in_0 . \qquad (2.6)$$

The general solutions for E_z in both regions are as follows,

$$E_z = \begin{cases} AI_0(\tau r) & 0 < r < a \\ BI_0(\tau_0 r) + CK_0(\tau_0 r) & a < r < b \end{cases} , \qquad (2.7)$$

where I_0 and K_0 are the zero order modified Bessel's functions of first and second kinds, respectively. The constants *A*, *B* and *C* are to be determined using appropriate boundary conditions at the plasma–vacuum and vacuum–metallic cylinder interfaces.

To find the unknown constants, we apply the continuity of E_z at *r* = *a* and at *r* = *b*. On *z* the metallic surface *r* = *b*, we use the impedance (Leontovich) boundary condition to account for the finite resistivity of the surface [20, 21, 22, 23]. The surface impedance of the metallic surface Z_m is [21, 22],

$$Z_m = \frac{1-i}{\sigma_w \delta_s}, \quad \delta_s = \sqrt{\frac{2}{\mu_0 \sigma_w \omega}} \qquad (2.8)$$

where σ_w is the wall conductivity and δ_s is the skin depth at frequency ω. The boundary conditions concerning the continuity of E_z at $r = a$ and the impedance boundary condition $E_{vz} = Z_m H_{v\theta}$ at $r = b$ results in the following expression for the longitudinal electric field E_z,

$$E_z = \begin{cases} I_0(\tau r) & 0 \le r \le a \\ I_0(\tau_0 a)\, \dfrac{I_0(\tau_0 r)\left[i\tau_0 k_0(\tau_0 b) - z_m \omega \in_0 k_0'(\tau_0 b)\right] - k_0(\tau_0 r)\left[i\tau_0 I_0(\tau_0 b) -_m \omega \in_0 I_0'(\tau_0 b)\right]}{I_0(\tau_0 a)\left[i\tau_0 K_0(\tau_0 b) -_m \omega \in_{00}' (\tau_0 b)\right] - K_0(\tau_0 a)\left[i\tau_0 I_0(\tau_0 b) - Z_m \omega \in_0 I_0'(\tau_0 b)\right]} & a < r < b, \end{cases} \quad (2.9)$$

where I_0' and K_0' are the derivatives with respect to the argument. Imposing the continuity of H_0 at $r = a$ results in the following dispersion relation,

$$\varepsilon_P = \frac{\tau I_0(\tau_0 a)}{\tau_0 I_0'(\tau_0 a)} \frac{I_0'(\tau_0 a)\left[i\tau_0 K_0(\tau_0 b) - Z_m \omega \in_0 K_0'(\tau_0 b)\right] - K_0'(\tau_0 a)\left[i\tau_0 I_0(\tau_0 b) - Z_m \omega \in_0 I_0'(\tau_0 b)\right]}{I_0(\tau_0 a)\left[i\tau_0 K_0(\tau_0 b) - Z_m \omega \in_0 K_0'(\tau_0 b)\right] - K_0(\tau_0 a)\left[i\tau_0 I_0(\tau_0 b) - Z_m \omega \in_0 I_0'(\tau_0 b)\right]} . \quad (2.10)$$

Equation (2.10) is the general dispersion relation for the TM electromagnetic surface waves on the interface between an axially symmetric plasma enclosed by vacuum in a resistive cylindrical pipe. Finite extent of plasma and bounding conductor in the transverse direction, and the finite surface impedance of the bounding wall are accounted for via a, b and Z_m, respectively.

For a perfectly conducting wall at $r = b$ such that $Z_m = 0$ and for a cold plasma, equation (2.10) reduces into the following dispersion relation [7, 14, 15, 16, 17, 18],

$$\varepsilon_P = \frac{\tau I_0(\tau_0 a)}{\tau_0 I_0'(\tau_0 a)} \frac{I_0'(\tau_0 a)K_0(\tau_0 b) - I_0(\tau_0 b)K_0'(\tau_0 a)}{I_0(\tau_0 a)K_0(\tau_0 b) - I_0(\tau_0 b)K_0(\tau_0 a)}. \quad (2.11)$$

With $I_0'(x) = I_1(x)$ *and* $K_0'(x) = -K_1(x)$, equation (2.11) can be written as follows,

$$\varepsilon_P = \frac{\tau I_0(\tau_0 a)}{\tau_0 I_0'(\tau_0 a)} \frac{I_1(\tau_0 a)\frac{K_0(\tau_0 b)}{I_0(\tau_0 b)} + K_1(\tau_0 a)}{I_0(\tau_0 a)\frac{K_0(\tau_0 b)}{I_0(\tau_0 b)} - K_0(\tau_0 a)}. \quad (2.12)$$

For a cold plasma column in free space such that $b \to \infty$, the ratio $K_0(\tau_0 b)/I_0(\tau_0 b)$ varies with b as $e^{-2\tau_0 b} \to 0$. Accordingly, equation (2.12) takes on the following form,

$$\varepsilon_P = -\frac{\tau I_0(\tau a)}{\tau_0 I_1(\tau a)} \frac{K_1(\tau_0 a)}{K_0(\tau_0 a)}. \quad (2.13)$$

For slow (electrostatic) wave conditions such that the phase velocity υ_ϕ of the modes is much less than the speed of light $\upsilon_\phi = \omega/k << c$, eq.(2.13) takes the following form for $\tau = \tau_0 = k$ [14],

$$\varepsilon_P = 1 - \frac{\omega_P^2}{\omega^2} = -\frac{I_0(ka)K_1(ka)}{K_0(ka)I_1(ka)} \to \frac{\omega}{\omega_P} = \pm \frac{1}{\sqrt{1 + \frac{I_0(ka)K_1(ka)}{K_0(ka)I_1(ka)}}}. \quad (2.14)$$

Numerical Examples

The dispersion relation of equation (2.14) is plotted in Fig. 1 for ω/ω_P versus ka .For the electrostatic surface waves in a cold plasma, and in the absence of the bounding conducting surface, typical characteristics of slow phase velocity SW's are observed [7, 3]. For a fixed plasma width a and very small wave numbers k the curve starts at $\omega = 0$, but for large k values it approaches the cutoff frequency of the plasma–vacuum interface $\omega = \omega_P/\sqrt{2}$. Low and large values of ω for a fixed k correspond, respectively, to two different physical situations, namely, thin and thick plasmas. The dispersion curve is uniquely determined by the product of k and a, namely, $x = ka$.

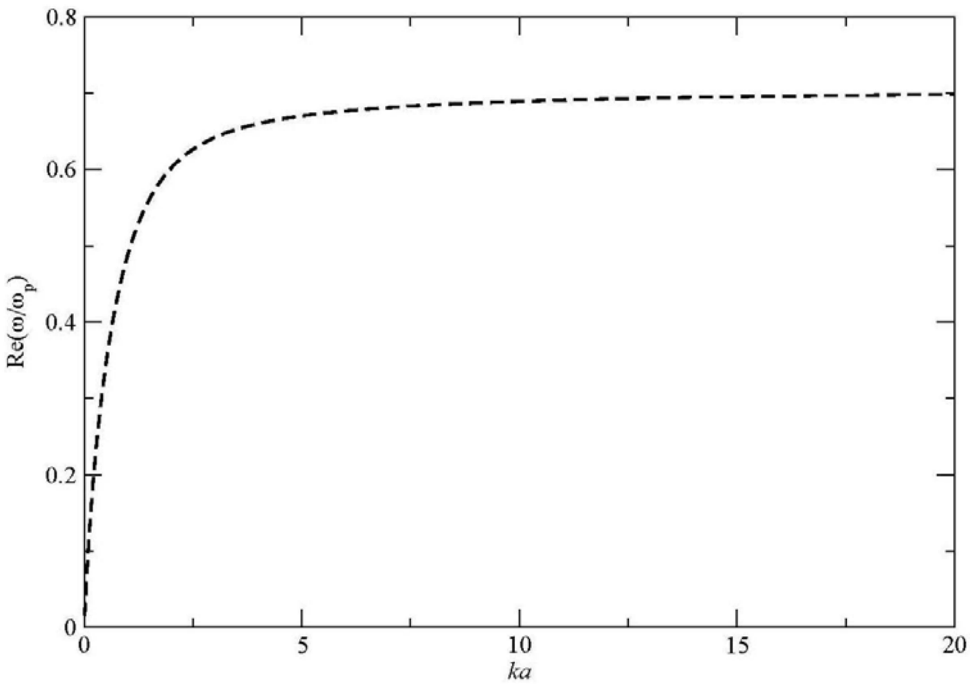

Fig. 1: Spectra of electrostatic SW's according to equation (2.14) for plasma in free space with the pipe wall moved to infinity.

When the radius of the bounding pipe is finite, and for a perfectly conducting wall, spectra of SW's for a cold plasma are described by equation (2.11). For the azimuthal symmetric mode with only TM modes being considered, characteristics of the dispersion curve are no longer determined by one parameter, as in the electrostatic case of equation (2.14). Plasma frequency and pipe radius, or the ratio (a/b) will affect the evolution of the curve. Presence of a good conducting bounding surface introduces additional dependence of the dispersion curve on the surface impedance Z_m of the metallic wall under consideration, as can be seen from dispersion relation of equation (2.10).

Possible spectra of surface waves resulting from the numerical solutions of the nonlinear dispersion relation (2.10), together with equation (2.6), are shown in Figs. 2 to 4. Fig. 2 shows the real part of ω versus ka for the representative parameters of wall conductivity of $\sigma_\omega 1.1\times10^6$ S/m (siemens per meter), plasma frequency $f_P = 300$ MHz ($\omega_P = 2\pi\, f_P$), and inner pipe radius $b = 10$ cm for different ratios a/b.

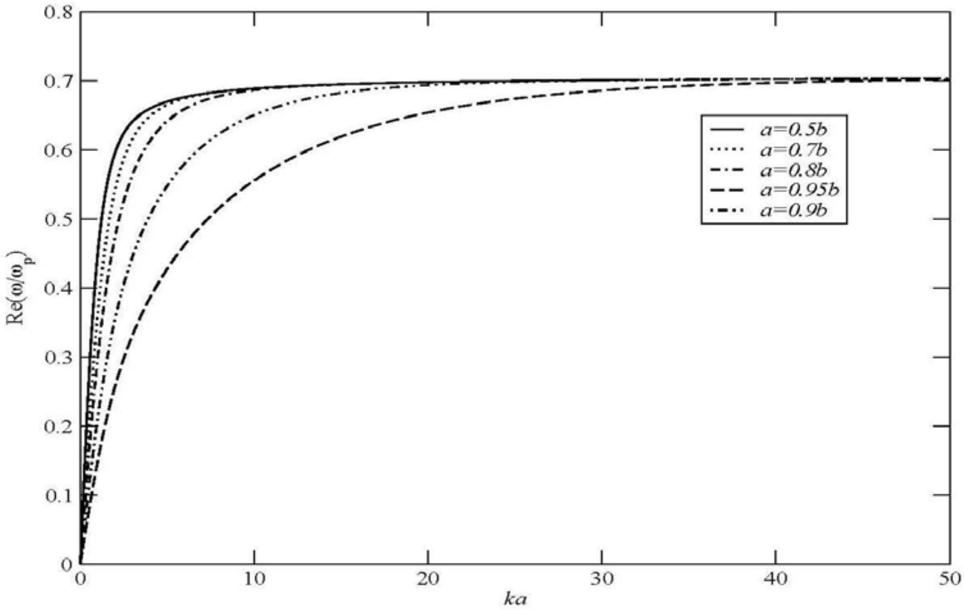

Fig. 2: Real part of the EMSW spectra according to equation (2.10) for pipe radius $b = 10$ cm, conductivity of stainless steel $\sigma_\omega = 1.1\times10^6$ S/m, plasma frequency $f_P = 300$ MHz, and for plasma widths $a = 0.5b$, $a = 0.7b$, $a = 0.8b$, $a = 0.9b$, and $a = 0.95b$.

Fig. 3 shows the imaginary part of the EMSW spectra according to equation (2.10) for pipe radius $b = 10$ cm, conductivity of stainless steel $\sigma_\omega = 1.1\times10^6$ S/m, plasma frequency $f_P = 300$ MHz with a values from low to high peaked curves $a = 0.5b$, $a = 0.8b$, $a = 0.9b$, and $a = 0.95b$. By changing the plasma frequency, the imaginary part of the EMSW spectra of equation (2.10) is shown in Fig. 4 for $b = 10$ cm, plasma width $a = 0.5b$, wall conductivity of $\sigma_\omega = 1.1\times10^6$ S/m, plasma frequencies f_P in MHz with values from low to high peaked curves, respectively, 200 MHz, 300 MHz, 2 GHz, and 3 GHz

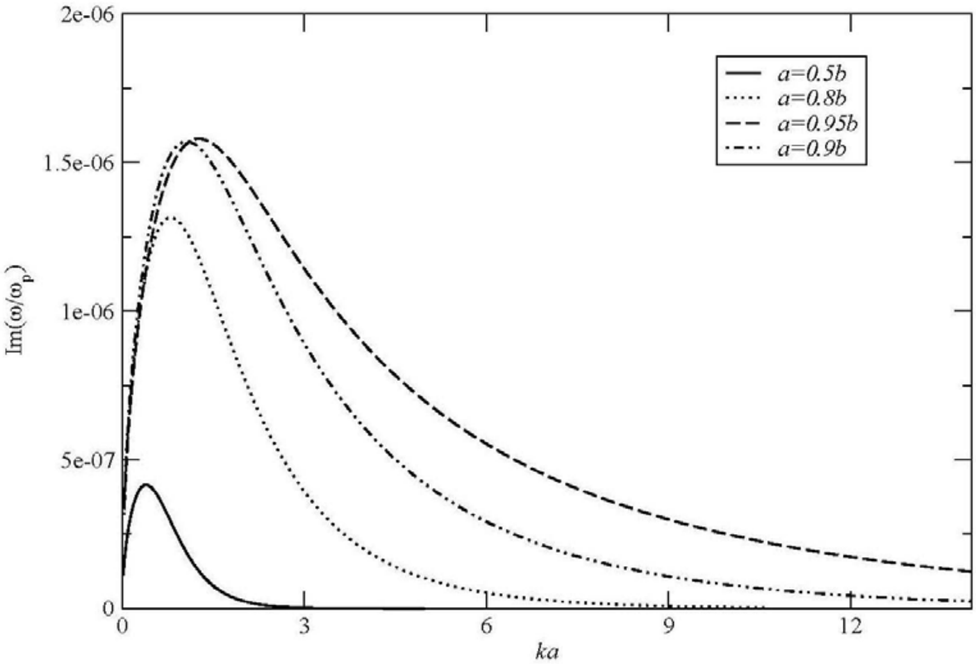

Fig. 3: Imaginary part of the EMSW spectra according to equation (2.10) for pipe radius $b = 10$ cm, conductivity of stainless steel $\sigma_\omega = 1.1\times 10^6$ S/m, plasma frequency $f_P = 300$ MHz, and for plasma widths $a=0.5b$, $a = 0.8b$, $a = 0.9b$, and $a = 0.95b$.

For small a/b values corresponding to thin plasma or a plasma surface at large distance from the conducting wall of the pipe, we observe the same spectra characteristics of Fig. 1 of the electrostatic surface wave modes for plasma–vacuum interface. By bringing the conducting wall closer and closer to the surface of the plasma, Fig. 2 shows for large ka (or for short wavelengths) that all curves converge toward the plasma–vacuum cutoff frequency $\omega = \omega_P/\sqrt{2}$. However, for moderate ka values, we observe considerable modifications of spectra curves, namely, by increasing the ratio a/b, surface wave frequencies shift down with the curves being shifted to the right. For small ka values, the dispersion curve gives a non–vanishing group velocity of the SW's indicating a transport of energy, while for large k values surface waves become localized oscillations with a vanishing group velocity.

The imaginary part of ω in Fig. 3 shows the opposite behavior, namely, all curves of different a/b ratio tend toward zero for large ka, and they show peaks that shift slightly to the right and become wider by increasing the ratio a/b. Including the finite, but large conductivity of the pipe wall, shows no observable effect on the real part of the surface wave spectra. Its effect manifests itself in introducing an imaginary part which has a peak value of 300 MHz for the parameters used to produce Fig. 3. For a fixed ratio $a/b = 0.5$ and varying plasma frequency, Fig. 4 shows that the imaginary part of ω can reach values of few tens of kilo Hz.

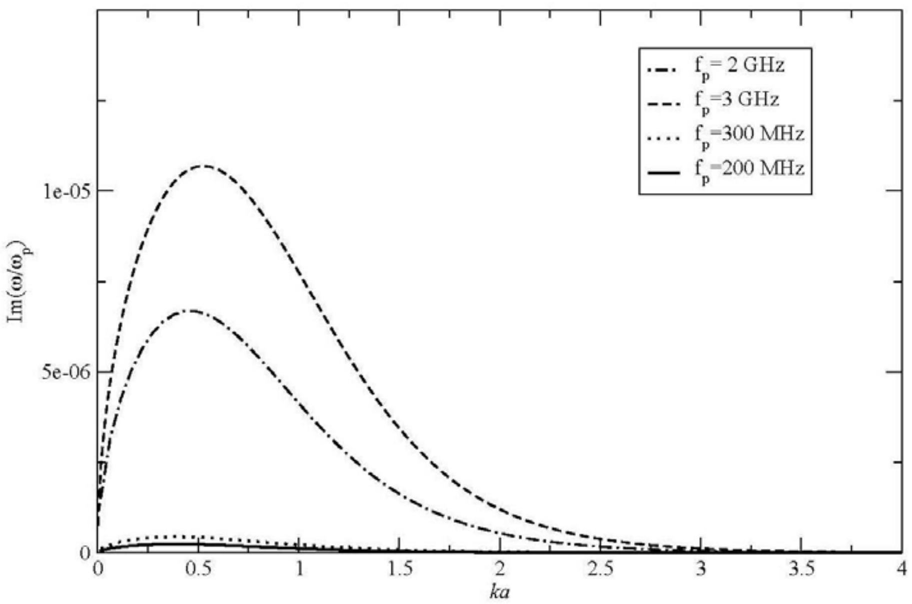

Fig. 4: Imaginary part of the EMSW spectra according to equation (2.10) for pipe radius $b = 10$ cm, Plasma width $a = 0.5b$, conductivity of stainless steel $\sigma_{\omega} = 1.1 \times 10^{6}$ S/m, plasma frequency $f_P =$ 200 MHz, 300 MHz, 2GHz, and 3GHz.

The observed characteristics of the surface oscillation is lower than that of the bulk oscillations $\omega = \omega_P$ because a part of the wave field is outside the plasma and the effective force of interaction among the plasma particles is weaker [24]. Electric charges whose fluctuations give rise to surface waves are localized at the plasma–dielectric interface. In the long wavelength limit, these charges interact with each other as if they were in a medium with an effective dielectric constant $\varepsilon_{eff} = \left(\varepsilon_P + \varepsilon_d\right)/2$. The condition $\varepsilon_{eff} = 0$ results in the surface wave spectra $\omega = \omega_P / \sqrt{1+\varepsilon_d}$, where $\varepsilon_d = 1$ for vacuum [see Figs. 1 to 4].

Conclusions

We have derived the general dispersion relation [equation (2.10)] for the TM electromagnetic surface waves on the interface separating an axially symmetric plasma and vacuum, both enclosed in a resistive cylindrical pipe. By assuming finite transverse width of a homogeneous cold plasma, whether in vacuum or in a resistive pipe, the spectra of surface waves resulting from the solution of the dispersion relation (2.10) have been found in terms a, ω_P, b *and* Z_m.

For the two equivalents physical situations of thin plasma or a conducting wall far away from the plasma surface, the observed spectra converge towards those of electrostatic surface wave modes existing at plasma–vacuum interface [see Fig. 1 and 2]. By bringing the conducting wall to a finite distance from the plasma–vacuum interface, large ka (short wavelengths) are not affected and all curves of Fig. 2 converge toward the plasma–vacuum cutoff frequency $\left(\omega = 0.707\omega_P\right)$. On the other hand, for moderate values of ka, we observe appreciable modifications of spectra curves; by increasing the ratio a/b, surface wave frequencies shift down with the curves being shifted to the right.

For conducting pipe–walls that can be represented by a surface impedance Z_m, and since Z_m is frequency dependent, the dispersion relation of waves existing in the structure under consideration becomes a function of the dispersive properties of the wall. Due to the large wall conductivity, the small value of surface impedance shows no observable effect on the real part of the surface wave spectra, and its effect is found to introduce an imaginary part of ω having a peak value of 300 Hz [see Fig. 3]. By varying the plasma frequency and keeping the ratio

a/b fixed at 0.5, the imaginary part of ω is found to reach values of few tens of kilo Hz [see Fig. 4].

Including surface wave spectra in modeling beam dynamics, especially, in beam instability analysis and impedance calculations, can improve modeling of longitudinal and transverse beam dynamics. When coupled to an external driver such as particle beams [25, 26, 27, 28, 29, 30], surface wave fields may be amplified gaining energy at the expense of the beam energy, coupled to the beam fields, and finally affecting the beam dynamics. Contributions from including higher order waveguide modes resulting from medium asymmetries or off–axis motion of transversally kicked beams are at hand and are topics of future investigation.

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