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ARTICLE

Design Principles for Quarter-Wave Retarders that Employ Total Internal Reflection and Light Interference in a Single-Layer Coating

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Abstract: Explicit equations are derived for the design of quarter-wave retarders (QWR) that exploit total internal reflection (TIR) and interference of light in a transparent thin-film coating at the base of a prism. The optimal refractive index and normalized thickness of QWR coatings on glass and ZnS prisms are determined as functions of the internal angle of incidence from 45° to 75°. An achromatic retarder that uses TIR by a Si_3N_4-coated N-BK10-Schott glass prism is also presented that achieves exact QWR at two wavelengths (409 and 500 nm) and exhibits a retardation error of < 1.5^{o} for wavelengths $375 \le \lambda \le 550$ nm in the near-UV and the violet-blue-green part of the visible spectrum.

Keywords: Polarization; Interference; Total internal reflection; Thin films; Wave retarders; Physical optics.

Introduction

Quarter-wave retarders (QWR) are versatile optical elements that are widely used for polarization-state generation and detection [1, 2]. The 90^{o} differential phase shift (or an odd multiple thereof) between the two orthogonal linear eigenpolarizations of a QWR is often obtained in transmission through a linearly birefringent crystalline plate or by employing total internal reflection (TIR) in a prism. QWR prisms of different designs that employ one or multiple TIR have been described [3 - 9].

To obtain QWR on single TIR at a dielectric-dielectric interface requires a refractive index ratio of $\ge \sqrt{2}+1 = 2.414$ [10]. For example, Ref. [11] describes a non-interference-type, broadband (1.2 – 4 µm), IR QWR that uses one TIR at a buried $Si\text{-}SiO_2$ interface. However, the presence of an optical interference coating (at the prism-air interface) significantly alters the phase shifts that accompany TIR, hence introduces flexibility that can be exploited in the design of QWR [12, 13]. The primary aim of this work is to present a new analytical approach for the design of such single-layer-coated TIR QWR and to provide several examples on its applications.

Analysis and Design Procedure

Fig. 1 shows TIR of a monochromatic light beam of wavelength λ at an angle of incidence ϕ at the base of a prism of refractive index n_0 which is coated with a transparent thin film of refractive index n_1 and thickness d. (Notice that the thickness d is greatly exaggerated on the scale of Fig. 1, and that the overlapping partial beams that are reflected from the front and back sides of the optically thin film are not shown.) The surrounding ambient medium is assumed to be air or an inert gas of refractive index $n_2 = 1$ and the entrance and exit faces of the prism are antireflection coated (ARC). The complex-amplitude reflection coefficients of

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the coated surface for the incident linear polarizations parallel p and perpendicular s to the plane of incidence, that account for light interference in the thin film, are given by the Airy-Drude formula [2]:

$$R_\nu = \frac{r_{01\nu} + r_{12\nu} X}{1 + r_{01\nu} r_{12\nu} X}, \nu = p, s. \tag{1}$$

In Eq. (1) $r_{01\nu}, r_{12\nu}$ are the Fresnel reflection coefficients at the prism-film (01) and film-ambient (12) interfaces for the ν polarization, respectively, and

$$\left.\begin{aligned} X &= \exp(-i\, 2\pi\varsigma), \\ \varsigma &= d / D_\phi, \\ D_\phi &= (\lambda/2)(n_1^2 - n_0^2 \sin^2\phi)^{-1/2}. \end{aligned}\right\} \tag{2}$$

TIR at the base of the prism requires that $\phi > \phi_{c02} = \arcsin(1/n_0); \phi_{c02}$ is the critical angle of the film-free prism-air (02) interface. To ensure partial reflection at the prism-film (01) interface, hence allow for light interference in the coating, we choose $n_1 > n_0$. Under these conditions, the overall reflection coefficients R_ν [Eq. (1)] are periodic functions of film thickness d with period D_ϕ given by Eq. (2).

The Fresnel interface reflection coefficients $r_{01\nu}, r_{12\nu}$ for the ν polarization can be conveniently expressed in terms of two angles α_ν, δ_ν,

$$\left.\begin{aligned} r_{01\nu} &= \tan\alpha_\nu, \\ r_{12\nu} &= \exp(i\,\delta_\nu), \\ \nu &= p, s. \end{aligned}\right\} \tag{3}$$

In the Nebraska-Muller conventions [2, 14] α_ν, δ_ν are restricted to the following ranges,

$$\left.\begin{aligned} &-45^\circ \le \alpha_p \le 45^\circ, -45^\circ \le \alpha_s \le 0, \\ &0 \le \delta_{p,s} \le \pi. \end{aligned}\right\} \tag{4}$$

By substitution of Eqs. (2) and (3) in Eq. (1), we obtain

$$\left.\begin{aligned} R_\nu &= \frac{\tan\alpha_\nu + \exp(-i\,\gamma_\nu)}{1 + \tan\alpha_\nu \exp(-i\,\gamma_\nu)}, \\ \gamma_\nu &= 2\pi\varsigma - \delta_\nu, \\ \nu &= p, s. \end{aligned}\right\} \tag{5}$$

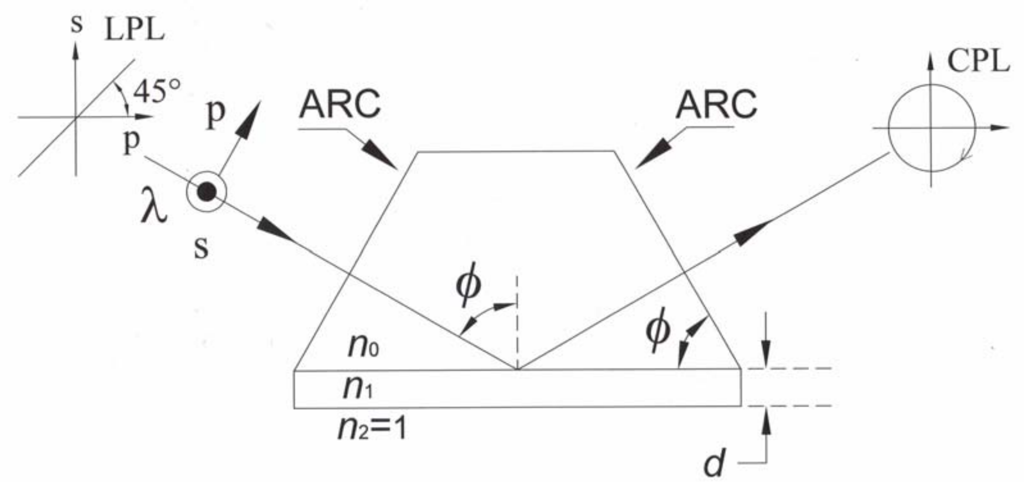

FIG. 1. TIR of a monochromatic light beam of wavelength λ at an angle of incidence ϕ at the base of a prism of refractive index n_0 which is coated with a transparent thin film of refractive index n_1 and thickness d. The entrance and exit faces of the prism are antireflection coated (ARC). p and s represent the orthogonal linear polarizations parallel and perpendicular to the plane of incidence, respectively. When QWR is achieved, incident linearly polarized light (LPL) at 45° azimuth from the plane of incidence is reflected as circularly polarized light (CPL).

From Eq. (5), it can be readily verified that $R_\nu R_\nu^* = 1$, which confirms that TIR takes place at the coated base of the prism. Consequently, R_ν is expressed as a pure phase factor

$$R_\nu = \exp(i\Delta_\nu), \nu = p, s, \qquad (6)$$

where Δ_ν is the overall TIR phase shift for the ν polarization. From Eqs. (5) and (6), the following important result is obtained:

$$\tan\Delta_\nu = \frac{-\cos(2\alpha_\nu)\sin\gamma_\nu}{\sin(2\alpha_\nu)+\cos\gamma_\nu}, \nu = p, s. \qquad (7)$$

The differential retardation on TIR is specified by the difference

$$\Delta = \Delta_p - \Delta_s. \qquad (8)$$

Taking the tangent of both sides of Eq. (8) gives

$$\tan\Delta = \frac{\tan\Delta_p - \tan\Delta_s}{1+\tan\Delta_p \tan\Delta_s}. \qquad (9)$$

To achieve QWR on TIR at the coated base of the prism, $\Delta = \pm\pi/2$, the denominator of the right-hand side of Eq. (9) must be zero, or

$$\tan\Delta_p \tan\Delta_s = -1. \qquad (10)$$

Substitution of Eq. (7) in Eq. (10) gives

$$\left.\begin{aligned} &\cos\gamma_p\cos\gamma_s + \sin(2\alpha_p)\cos\gamma_s + \\ &\sin(2\alpha_s)\cos\gamma_p + \\ &\cos(2\alpha_p)\cos(2\alpha_s)\sin\gamma_p\sin\gamma_s \\ &+\sin(2\alpha_p)\sin(2\alpha_s) = 0. \end{aligned}\right\} \qquad (11)$$

Equation (11) is the essential design condition for achieving QWR using TIR by a single-layer-coated prism (Fig. 1). The angles $\alpha_p, \alpha_s, \gamma_p, \gamma_s$ that appear in Eq. (11) are expressed as explicit functions of the prism and film refractive indices n_0, n_1, angle of incidence ϕ and normalized film thickness ς by

$$\left.\begin{aligned} \alpha_p &= (\pi/4) - \\ &\arctan[n_0(n_1^2 - n_0^2\sin^2\phi)^{1/2} / \\ &\qquad (n_1^2\cos\phi)], \\ \alpha_s &= (\pi/4) - \\ &\arctan[(n_1^2 - n_0^2\sin^2\phi)^{1/2} / \\ &\qquad (n_0\cos\phi)]; \end{aligned}\right\} \qquad (12)$$

$$\left.\begin{aligned} \gamma_p &= 2\pi\varsigma - \\ &2\arctan[n_1^2(n_0^2\sin^2\phi - 1)^{1/2} / \\ &\qquad (n_1^2 - n_0^2\sin^2\phi)^{1/2}], \\ \gamma_s &= 2\pi\varsigma - \\ &2\arctan[(n_0^2\sin^2\phi - 1)^{1/2} / \\ &\qquad (n_1^2 - n_0^2\sin^2\phi)^{1/2}]. \end{aligned}\right\} \qquad (13)$$

For a given prism with refractive index n_0 and angle of incidence ϕ, Eq. (11) can be cast in the simple functional form

$$f(n_1, \varsigma) = 0. \qquad (14)$$

Eq. (14) represents the constraint on the refractive index n_1 and normalized thickness ς of the interference coating such that QWR is realized in TIR. As will be seen in the next section, Eq. (14) has infinitely many solutions, but only one is optimal in that QWR is achieved with minimum sensitivity to small changes of film thickness or wavelength.

Total Internal Reflection QWR Using Coated Glass Prism

Consider a prism of refractive index $n_0 = 1.5$ (e.g. glass) and an angle of incidence $\phi = 60^\circ$. Fig. 2 shows a plot of the function $f(n_1, \varsigma)$ versus ς, $0 \le \varsigma \le 1$, for constant values of film refractive index n_1 from 1.9 to 6.1 in equal steps of 0.2 that represent numerous transparent optical coating materials in the visible and IR light. For each n_1 in this range, Eq. (14) has two solutions for ς (represented by points A and B for the case of n_1 = 1.9). The minimum value of n_1 for which the two solutions of Eq.

(14) coincide is $n_{1\min} = 1.7924$, and no solutions exist if $n_1 < n_{1\min}$. This corresponds to the point of tangency C ($\varsigma_C = 0.355$) of the topmost curve in Fig. 2 and the dashed line f = 0. Point C represents the optimal choice of film refractive index and thickness as it indicates insensitivity to small changes of ς, hence tolerance to small film-thickness errors or wavelength shifts. Suitable optical coating materials whose refractive index can be tuned to the optimal value $n_{1\min} = 1.7924$ (e.g. by appropriate control of stoichiometry and deposition conditions) include SiON [15] and Y_2O_3 [16].

Fig. 3 represents the locus of all possible solutions of Eq. (14) for coatings on a prism with refractive index $n_0 = 1.5$ that achieve QWR on TIR at an internal angle of incidence $\phi = 60^\circ$. Again, the optimal coating refractive index and thickness correspond to point C at the bottom of the curve.

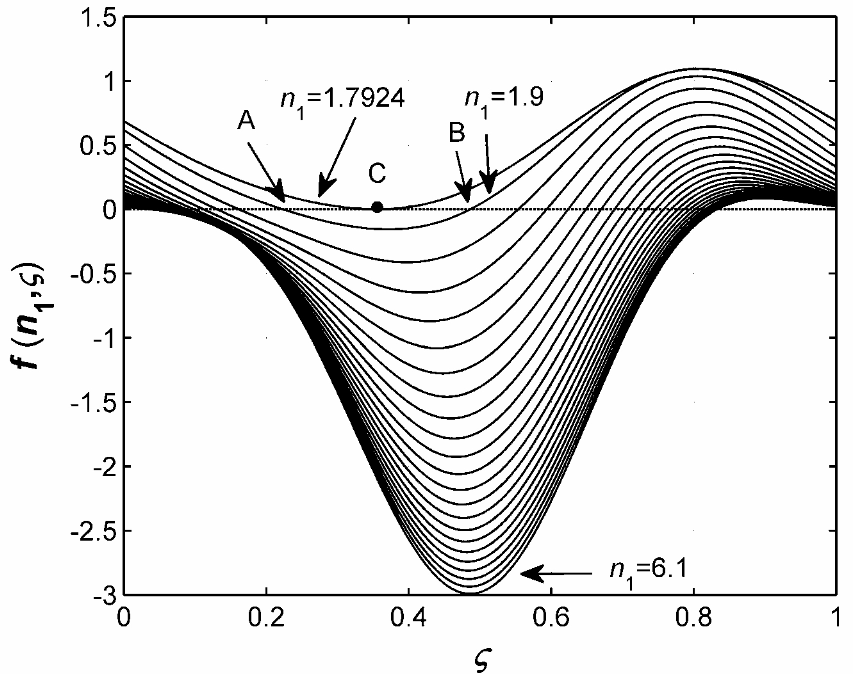

FIG. 2. Plot of the function $f(n_1, \varsigma)$ versus ς for constant values of n_1 from 1.9 to 6.1 in uniform steps of 0.2 for $n_0 = 1.5$ and $\phi = 60^\circ$. For each n_1 in this range, Eq. (14) has two solutions for ς (represented by points A and B on the curve for n_1 = 1.9). The minimum value of n_1 for which the two solutions of Eq. (14) coincide is $n_{1\min} = 1.7924$ and corresponds to the point of tangency C of the topmost curve with the f = 0 line.

Fig. 4 shows the optimal refractive index $n_{1\min}$ and associated normalized thickness ς of thin-film coatings on a prism with $n_0 = 1.5$ that achieve QWR on TIR at different angles of incidence in the range $45^\circ \le \phi \le 75^\circ$.

Figure 5 shows the corresponding results for optimal QWR coatings on Cleartran ZnS [17] prism with refractive index $n_0 = 2.35$ for red light ($\lambda \approx$ 633 nm).

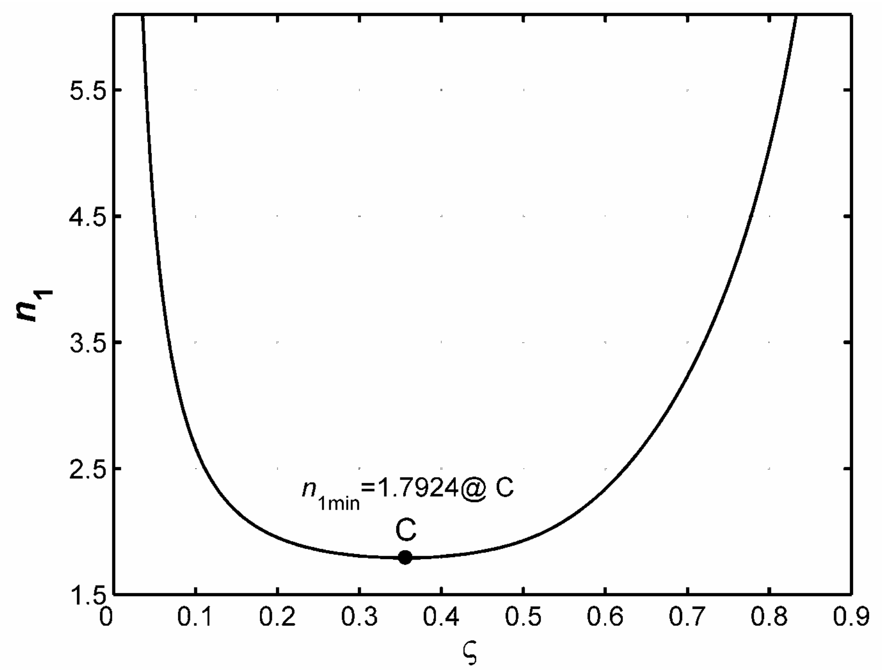

FIG. 3. Locus of all possible solutions of Eq. (14) is shown for coatings on prism of refractive index $n_0 = 1.5$ (e.g. glass) that achieve QWR on TIR at an angle of incidence $\phi = 60^\circ$. The optimal coating is represented by point C.

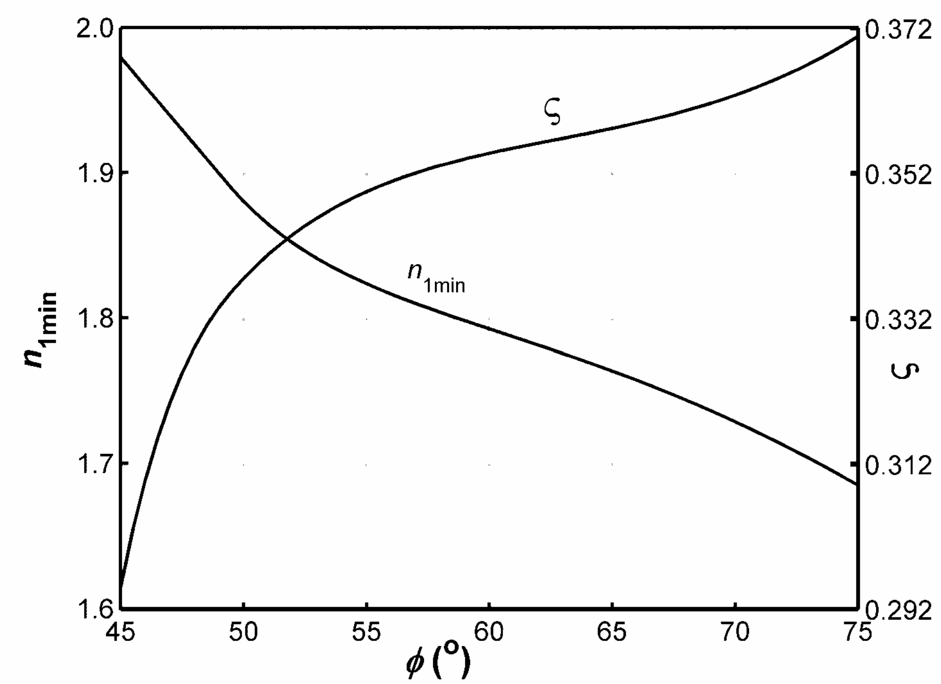

FIG. 4. Optimal (minimum) refractive index $n_{1\min}$ and associated normalized thickness ς of thin-film coatings on a prism of refractive index $n_0 = 1.5$ that achieve QWR on TIR at angles of incidence ϕ that vary from 45° to 75°.

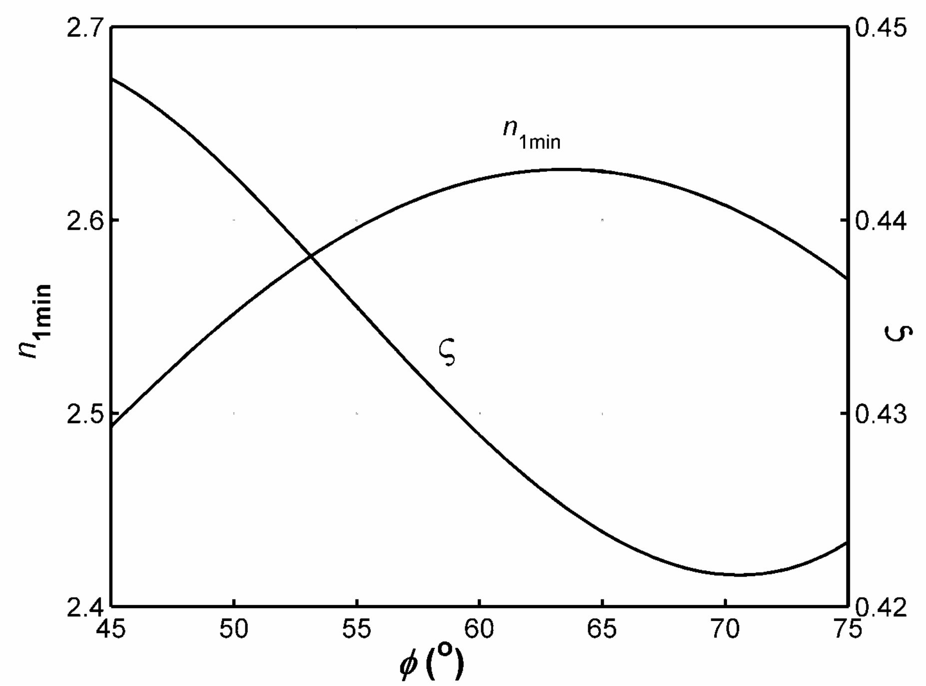

FIG. 5. Optimal (minimum) refractive index $n_{1\min}$ and associated normalized thickness ς of thin-film coatings on ZnS prism ($n_0 = 2.35$) that achieve QWR on TIR at angles of incidence ϕ that vary from 45° to 75°.

Spectral Response of TIR QWR that Uses Right-Angle Prism of N-BK10 Glass Coated with Si_3N_4 Film

We now consider a specific TIR QWR made of a right-angle prism ($\phi = 45^\circ$) of N-BK10 Schott glass with $n_0 = 1.5021$ [18] at the preselected design wavelength $\lambda = 500$ nm. The calculated optimal coating refractive index $n_{1\min} = 2.0607$ matches that of silicon nitride [15] at $\lambda = 500$ nm, and the associated optimum film thickness is d = 43.66 nm.

Figure 6 shows the retardation Δ *versus* λ of this QWR for 375 $\leq \lambda \leq$ 550 nm, which includes near-UV and the violet-blue-green part of the visible spectrum. In these calculations, the dispersion of the coating and prism materials is accounted for [15, 18]. Notice that exact QWR ($\Delta = 90^\circ$) is achieved at the design wavelength $\lambda = 500$ nm (point P), as expected, and also at the shorter wavelength $\lambda = 409$ nm (point Q). The deviation of Δ from 90° is < 1.5^o over the $375 \leq \lambda \leq 550$ nm spectral range and is < 0.55^o over the 400-500 nm band. This retardation error is less than that of commercial achromatic crystal wave plates over comparable bandwidths [19].

Conclusions

We provided detailed explicit analysis of the conditions that are required to achieve QWR (90^o differential phase shift between the *p* and *s* linear polarizations) on total internal reflection (TIR) at the base of a prism which is coated with a single-layer optical interference film. For given prism refractive index and angle of incidence, optimal values of the refractive index and thickness of the thin-film coating for QWR are obtained. Specific results are shown over a range of incidence angles for TIR QWR using transparent coatings on glass and ZnS substrates. A TIR retarder that employs Si_3N_4-coated N-BK10-Schott glass prism is also presented that exhibits exact QWR at two wavelengths (409 and 500 nm) and has a retardation error of < 1.5^o over the 375 - 550 nm spectral range.

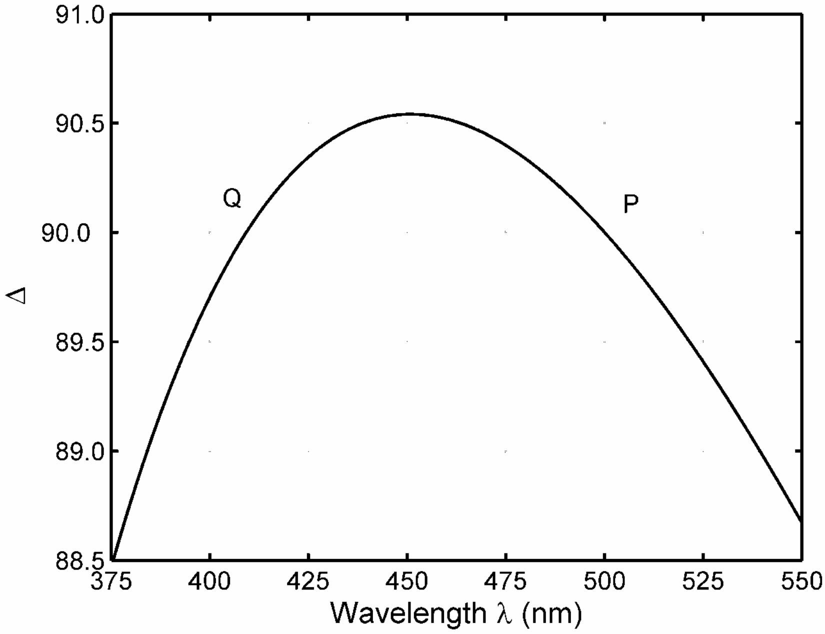

FIG. 6. Spectral response (Δ versus λ) of TIR QWR made of right-angle prism (ϕ = 45°) of N-BK10 Schott glass with refractive index $n_0 = 1.5021$ at $\lambda = 500$ nm whose base is coated with a silicon-nitride layer of refractive index $n_1 = 2.0607$ ($\lambda = 500$ nm) and thickness d = 43.66 nm. QWR is exactly achieved at two wavelengths λ = 500 and 409 nm at points P and Q, respectively.

References

[1]Bennett, J.M., "Polarization", in: M. Bass (ed.), Handbook of Optics, Vol. I (McGraw-Hill, New York, 1995), Chap. 5.

[2]Azzam, R.M.A. and Bashara, N.M., "Ellipsometry and Polarized Light" (North-Holland, Amsterdam, 1987).

[3]King, R.J., J. Sci. Instrum., 43 (1966) 617.

[4]Bennett, J.M., Appl. Opt., 9 (1970) 2123.

[5]Filinski, I. and Skettrup, T., Appl. Opt., 23 (1984) 2747.

[6]Kan'an, A. M. and Azzam, R. M. A., Opt. Eng., 33 (1994) 2029.

[7]Azzam, R.M.A. and Howlader, M.M.K., J. Lightwave Technol., 14 (1996) 873.

[8]Azzam, R.M.A. and Khanfar, H.K., Appl. Opt., 47 (2008) 359.

[9]Azzam, R.M.A. and Khanfar, H.K., Appl. Opt., 47 (2008) 4878.

[10]Azzam, R.M.A., J. Opt. Soc. Am., A, 21 (2004) 1559.

[11]Azzam, R.M.A. and Spinu, C.L., J. Opt. Soc. Am., A, 21 (2004) 2019.

[12]Spiller, E., Appl. Opt., 23 (1984) 3544.

[13]Cojocaru, E., Appl. Opt., 28 (1989) 211.

[14]Muller, R.H., Surf. Sci., 16 (1969) 14.

[15]Bååk, T., Appl. Opt., 21 (1982) 1069.

[16]www.cerac.com/pubs/proddata/y2o3.htm.

[17]Debenham, M., Appl. Opt., 23 (1984) 2238.

[18]www.us.schott.com/optics_devices/english/products/flash/abbediagramm_flash.html.

[19]www.edmundoptics.com.