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Effect of Magnetic Anisotropy on a One - Dimensional System of Magnetic Particles

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Abstract: Using a statistical approach, the magnetization and the initial susceptibility for a one – dimensional chain of a dilute ferromagnetic fluid have been investigated. Our assembly consisted of an *N* – particle chain with *N/3* non - interacting systems. We have studied three distinct cases: case 1 with randomly oriented easy axis of the particle assembly and the applied magnetic field is parallel to the chain. We found that the initial susceptibility follows Curie – Weiss behavior with positive ordering temperature T_o that does not depend on the anisotropy constant *K* of the particles. In case 2, the applied field is perpendicular to the chain with randomly oriented easy axis. In this case, we found an antiferromagnetic transition with no dependence on *K*. In case 3, when the easy axis is fixed at an angle ξ relative to *H*, we found that whether *H* parallel or perpendicular to the chain there is an interplay between ferromagnetic-like and antiferromagnetic-like behavior, depending on *K*, particle separation within the system and the angle ξ.

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Introduction

Many scientists have been interested in studying the magnetic behavior of the magnetic fine particle systems in liquid and solid states [1–4]. Many different models have been introduced to study the magnetic behavior analytically or numerically. Morup [5] showed that the magnetic interaction leads to spontaneous ordering, and he was able to give a good description of the magnetic properties above the ordering temperature. The effect of orientation texture has been considered analytically by Chantrell *et al.* [6] and numerically be Raikher [7]. Odeh *et al.* [8] used the Dimer model and found that the magnetic behavior of ferro-fluids depends on the direction of the external magnetic field with respect to the particle chain. Ayoub *et al.* [9] suggested interactions other than the dipole – dipole interaction to explain the Curie – Weiss behavior for a three - dimensional Dimer model. Smirnov and Komogortsev [10] used the generalized SW model [11, 12] in order to numerically investigate the magnetization curves of an ensemble of randomly oriented ferromagnetic single-domain nanoparticles. They discussed the possibility of estimating the ratio of the uniaxial anisotropy energy to the total magnetic anisotropy energy at low fields. For the case of transition between different symmetries of local magnetic anisotropy energy, they suggested that a more complicated form of nonlinear variations of the magnetic properties must be considered. A hint of possible ferromagnetic transition was shown experimentally by Mamiya *et al.* [13]. They have conducted their experiments

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on strongly interacting Fe_3N ferrofluids. Magnetic properties of two-dimensional layers of interacting nanoparticles with random anisotropy were calculated using Monte Carlo simulations [14]. The model refers to thin granular antiferromagnetic films. The effect of shape anisotropy on magnetic properties of ferrofluids has been calculated [15]. Two particle interactions were considered, and it was found that the anisotropy plays a major role in determining the magnetic state.

In this paper, we will use a statistical mechanical approach to investigate the effect of anisotropy energy dominated by the dipole – dipole interaction on the initial susceptibility of a ferro-fluid chain. We will consider an assembly consisting of *N* single domain fine magnetic particles constrained to move in one dimension. We will propose a three particle interaction that we shall call the Trimer model. In the Trimer model, the particles are approximated to a set of *N/3* independent systems. Each system consists of three interacting identical spherical particles. Each particle has an average magnetic dipole moment μ which is randomly oriented and each particle has an easy axis. We will investigate three distinct configurations. The first two are with randomized easy axis and applied field *H* parallel or perpendicular to the assembly. The last case is with a fixed orientation of the easy axis and *H* parallel or perpendicular to the assembly. The first two cases are applicable for the ferromagnetic fluids, while the third is applicable for a solid matrix.

Results and Discussion

Consider an assembly of *N/3* systems; the total partition function is given by:

$$Z_T = \frac{Z^{N/3}}{\left(N/3\right)!}\ , \tag{1}$$

where *Z* is the partition function for a single system. A single system is consisting of three interacting particles. The single system partition function is given by:

$$Z = \int \mathrm{Exp}[-\frac{E_T}{k_B T}]\ \mathrm{d}\Gamma . \tag{2}$$

The integral is to be taken over the volume phase space $\mathrm{d}\Gamma$. The total energy E_T of our system can be represented as:

$$E_T = E_0 + E_{p-p} + E_a\ , \tag{3}$$

where E_0 is the external field dipole interaction energy, $E_{p\text{-}p}$ is the total dipole – dipole interaction energy among the particles of a single system and E_a is the anisotropic energy of the system. The external energy is given by:

$$\left.\begin{aligned} E_0 &= -\sum_1^3 \vec{\mu}_i \cdot \vec{H} \\ &= -\mu H\left(\cos\theta_1 + \cos\theta_2 + \cos\theta_3\right) \end{aligned}\right\}, \tag{4}$$

where θ_i is the angle between the direction of the external applied magnetic field *H* and the magnetic dipole moment of each particle within the single system. The dipole – dipole interaction energy is given by:

$$E_{p-p} = \sum_{i>j}^{3}\sum_{j=1}^{3} \frac{\vec{\mu}_i \cdot \vec{\mu}_j - 3(\hat{r}_{ij} \cdot \vec{\mu}_i)(\hat{r}_{ij} \cdot \vec{\mu}_j)}{r_{ij}^3}\ , \tag{5}$$

where μ_i and μ_j are the dipole moments of particle *i* and particle *j*, respectively and r_{ij} is the separation between the two particles. The anisotropy energy is given by:

$$E_a = KV\sum_{i=1}^{3}\sin^2\beta_i\ , \tag{6}$$

where *K*, *V* and β_i are, respectively, the anisotropy constant, the volume of each particle and the angle between the magnetic dipole moment of the particle and its easy axis.

The magnetization M of the system can be calculated using the equation:

$$M = \frac{Nk_B T}{3Z}\frac{\partial Z}{\partial H}\ . \tag{7}$$

The initial susceptibility of the system can be calculated as:

$$\chi = \lim_{H\to 0}\frac{\partial M}{\partial H}\quad . \tag{8}$$

Now, we will discuss in detail each of the three cases mentioned above. We will use

Eq.7 to calculate the magnetization and Eq. 8 to calculate the susceptibility.

Parallel Configuration

In this case the applied magnetic field is parallel to the chain axis (Fig. 1). In the limit that the particle – particle interaction energy and the anisotropy energy are very small, one can expand the exponential term in the partition function as:

$$Z = e^{\frac{3KV}{k_B T}} \int \mathrm{Exp}\left[\frac{\mu H}{k_B T}\left(\cos\theta_1 + \cos\theta_2 + \cos\theta_3\right)\right] \times \left\{1 + \frac{\mu^2}{k_B T}\left(\frac{g_1}{x_1^3} + \frac{g_2}{x_2^3} + \frac{g_3}{(x_2 - x_1)^3}\right) + \frac{KV}{k_B T}\left(J_1^2 + J_2^2 + J_3^2\right) + \frac{1}{2}\left(\frac{\mu^2}{k_B T}\left(\frac{g_1}{x_1^3} + \frac{g_2}{x_2^3} + \frac{g_3}{(x_2 - x_1)^3}\right) + \frac{KV}{k_B T}\left(J_1^2 + J_2^2 + J_3^2\right)\right)^2\right\} \mathrm{d}\Gamma \quad (9)$$

where

$$\begin{aligned} g_1(\theta_1,\theta_2,\varphi_1,\varphi_2) &= 2\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2) \\ g_2(\theta_1,\theta_3,\varphi_1,\varphi_3) &= 2\cos\theta_1\cos\theta_3 - \sin\theta_1\sin\theta_3\cos(\varphi_1 - \varphi_3) \\ g_3(\theta_2,\theta_3,\varphi_2,\varphi_3) &= 2\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3\cos(\varphi_2 - \varphi_3) \\ J_i &= \cos\beta_i = \cos\theta_i\cos\eta_i + \sin\theta_i\sin\eta_i\cos(\varphi_i - \psi_i) \end{aligned} \quad (10)$$

In Eq. 9, x_1 is the separation between the first and the second particles, while x_2 is the separation between the first and the third particles. Performing the necessary calculations, the initial susceptibility for the system in the limit $\alpha = \frac{\mu H}{k_B T} < 1$ is obtained having the following expression:

$$\chi = \frac{\frac{N\mu^2}{3k_B}}{T - \frac{2\mu^2}{9k_B}U\left(x_1^f, x_1^i, x_2^f, x_2^i\right)} . \quad (11)$$

Here, the function $U(x_1^i, x_1^f, x_2^i, x_2^f)$ is defined as:

$$U(x_1^f, x_1^i, x_2^f, x_2^i) = \frac{L_1(x_1^f, x_1^i, x_2^f, x_2^i)}{(x_1^f - x_1^i)(x_2^f - x_2^i)} \quad (12)$$

where x_1^f and x_1^i are the maximum and minimum separation between the first and the second particles. x_2^f and x_2^i are the maximum and minimum separation between the first and the third particles. The function L_1 is defined as:

$$L_1 = 2\int_{x_1^i}^{x_1^f}\int_{x_2^i}^{x_2^f}\left(\frac{1}{x_1^3} + \frac{1}{x_2^3} + \frac{1}{(x_2 - x_1)^3}\right)\mathrm{d}x_1\,\mathrm{d}x_2 . \quad (13)$$

By comparing Eq. 11 with the well-known Curie-Weiss law $\chi = \frac{C}{T - T_o}$, one can obtain the following expressions for the constant C and the ordering temperature T_o:

$$C = \frac{N\mu^2}{3k_B} \quad (14)$$

$$T_o = \frac{2\mu^2}{9k_B}U . \quad (15)$$

T_o does not depend on the anisotropy constant. Eq. 11 suggests a ferromagnetic behavior of the system. Therefore, a ferromagnetic state will exist in this type of configuration. Moreover, a three - body interaction leads to the same behavior as for a two - body interaction. Similar magnetic behavior has been found by Odeh *et al.* [8] and Obeidat *et al.* [15] for the Dimer model.

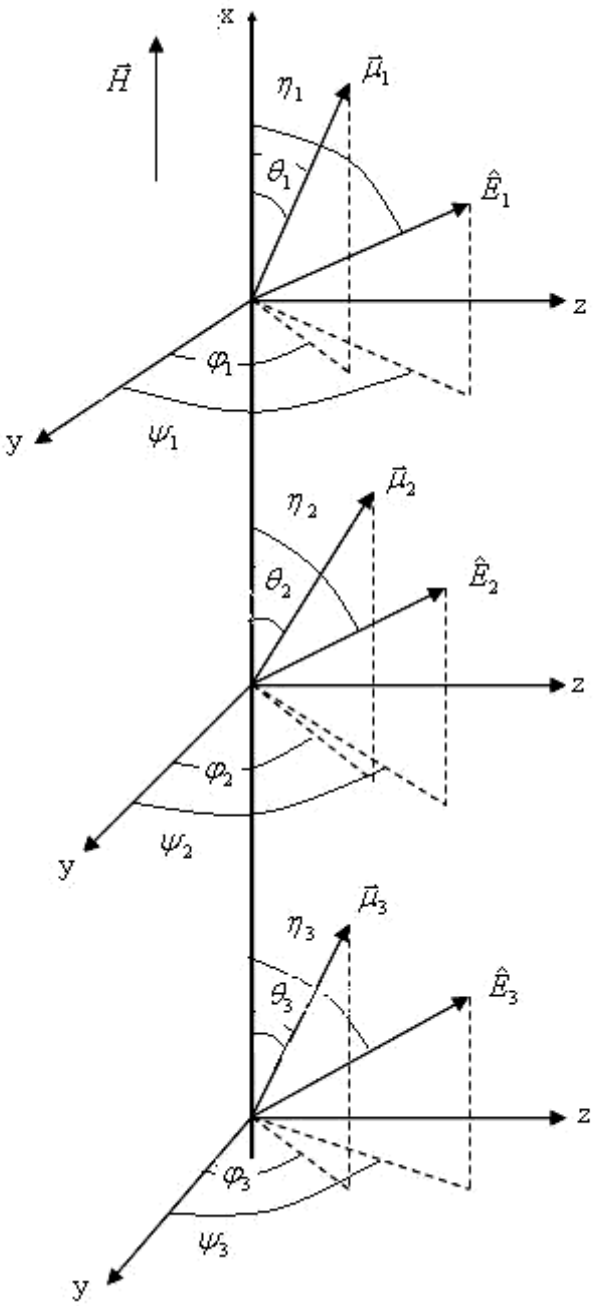

FIG. 1. Parallel configuration: Trimer model in a parallel applied magnetic field. The angles θ and φ are the polar and the azimuthal angles of the vector μ. The angles β and ψ are the polar and the azimuthal angle of the easy axis E.

Perpendicular Configuration

Fig. 2 shows the second case, where the applied magnetic field is perpendicular to the chain. In this case, the interaction energy is given by:

$$\left.\begin{aligned} E_{p-p} = & \frac{\mu^2}{y_1^2} q_1(\theta_1,\theta_2,\varphi_1,\varphi_2) \\ & + \frac{\mu^2}{y_2^2} q_2(\theta_1,\theta_3,\varphi_1,\varphi_3) \\ & + \frac{\mu^2}{(y_2-y_1)^2} q_3(\theta_2,\theta_3,\varphi_2,\varphi_3) \end{aligned}\right\} \quad (16)$$

where y_1 is the separation between the first and the second particles, while y_2 is the separation between the first and the third particles. The functions q_1, q_2 and q_3 are given by:

$$\left.\begin{aligned} q_1(\theta_1,\theta_2,\varphi_1,\varphi_2) = {} & \\ & 3\sin\theta_1\sin\theta_2\sin\varphi_1\sin\varphi_2 \\ & -\sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2) \\ & -\cos\theta_1\cos\theta_2 \\ q_2(\theta_1,\theta_3,\varphi_1,\varphi_3) = {} & \\ & 3\sin\theta_1\sin\theta_3\sin\varphi_1\sin\varphi_3 \\ & -\sin\theta_1\sin\theta_3\cos(\varphi_1-\varphi_3) \\ & -\cos\theta_1\cos\theta_3 \\ q_3(\theta_2,\theta_3,\varphi_2,\varphi_3) = {} & \\ & 3\sin\theta_2\sin\theta_3\sin\varphi_2\sin\varphi_3 \\ & -\sin\theta_2\sin\theta_3\cos(\varphi_2-\varphi_3) \\ & -\cos\theta_2\cos\theta_3 . \end{aligned}\right\} \quad (17)$$

We have calculated the initial susceptibility by applying the same assumption as above ($\alpha < 1$), and we have found that:

$$\chi = \frac{\frac{N\mu^2}{3k_B}}{T + \frac{\mu^2}{9k_B}U\left(y_1^f, y_1^i, y_2^f, y_2^i\right)} . \quad (18)$$

In this form, the ordering temperature is given by:

$$T_o = -\frac{\mu^2}{9k_B}U . \quad (19)$$

Here, U is the same function as for the previous case. Eq. 18 and Eq. 19 show that the initial susceptibility and the ordering temperature do not depend on the anisotropy constant. However, in this case, the ordering temperature T_o has a negative sign and its magnitude is half that of the parallel case. The negative sign of the ordering temperature suggests an anti-ferromagnetic behavior of the system. Therefore, an antiferromagnetic state exists for such configuration. Also, our results indicate that the system has the same behaviour as for the two body interaction [8, 15].

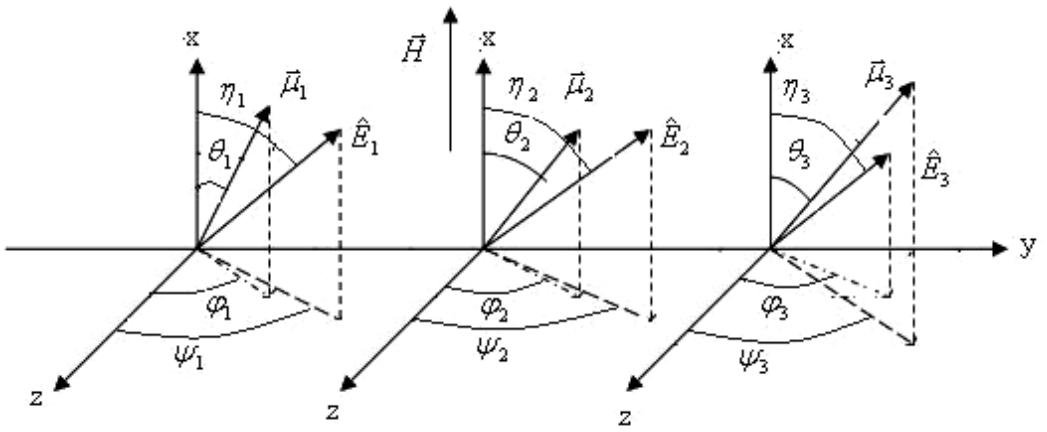

FIG. 2. Perpendicular configuration: Trimer model in a perpendicular applied magnetic field. The angles have the same meaning as in Fig. 1.

Easy Axis is Fixed Relative to the Applied Magnetic Field

As a last case, we will assume that the easy axis of the fine magnetic particles has a fixed orientation with respect to the external magnetic field. Fig. 3 shows the parallel applied magnetic field with respect to the assembly. The angle between the field and the easy axis for each particle is considered fixed at ξ. Using the same approximations as before, we have calculated the ordering temperature and found that:

$$T_o = \frac{2\mu^2}{9k_B}U + \frac{KV}{3k_B}\left(2 - 3\sin^2\xi\right) . \quad (20)$$

As can be seen form Eq. 20, the ordering temperature depends on the anisotropic constant *K* and the angle between the easy axis and the applied field ξ. The magnetic behavior of the system interplays between ferromagnetic and antiferromagnetic states.

Under the same condition, we have calculated the ordering temperature when the applied magnetic field is perpendicular to the chain. We found that the initial susceptibility and the ordering temperature depend on the anisotropic constant *K*. Our results of the ordering temperature is:

$$T_o = -\frac{\mu^2}{9k_B}U + \frac{KV}{3k_B}\left(2 - 3\sin^2\xi\right). \qquad (21)$$

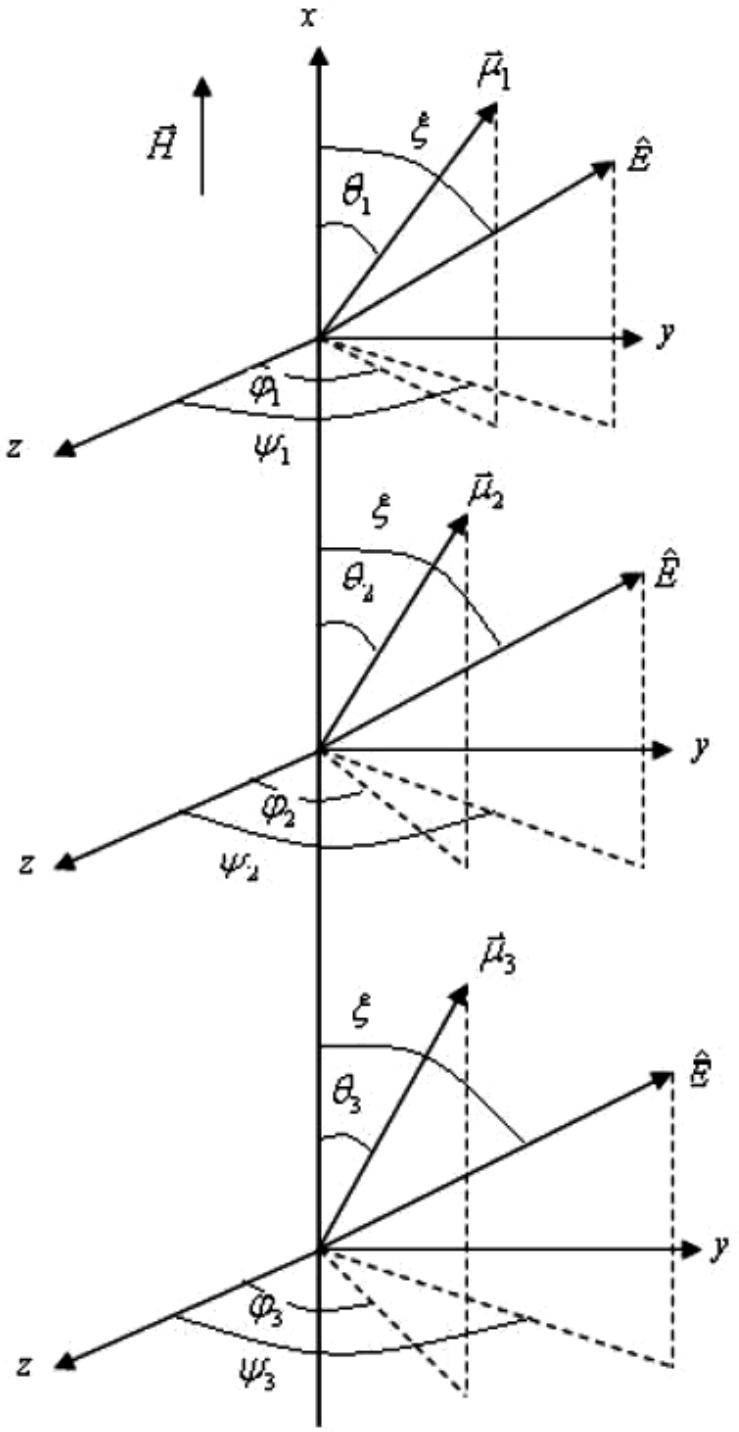

FIG. 3. Trimer model with fixed easy axis in a parallel applied magnetic field. The angles have the same meaning as in Fig. 1.

In Eq. 21, an antiferromagnetic state or a ferromagnetic state can be established depending on the parameters *U*, *K* and the angle ξ. In case of $\xi = \pi/2$, the ordering temperature is negative and an anti-ferromagnetic state is possessed by the system which reduces to case 2.

Table 1 shows the calculated results of the ordering temperature for Fe_3O_4. We have taken the average diameter to be 7.4 nm and the surfactant layer to be 2 nm. In the Table, the calculated values have been compared with the experimental values obtained by Popplewell *et al.* [3] and the calculated values for the Dimer model using expressions obtained by Obeidat *et al.* [15]. One can see that our model has much better results than those of the two - body particle interaction.

TABLE 1. The calculated values for the ordering temperature compared with the experimental values (Ref. [3]) and the calculated values from the expression obtained in Ref. [15]

Packing fraction		0.03	0.05	0.07
Ordering Temperature	Trimer model	19	32	45.4
	Dimer model Ref. [15]	11.37	13.26	15.5
	Exp. Values Ref. [3]	19	38	48

Conclusion

We have studied a statistical assembly consisting of a chain of *N* identical spherical particles each with a magnetic moment μ and a preferred easy magnetization axis. Our approach was carried out by considering three - body magnetic interactions (Trimer model). We have investigated three different cases. In cases 1 and 2, we have assumed a randomly oriented easy axis with the applied magnetic field parallel to the chain in case 1 and perpendicular to the chain in the second case. Our results for case 1 showed that a transition into ferromagnetic state can occur, where the ordering temperature was found to be positive. In case 2, we found that an antiferromagnetic state exists with negative ordering temperature. In the third case, where we assumed that the easy axis has a fixed direction relative to the applied magnetic field, our calculations showed that whether the applied magnetic field is parallel or perpendicular to the chain, there is a presence of both ferromagnetic and antiferromagnetic states depending on the anisotropy constant *K*, the particle separations through the parameter *U* and the orientation of the easy axis relative to the applied field ξ. Moreover, one can adjust the parameters ξ and *U* in a solid matrix and predict the magnetic state. The calculated ordering temperature for Fe_3O_4 shows much better values than those of the Dimer model.

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References

[1]Menear, S., Bradbury, A. and Chantrell, R.W., J. Magn. Magn. Mater. 39 (1983) 17.

[2]Ayoub, N.Y., Kobeissi, M.A., Chantrell, R.W., O'Grady, K. and Popplewell J., J. Phys. F: Metal Phys. 15 (1985) 2229.

[3]Popplewell, J., Abu-Aisheh, B. and Ayoub, N.Y., J. Appl. Phys. 64 (1988) 5852.

[4]Ayoub, N.Y., Abu-Aisheh, B., Laham, N. and Popplewell, J., IEEE Tran. Magn. 25 (1989) 3860.

[5]Morup, S.J., IEEE Tran. Magn. 23 (1987) 5.

[6]Chantrell, R.W., Ayoub, N.Y. and Popplewell, J., J. Magn. Magn. Mater. 53 (1985) 199.

[7]Raikher, Yu. L., J. Magn. Magn. Mater. 39 (1983) 11.

[8]Odeh, I., Abdallah, A.K., Dababneh, M.S., Laham, N.M. and Ayoub, N.Y., The Arabian Journal for Science and Engineering, 19 (1994) 697.

[9]Ayoub, N.Y., Shamoun, A., Abdallah, A.K., Dababneh, M.S., Odeh, I.M. and Laham, N.M., J. Magn. Magn. Mater. 127 (1993) 75.

[10]Smirnov, S.I. and Komogortsev, S.V., J. Magn. Magn. Mater. 320 (2008) 1123.

[11]Thiaville, A., J. Magn. Magn. Mater. 182 (1998) 5.

[12]Thiaville, A., Phys. Rev. B, 61 (2000) 12221.

[13]Mamiya, H., Nakatani, I. and Furubayashi, T., Phys. Rev. Lett. 84 (2000) 6106.

[14]Gruyters, M., J. Magn. Magn. Mater. 320 (2008) 407.

[15]Obeidat, A.A., Gharaibeh, M.A., Al-Safadi, D., Al-Samarah, D.H., Qaseer, M.K.H., Ayoub, N.Y., J. Supercond. Nov. Magn. 22 (2009) 805.