Volume 4, Number 1, 2011, pp. 51-58

Jordan Journal of Physics

ARTICLE

IBM-1 Calculations of Low-Lying Excited Levels and Electric Transition Probabilities B(E2) on the Even-Even "$^{174-180}$Hf Isotopes"

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Received on: 21/10/2010; Accepted on: 3/4/2011

Abstract: In this paper, the interacting boson model (IBM-1) is discussed and employed for calculating the energy levels and the electromagnetic transition probabilities B(E2) of the even – even "$^{174-180}$Hf isotopes". These isotopes have been investigated based on two different arrangements; i.e., the dynamical symmetry of "$^{176-180}$Hf isotopes", SU(3) (deformed nuclei) and the dynamical symmetry of "^{174}Hf isotope" in transition region SU(3)-O(6). The determined values using the IBM-1 Hamiltonian showed significant agreement with the experimentally reported energy levels and B(E2) values. The model provides a fast and accurate prediction method of energy levels and B(E2) values.
Keywords: Hafnium isotopes; Energy levels; Interacting boson model.

Introduction

The nucleus consists of many nucleons (protons and neutrons). Each nucleon is interacting with all others and moving within a complex structure. This structure could be described by the analytical solution of their wave function. As a consequence, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell. The interacting boson model of Arima and Iachello [1-6] has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. In this model, it was assumed that low-lying collective states of even-even nuclei could be described as states of a given (fixed) number N of bosons. Each boson could occupy two levels; one with angular momentum L = 0 (s-boson) and the other, usually with higher energy, with L = 2 (d-boson). In the original form of the model known as the Interacting Boson Model (IBM-1), proton- and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure, which is associated with it in terms of s- and d- boson operators.

The even-mass Hafnium "$^{174-180}$Hf isotopes" have been extensively investigated experimentally using a wide variety of reactions. From these studies, the excited states in the "$^{174-180}$Hf isotopes" have been investigated from (γ,γ'), (d,p), (α,2nγ), (α,4nγ), (n,γ), (n,n'γ) and coulomb excitation reactions which gave information about the experimental energy levels and the electromagnetic transition probabilities B(E2) in these isotopes [7-16].

The aim of the present work is to investigate the dynamical symmetry of "$^{176-180}$Hf isotopes" and study the energy levels and the electromagnetic transition probabilities B(E2) of these isotopes (proton number 72) within the framework of the (IBM-1) model.

Theoretical Basics of Interacting Boson Model (IBM-1)

The IBM-1 Hamiltonian can be expressed as [5, 17]:

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$$\left.\begin{aligned} H &= \varepsilon_s s^+ .s + \varepsilon_d (d^+ .d) \\ &+ \sum_{L=0,2,4} c_L \left[(d^+ d^+)^{(L)} .(dd)^{(L)} \right] \\ &+ \frac{1}{2} v_0 \left[(d^+ d^+)_0^{(0)} s^2 + (s^+)^2 (dd)_0^{(0)} \right] \\ &+ \sqrt{\frac{1}{2}} v_2 \left[\left[(d^+ d^+)^{(2)} ds \right]_0^{(0)} + \left[s^+ d^+ (dd)^{(2)} \right]_0^{(0)} \right] \\ &+ \frac{1}{2} u_0 (s^+)^{(2)} s^+ + \frac{1}{\sqrt{5}} u_2 s^+ s (d^+ d) \end{aligned}\right\} \quad (1)$$

This Hamiltonian contains 2 terms of one-body interactions, (ε_s and ε_d), and 7 terms of two-body interactions [c_L (L = 0,2,4), v_L (L = 0,2), u_L (L = 0,2)], where ε_s and ε_d are the single-boson energies, and c_L, v_L and u_L describe the two-boson interactions. However, it turns out that for a fixed boson number N, only one of the one-body terms and five of the two-body terms are independent, as it can be seen by noting $N = n_s + n_d$.

The IBM-1 Hamiltonian equation (1) can be written in general form as [3,18,19,,20,21]:

$$\left.\begin{aligned} \hat{H} &= \varepsilon (\hat{n}_s + \hat{n}_d) + a_o \hat{P} .^+ \hat{P} + a_1 \hat{L} .\hat{L} \\ &+ a_2 \hat{Q} .\hat{Q} + a_3 \hat{T}_3 \hat{T}_3 + a_4 \hat{T}_4 .\hat{T}_4 \end{aligned}\right\} \quad (2)$$

where ε is the bosons energy, and the operators are:

$$\left.\begin{aligned} \hat{n}_s &= \hat{s}^+ .\hat{\tilde{s}} \quad , \quad \hat{n}_d = \hat{d}^+ .\hat{\tilde{d}} \\ \hat{P} &= \frac{1}{2} (\hat{\tilde{d}} .\hat{\tilde{d}}) - \frac{1}{2} (\hat{s} .\hat{s}) \\ \hat{L} &= \sqrt{10} \left[\hat{d}^+ \times \hat{\tilde{d}} \right]^{(1)} \\ \hat{Q} &= \sqrt{5} \left[(\hat{d}^+ \times \hat{\tilde{s}}) + (\hat{s}^+ \times \hat{\tilde{d}}) \right]^{(2)} + \chi \left[\hat{d}^+ \times \hat{\tilde{d}} \right]^{(2)} \\ \hat{T}_3 &= \left[\hat{d}^+ \times \hat{\tilde{d}} \right]^{(3)} \\ \hat{T}_4 &= \left[\hat{d}^+ \times \hat{\tilde{d}} \right]^{(4)} \end{aligned}\right\} \quad (3)$$

The phenomenological parameters a_0, a_1, (a_2,χ), a_3, a_4, represent the strengths of the pairing, angular momentum, quadrupole, octoupole and hexadecoupole interaction between bosons, respectively.

The Hamiltonian equations (1, 2) can be rewritten in terms of the Casimir operators of U(6) group. In that case, one says that the Hamiltonian H has three dynamical symmetries. These symmetries are called SU(5) vibrational, SU(3) rotational and O(6) γ-unstable. The three dynamical symmetries of IBM-1 are:

Group Chin I: The Vibrational Symmetry SU(5)

This dynamical symmetry group describes the vibrational nuclei which have a spherical shape. The Hamiltonian of this symmetry can be written as [18, 20]:

$$\left.\begin{aligned} \hat{H}^{(I)} &= \varepsilon (\hat{n}_s + \hat{n}_d) \\ &+ a_1 \hat{L} .\hat{L} + a_3 \hat{T}_3 .\hat{T}_3 + a_4 \hat{T}_4 .\hat{T}_4 \end{aligned}\right\} \quad (4)$$

$\varepsilon (\hat{n}_s + \hat{n}_d)$ is the energy of s and d boson, a_1 is the angular momentum, a_3 is the octoupole and a_4 the hexadecoupole parameters.

Group Chin II: The Rotational Symmetry SU(3)

This symmetry is used to describe the rotational spectra of nuclei that possess axial symmetrical rotor. The Hamiltonian of this symmetry used the angular momentum and the quadrupole parameters (a_1, a_2). This Hamiltonian is given by [18, 20]:

$$\hat{H}^{(II)} = a_1 \hat{L} .\hat{L} + a_2 \hat{Q} .\hat{Q} \quad (5)$$

Group Chin III: The Γ – Unstable Symmetry O(6)

This symmetry is used to describe the asymmetric (γ -soft) deformed rotor of nuclei. The Hamiltonian parameters of this symmetry are the strengths of the pairing a_0, angular momentum a_1 and octoupole a_3. The Hamiltonian equation of this symmetry is [18,20]:

$$\hat{H}^{(III)} = a_0 \hat{P}^+ .\hat{P} \ + a_1 \hat{L} .\hat{L} + a_3 \hat{T}_3 .\hat{T}_3 \quad (6)$$

Table 1 shows the typical conditions of the energy ratios for these three dynamical symmetries.

TABLE 1.The typical conditions of the energy ratios for three dynamical symmetries in IBM-1 [18, 20]

symmetry	$E(\frac{4_1^+}{2_1^+})$	$E(\frac{6_1^+}{2_1^+})$	$E(\frac{8_1^+}{2_1^+})$
SU(5)	2	3	4
O(6)	2.5	4.5	7
SU(3)	3.33	7	12

The interferences between these three dynamical symmetries give three transitional regions. These regions are as follows *SU(3) - SU(5):* This transitional region can be treated by breaking SU(3) symmetry in the direction of SU(5) by adding $\varepsilon(\hat{n}_s+\hat{n}_d)$, $a_3\hat{T}_3.\hat{T}_3$, $a_4\hat{T}_4.\hat{T}_4$ terms. The Hamiltonian of this region can be written as [18, 20]:

$$\left.\begin{aligned}\hat{H}&=\varepsilon(\hat{n}_s+\hat{n}_d)+a_1\hat{L}.\hat{L}\\&+a_2\hat{Q}.\hat{Q}+a_3\hat{T}_3.\hat{T}_3+a_4\hat{T}_4.\hat{T}_4\end{aligned}\right\} \quad (7)$$

SU(3) - O(6): The nuclei in this transitional region can be treated by breaking SU(3) symmetry in the direction of O(6) by adding $a_0\hat{P}^+.\hat{P}$, $a_3\hat{T}_3.\hat{T}_3$ terms. The Hamiltonian of this region can be written as [18, 20]:

$$\hat{H}=a_0\hat{P}^+.\hat{P}+a_1\hat{L}.\hat{L}+a_2\hat{Q}.\hat{Q}+a_3\hat{T}_3.\hat{T}_3 \quad (8)$$

O(6) - SU(5): The nuclei in this transitional region can be treated by a Hamiltonian containing $\varepsilon(\hat{n}_s+\hat{n}_d)$ and $a_0\hat{P}^+.\hat{P}$ terms as [18, 20]:

$$\left.\begin{aligned}\hat{H}&=\varepsilon(\hat{n}_s+\hat{n}_d)+a_0\hat{P}^+.\hat{P}\\&+a_1\hat{L}.\hat{L}+a_3\hat{T}_3.\hat{T}_3+a_4\hat{T}_4.\hat{T}_4\end{aligned}\right\} \quad (9)$$

The Electromagnetic Transitions' Operator

The general form of the electromagnetic transitions' operator in IBM-1is [18, 20, 21]:

$$\left.\begin{aligned}\hat{T}^{(L)}&=\gamma_0\left[\hat{s}^+\times\tilde{\hat{s}}\right]^{(0)}\\&+\alpha_2\left[\hat{d}^+\times\tilde{\hat{S}}+\hat{S}^+\times\tilde{\hat{d}}\right]^{(2)}\\&+\beta_L\left[\hat{d}^+\times\tilde{\hat{d}}\right]^{(L)}\end{aligned}\right\} \quad (10)$$

where γ_0, α_2 and β_L (L = 0, 1, 2, 3, 4) are parameters specifying the various terms in the corresponding operators. Equation (10) yields transition operators for E0, M1, E2, M3 and E4 transitions with appropriate values of the corresponding parameters. The electric quadrupole operator E2 has a widespread application in the analysis of **γ**-ray transitions and it is deduced from equation (10) as [18, 20, 21]:

$$\left.\begin{aligned}\hat{T}^{(E2)}&=\alpha_2\left[\hat{d}^+\times\tilde{\hat{S}}+\hat{S}^+\times\tilde{\hat{d}}\right]^{(2)}\\&+\beta_2\left[\hat{d}^+\times\tilde{\hat{d}}\right]^{(2)}\end{aligned}\right\} \quad (11)$$

It is clear that, for the E2 polarity, two parameters, α_2 and β_2, are needed in addition to the wave functions of initial and final states.

The electric transition probabilities' B(E2) values are defined in terms of reduced matrix elements by Iachello and Arima (1987) as [20, 21]:

$$\left.B(E2;L_i\to L_f)=\frac{1}{2L_i+1}\left|\left\langle L_f\left\|\hat{T}^{(E2)}\right\|L_i\right\rangle\right|^2(eb)^2\right\} \quad (12)$$

Results and Discussion

The obtained results can be discussed separately for both energy levels and the electric transition probabilities B(E2) as follows:

Energy Levels

The IBM-1 model has been used in the calculation of the energy levels of the "$^{174-180}$Hf isotopes", using the experimental energy ratios:

$E(\frac{4_1^+}{2_1^+})$ = 3.3, 3.29, 3.3, 3.3,

$E(\frac{6_1^+}{2_1^+})$ = 6.76, 6.77, 7, 6.88 and

$E(\frac{8_1^+}{2_1^+})$ = 11.21, 11.33, 11.67, 11.645,

respectively. It has been found that the "$^{176-180}$Hf isotopes" are rotational (deformed nuclei) and that they have a dynamical symmetry SU(3) and the "^{174}Hf isotope" is found in the transitional region SU(3)-O(6) respecting to IBM-1.

According to the Hamiltonian of the IBM-1 model, the energy levels of the "^{174}Hf isotope" (total number of bosons 15 in the transitional region SU(3) - O(6), Eq. 8) and the "$^{174\text{-}180}$Hf isotopes" (total number of bosons 16, 15 and 14, respectively, in dynamical symmetry SU(3), Eq. 5) have been calculated using the angular momentum, quadrupole and octoupole parameters [a_1, (a_2, χ) and a_3]. The best fit values of these parameters are given in Table 2, which shows the values of the relevant parameters. These values are obtained by fitting to get results of the energy levels that match with the experimentally reported data [22-25], whereas, the first two terms and the last term in Eq. 2 have now been included because they are irrelevant to the case of fully and weakly deformed nuclei (rotational nuclei).

TABLE 2. The best fit values of the Hamiltonian parameters for "$^{174\text{-}180}$Hf isotopes"

Isotopes	a_1	a_2	a_3	χ
	MeV	MeV	MeV	
^{174}Hf	0.0450	-0.0105	0.0640	-0.06
^{176}Hf	0.0097	-0.1130	0.000	-0.6
^{178}Hf	0.0960	-0.0146	0.000	-0.11
^{180}Hf	0.0100	-0.0140	0.000	-0.26

The obtained results by this process are given in Table 3; this table shows the ground state, β and γ bands of the experimental and the IBM-1 calculations energy levels of "$^{174\text{-}180}$Hf isotopes". It shown that there is a good agreement between experimental energy values and the IBM-1 calculations.

TABLE 3. The experimental and calculated energy levels of $^{174\text{-}180}$Hf isotopes

^{174}Hf Isotope

g-band			β band			γ- band		
E (Mev)			E (Mev)			E (Mev)		
L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1
$^{+}0$	0	0	$^{+}0$	0.828	0.808	2^{+}	1.226	1.220
$^{+}2$	0.090	0.090	$^{+}2$	0.900	0.903	3^{+}	1.336	1.343
$^{+}4$	0.297	0.294	$^{+}4$	1.062	1.110	4^{+}	1.448	1.478
$^{+}6$	0.608	0.602	$^{+}6$	1.307	1.419	5^{+}	1.658	1.667
$^{+}8$	1.009	1.004						
$^{+}10$	1.485	1.492						

^{176}Hf Isotope

g-band			β band			γ band		
E (Mev)			E (Mev)			E (Mev)		
L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1
$^{+}0$	0	0	$^{+}0$	1.149	1.152	2^{+}	1.341	1.331
$^{+}2$	0.088	0.085	$^{+}2$	1.226	1.237	3^{+}	1.445	1.415
$^{+}4$	0.29	0.283	$^{+}4$	1.39	1.435	4^{+}	1.54	1.528
$^{+}6$	0.596	0.595	$^{+}6$	1.628	1.747	5^{+}	1.727	1.67
$^{+}8$	0.997	1.02						
$^{+}10$	1.481	1.558						

^{178}Hf Isotope

g-band			γ band			β band		
E (Mev)			E (Mev)			E (Mev)		
L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1
$^{+}0$	0	0	2^{+}	1.17	1.18	$^{+}0$	1.19	1.18
$^{+}2$	0.09	0.09	3^{+}	1.26	1.27	$^{+}2$	1.27	1.27
$^{+}4$	0.30	0.30	4^{+}	1.38	1.39	$^{+}4$	1.45	1.48
$^{+}6$	0.63	0.63	5^{+}	1.53	1.54	$^{+}6$	1.73	1.82
$^{+}8$	1.05	1.08						
$^{+}10$	1.57	1.65						

^{180}Hf Isotope								
g-band			β band			γ-band		
E (Mev)			E (Mev)			E (Mev)		
L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1	L^{π}	Exp	IBM-1
$^{+}0$	0	0	$^{+}0$	1.102	1.102	2^{+}	1.199	1.193
$^{+}2$	0.093	0.090	$^{+}2$	1.183	1.169	3^{+}	1.291	1.260
$^{+}4$	0.308	0.302	$^{+}4$	1.369	1.381	4^{+}	1.409	1.405
$^{+}6$	0.640	0.632	$^{+}6$	1.703	1.715	5^{+}	1.557	1.533
$^{+}8$	1.083	1.089						
$^{+}10$	1.630	1.664						

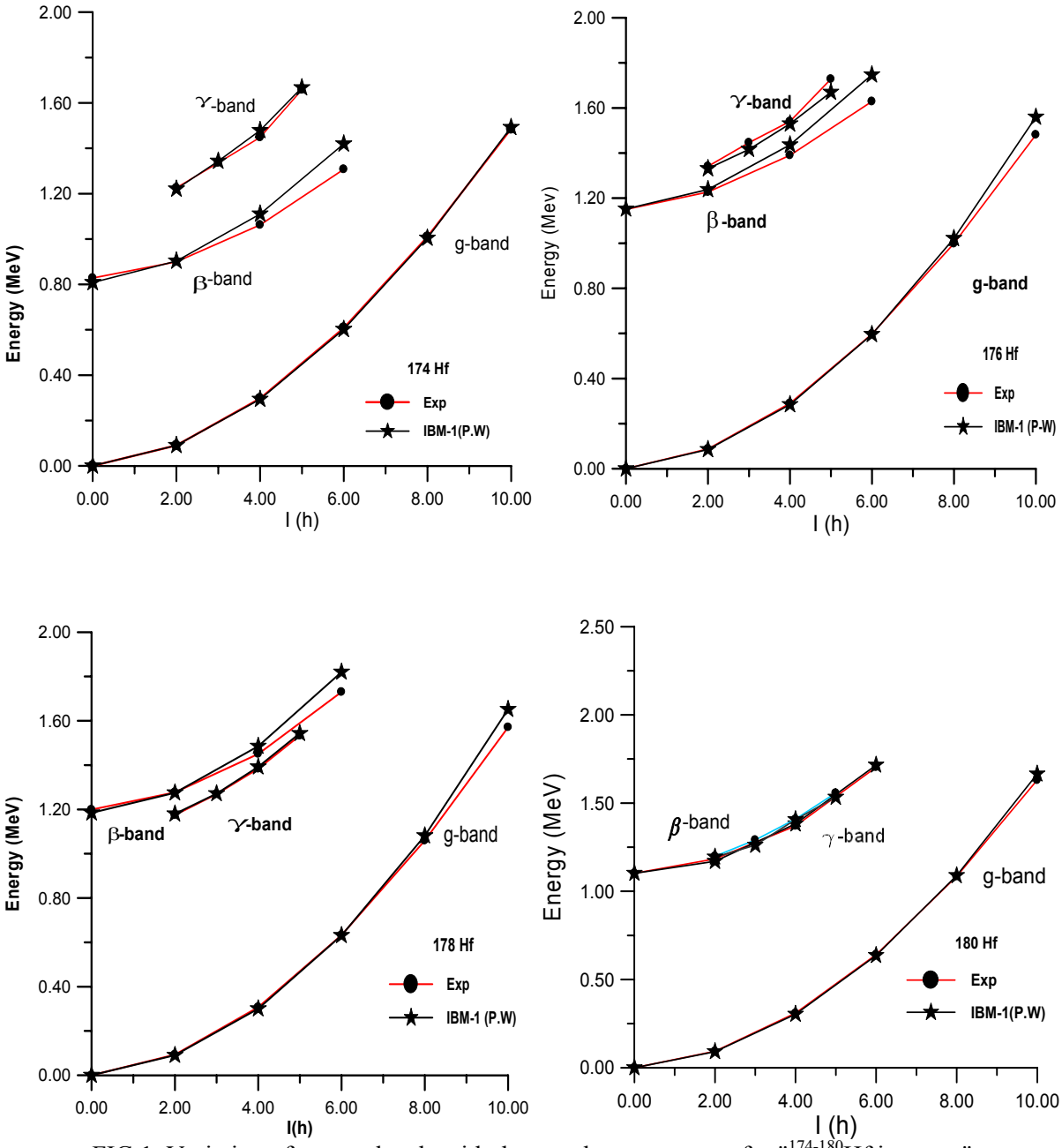

FIG.1. Variation of energy levels with the angular momentum for "$^{174\text{-}180}$Hf isotopes".

The root mean square deviation [26] $RMSD = \left[\frac{1}{N}\sum (E_{cal} - E_{Exp})^2\right]^{\frac{1}{2}}$ is used to compare experimental and calculated energy levels (N is the number of levels), Table 4. In the ground state bands the best agreement was found in ^{174}Hf where the smallest value of RMSD is equal to 0.0045 and equal to 0.01 for γ-bands in ^{178}Hf. However, RMSD = 0.011 for β-bands in ^{180}Hf.

TABLE 4. The root mean square deviation RMSD between experimental and calculated energy levels

Isotopes	RMSD		
	g- band	β- band	γ-band
^{174}Hf	0.0045	0.062	0.016
^{176}Hf	0.033	0.064	0.033
^{178}Hf	0.035	0.048	0.010
^{180}Hf	0.015	0.011	0.020

A very good agreement has been obtained between the experimentally reported and the calculated energy level variations with the angular momentum as depicted in Fig.1. This figure shows that the experimental and the theoritical energy levels of the ground state, β and γ bands in "$^{174-180}$Hf isotopes" increase with increasing angular momentum. The very good agreement was also found between the experimental and the theoritical energy levels of the ground state bands in all studied isotopes, and good agreement was achieved in energy levels of β and γ bands. In general, the experimental and the IBM-1 calculated energy levels in "$^{174-180}$Hf isotopes" increase with angular momentum as I(I+1) because these isotopes are of rotational nuclei (deformed nuclei). Comparison between the ground state, β and γ bands of the experimentally determined energy levels and the IBM-1 calculations is shown in Fig. 2.

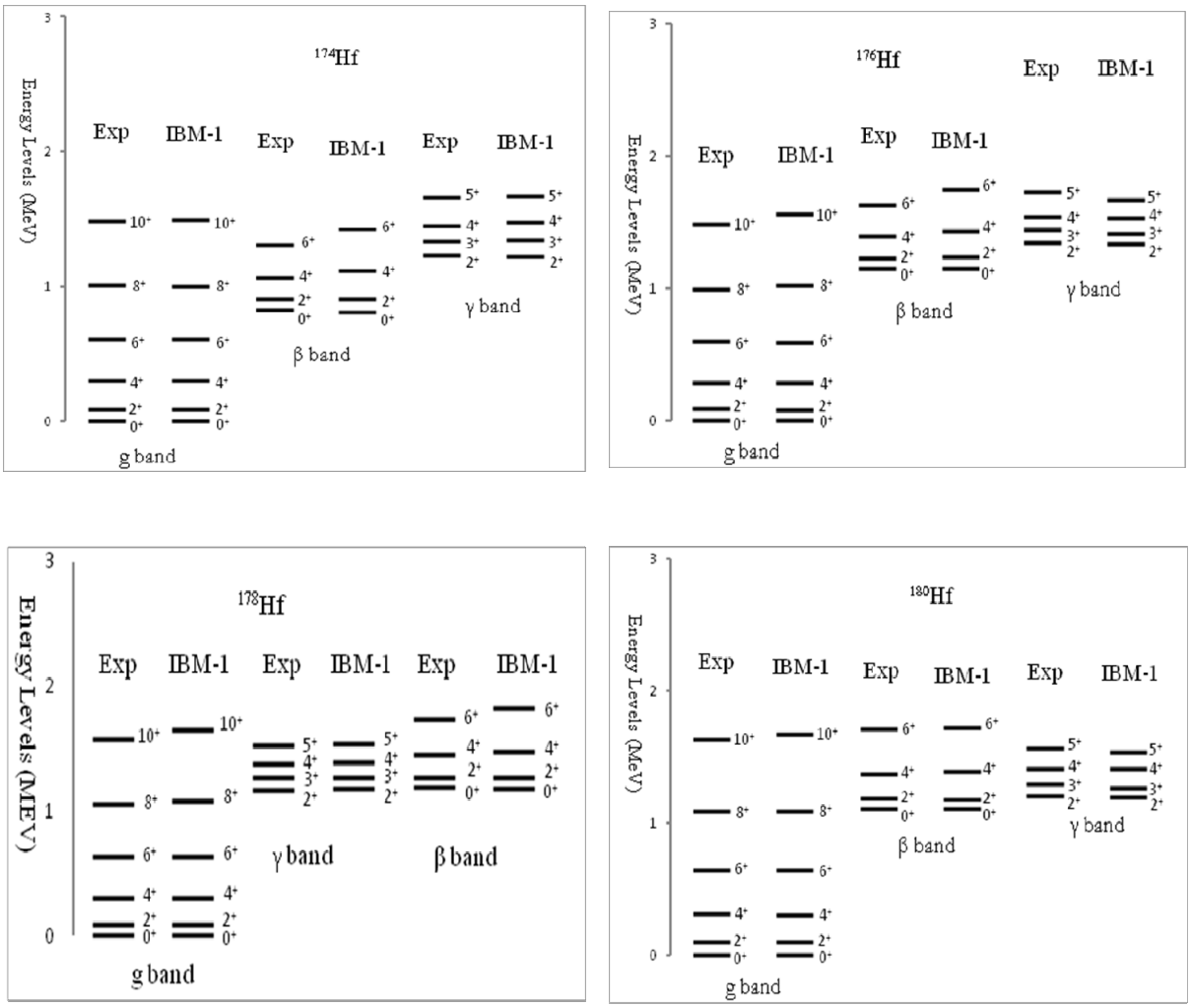

FIG. 2. The ground state, β and γ bands of the experimental and calculated energy levels of "$^{174-180}$Hf isotopes".

Electric Transition Probabilities B(E2)

Once the wave functions have been fixed by fitting the energy levels, one could determine the electric transition probabilities B(E2) between these levels. The calculated B(E2) values for transitions in "$^{174-180}$Hf isotopes" were obtained by employing Eq. 12. The used $T^{(E2)}$ parameters of "$^{174-180}$Hf isotopes" are given in Table 5.

TABLE 5. The used $T^{(E2)}$ parameters α_2 and β_2 for "$^{174-180}$Hf isotopes"

Isotopes	α_2 (eb)	β_2 (eb)
^{174}Hf	0.043	-0.3000
^{176}Hf	0.128	-0.0560
^{178}Hf	0.129	-0.0381
^{180}Hf	0.140	-0.0352

A comparison between the experimentally determined values of B(E2) [22-25] and those calculated by the IBM-1 model is given in Table 6, which shows that the B(E2) transitions between β-band and g-band and between γ-band and g-band are smaller than the B(E2) transitions between g-band and g-band, and this table shows also that, in general, there is a good agreement between the experimentally reported and calculated B(E2) values in ground state bands in "$^{174-180}$Hf isotopes" except the transition 6_g^+ to 4_g^+ in "$^{178-180}$Hf isotopes", where the experimental and calculated B(E2) values of this transition are weak in agreement. The experimental and calculated B(E2) transitions between β-band and g-band and between γ-band and g-band in general are weakly in agreement except the transitions 2_β^+ to 0_g^+ in ^{176}Hf isotope and 2_γ^+ to 0_g^+ in ^{178}Hf isotope which gave a very good agreement. The weak agreement between the experimental and calculated values in some B(E2) in those isotopes can be explained by the fact that many small components of the initial and final wave functions contribute coherently to the value of this reduced E2 transition probability, since these small components are not stable enough against small changes in the model parameters [17]. There is no available experimental transition data to many transitions in Table 6; therefore, it has been predicted by using IBM-1.

Table 6 shows that in general there is a good agreement between the experimentally reported B(E2) values and the theoretically calculated ones.

TABLE 6. Experimental and calculated B(E2) values of g-band to g-band, β-band to g-band and γ-band to g-band transitions in "$^{174-180}$Hf isotopes"

I_i I_f	^{174}Hf B(E2)(e^2b^2)		^{176}Hf B(E2)(e^2b^2)		^{178}Hf B(E2)(e^2b^2)		^{180}Hf B(E2)(e^2b^2)	
	Exp	IBM-1	Exp	IBM-1	Exp	IBM-1	Exp	IBM-1
2_g^+ 0_g^+	0.877	0.870	1.07	1.06	0.95	0.95	0.936	0,939
4_g^+ 2_g^+	-	1.218	-	1.50	-	1.34	1.38	1.33
6_g^+ 4_g^+	-	1.290	-	1.63	1.30	1.45	1.32	1.43
8_g^+ 6_g^+	-	1.750	-	1.66	1.41	1.48	1.50	1.45
10_g^+ 8_g^+	-	1.215	-	1.65	1.51	1.48	1.44	1.43
2_β^+ 0_g^+	0.012	0.0013	0.00057	0.00057	0.0043	0.002	-	0.0073
2_β^+ 2_g^+	-	0.018	-	0.0091	-	0.0031	-	0.04
2_γ^+ 0_g^+	0.027	0.104	0.023	0.012	0.023	0.023	0.023	0.0033
2_γ^+ 2_g^+	0.042	0.113	-	0.019	0.026	0.037	0.031	0.006

Conclusions

Theoretical calculations using IBM-1 model were performed for "$^{174-180}$Hf isotopes" with proton number of 72. The "^{174}Hf isotope" has a total number of 15 bosons and is considered as weakly rotational **(**weakly deformed). This nucleus is located within the transitional region SU(3) - O(6) with respect to the IBM-1 model. The "$^{176-180}$Hf isotopes" have bosons' total numbers of 16, 15 and 14, and are considered fully rotational (fully deformed) nuclei, and the dynamical symmetry of these isotopes is SU(3). The low-lying positive parity states (energy levels) and the theoretically obtained values of electric transition probabilities B(E2) for these isotopes using IBM-1 model are compared with the experimentally reported values. A very good agreement was obvious. Therefore, it is possible to describe the energy levels of "$^{176-180}$Hf isotopes" by using the IBM-1 model. Moreover, it is worth mentioning that more experimental investigation on "$^{176-180}$Hf isotopes" B(E2) values is required in order to identify the strength of E2 transitions within the ground state band, from β-band to ground band and from γ-band to ground band.

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