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IBM-1Calculations of Energy Levels and Electric Transition Probabilities B(E2) in "$^{158-160}$Gd Isotopes"

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Abstract: In this paper, the interacting boson model (IBM-1) is described and employed for calculating the energy levels and electric transition probabilities B(E2) for "$^{158-160}$Gd isotopes". The values of the parameters have been determined using the IBM-1 Hamiltonian which yield the best fit to the available experimental energy levels. Particular attention has been paid to the B(E2) transition probabilities. In general, a good agreement has been achieved between the IBM-1 calculations and the available experimental results of energy levels and B(E2) values. The results showed that the "$^{158-160}$Gd isotopes" are rotational (deformed) nuclei and that they have dynamical symmetry SU(3) in the interacting boson model IBM-1.

Keywords: Isotope; Energy levels; Interacting boson model; Transition probabilities.

Introduction

The interacting boson model (IBM) is a nuclear model proposed by Iachello and Arima in (1974) [1].The basic idea of the IBM [1-5] is used to describe the low-lying collective states in even-even nuclei employing a system of interacting s-and d-bosons carrying angular momentums 0 and 2, respectively.(It is reasonable to view the boson states as being constructed from the valence space only and to identify the bosons as correlated pairs of like nucleons). The bosons number N= n_s+n_d is finite and conserved in a given nucleus and simply given as half of the total number of valence nucleons. There are four versions of this model (IBM-1, IBM-2, IBM-3 and IBM-4). In the IBM-1 version, no distinction is made between protons and neutrons. Moreover, the valence number counting is always done relative to the nearest closed shells. Since the bosons could be pairs of holes or particles, the s (L=0) and d (L=2) bosons of the IBM-1 have six sub - states and therefore define six-dimensional space. This leads to a description in terms of the unitary group in six dimensions, U(6).Thus, many of the characteristic properties of the IBM-1 can be derived by group- theoretical methods and analytically expressed. The different reductions of U(6) lead to three dynamical symmetries known as SU(5), SU(3) and O(6), which are related to the spherical vibrator, deformed rotor and asymmetric (γ -soft), respectively. The three corresponding dynamical symmetries, group chains of U(6) can be written as [5]:

$$U(6)\supset SU(5)\supset O(5)\supset O(3)\supset O(2) \qquad (I)$$

$$U(6)\supset SU(3)\supset O(3)\supset O(2) \qquad (II)$$

$$U(6)\supset O(6)\supset O(5)\supset O(3)\supset O(2) \qquad (III)$$

The energy levels of "$^{158-160}$Gd isotopes" have been extensively experimentally investigated using a wide variety of reactions. From these studies, the excited states in the "$^{158-160}$Gd isotopes" have been investigated from (γ, γ'), (d, p), (α, 2nγ), (α, 4nγ), (n, γ), (n, n'γ) and coulomb excitation reactions which gave information about the experimental energy levels and the B(E2) values in these isotopes [6-28].

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The aim of the present work is to investigate the dynamical symmetry of "[158-160]Gd isotopes" and study the energy levels and the B(E2) values of these isotopes within the framework of the (IBM-1) model.

Theoretical Basics of IBM-1 Model

The most general Hamiltonian of IBM-1 is [5]:

$$\hat{H} = \begin{bmatrix} \varepsilon(\hat{n}_s + \hat{n}_d) + a_o \hat{P}^{+}.\hat{P} + a_1 \hat{L}.\hat{L} \\ +a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \end{bmatrix} \quad (1)$$

where ε is the bosons energy and the operators are [5]:

$$\left.\begin{aligned} &\hat{n}_s = \hat{s}^{+}.\hat{\tilde{s}} \quad , \quad \hat{n}_d = \hat{d}^{+}.\hat{\tilde{d}} \\ &\hat{P} = \frac{1}{2}(\hat{\tilde{d}}.\hat{\tilde{d}}) - \frac{1}{2}(\hat{s}.\hat{s}) \\ &\hat{L} = \sqrt{10}\left[\hat{d}^{+} \times \hat{\tilde{d}}\right]^{(1)} \\ &\hat{Q} = \sqrt{5}\left[(\hat{d}^{+} \times \hat{\tilde{s}}) + (\hat{s}^{+} \times \hat{\tilde{d}})\right]^{(2)} + \chi\left[\hat{d}^{+} \times \hat{\tilde{d}}\right]^{(2)} \\ &\hat{T}_3 = \left[\hat{d}^{+} \times \hat{\tilde{d}}\right]^{(3)} \\ &\hat{T}_4 = \left[\hat{d}^{+} \times \hat{\tilde{d}}\right]^{(4)} \end{aligned}\right\} \quad (2)$$

The phenomenological parameters a_0, a_1, (a_2, χ), a_3, a_4, represent the strengths of the pairing, angular momentum, quadrupole, octoupole, hexadecoupole interactions between bosons, respectively.

The three chains of the dynamical symmetries of IBM-1 are:

Group Chain I: The Vibrational SU(5) Limit

This limit of dynamical symmetry group describes the vibrational nuclei which have a spherical shape. The Hamiltonian of this chain can be written as [3, 5]:

$$\hat{H}^{(I)} = \begin{bmatrix} \varepsilon(\hat{n}_s + \hat{n}_d) \; + a_1 \hat{L}.\hat{L} \\ +a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \end{bmatrix} \quad (3)$$

Group Chain II: The Rotational SU(3) Limit

This limit is used to describe the rotational spectra of nuclei that possess axial symmetrical rotor. The Hamiltonian of this limit is given by [3, 5, 29]:

$$\hat{H}^{(II)} = a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} \quad (4)$$

Group Chain III: The γ -Unstable O(6) Limit

This limit is used to describe the asymmetric (γ -soft) deformed rotor of nuclei, the Hamiltonian of this limit is [3, 5, 30, 31]:

$$\hat{H}^{(III)} = a_0 \hat{P}^{+}.\hat{P} \; + a_1 \hat{L}.\hat{L} + a_3 \hat{T}_3.\hat{T}_3 \quad (5)$$

Table 1 shows the behavior of these limits.

TABLE 1. Energy ratios and the basic conditions of the B(E2) values of the corresponding limits [3, 5].

Limit	$E(\frac{4_1^+}{2_1^+})$	$E(\frac{6_1^+}{2_1^+})$	$\frac{B(E2;4_1^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)}$	$\frac{B(E2;0_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)}$	$\frac{B(E2;2_2^+ \to 2_1^+)}{B(E2;2_1^+ \to 0_1^+)}$
SU(5)	2	3	< 2	< 2	< 2
SU(3)	3.33	7	< (10/7)	≈ 0	≈ 0
O(6)	2.5	4.5	< (10/7)	≈ 0	< (10/7)

The Electromagnetic Transitions Operator

The general form of the electromagnetic transitions operator in IBM-1 is [3. 5, 32]:

$$\left.\begin{aligned}\hat{T}^{(L)} &= \gamma_0\left[\hat{s}^{+}\times\hat{\tilde{s}}\right]^{(0)}\\ &+\alpha_2\left[\hat{d}^{+}\times\hat{\tilde{S}}+\hat{S}^{+}\times\hat{\tilde{d}}\right]^{(2)}\\ &+\beta_L\left[\hat{d}^{+}\times\hat{\tilde{d}}\right]^{(L)}\end{aligned}\right\} \quad (6)$$

where γ_0, α_2 and β_L (L = 0, 1, 2, 3, 4) are parameters specifying the various terms in the corresponding operators. Eq. 6 yields transition operators for E0, M1, E2, M3 and E4 transitions with appropriate values of the corresponding parameters.

The electric quadrupole operator E2 has a widespread application in the analysis of **γ**-ray transitions and it is deduced from Eq. 6 as [3, 5, 31]:

$$\left.\begin{aligned}\hat{T}^{(E2)} &= \alpha_2\left[\hat{d}^{+}\times\hat{\tilde{S}}+\hat{S}^{+}\times\hat{\tilde{d}}\right]^{(2)}\\ &+\beta_2\left[\hat{d}^{+}\times\hat{\tilde{d}}\right]^{(2)}\end{aligned}\right\} \quad (7)$$

It is clear that, for the E2 polarity, two parameters $\boldsymbol{\alpha}_2$ and β_2 are needed in addition to the wave functions of initial and final states.

The B(E2) values are defined in terms of reduced matrix elements by Iachello and Arima (1987) as [5, 31, 33]:

$$\left.\begin{aligned}&B(E2;L_i\rightarrow L_f)=\\ &\frac{1}{2L_i+1}\left|\left\langle L_f\left\|\hat{T}^{(E2)}\right\|L_i\right\rangle\right|^2 \quad (eb)^2\end{aligned}\right\} \quad (8)$$

Results and Discussion

Energy Levels

The IBM-1 model has been used in the calculation of the energy levels of the "158-160Gd isotopes", using the experimental energy ratios $E(\frac{4_1^+}{2_1^+})$=3.29, 3.3 and $E(\frac{6_1^+}{2_1^+})$= 6.78, 6.84, respectively. It has been found that the "158-160Gd isotopes" are rotational (deformed) nuclei and that they have a dynamical symmetry SU(3) respecting to the IBM-1.

According to the Hamiltonian of the dynamical symmetry SU(3) limit (Eqs. 1 and 4), the energy levels of the "158-160Gd isotopes" (total number of bosons are 13,14, respectively) have been calculated using the angular momentum and quadrupole parameters [a_1, (a_2, χ)]. The best fit values of these parameters are given in Table 2, which shows the values of the relevant parameters, these values are obtained by fitting to get results of the energy levels that match with the experimentally reported data [12-27], whereas, the first two and the last term in equation (1) have not been included because they are irrelevant to the case of fully and weakly deformed nuclei (rotational nuclei).

TABLE 2. The best fit values of the Hamiltonian parameters for "158-160Gd isotopes".

Isotopes	a_1 MeV	a_2 MeV	X MeV
^{158}Gd	0.0065	-0.0167	-1.16
^{160}Gd	0.00425	-0.0213	-0.61

Comparison between the experimentally determined energy levels and the IBM-1 calculations is shown in Fig. 1. Fig.1 shows the experimental [12, 27] and the IBM-1 calculations of ground state, **γ** and β bands of "158-160Gd isotopes". It shows that there is a good agreement between experimental and the IBM-1 calculations.

Root mean square deviation [34] $RMSD=\left[\frac{1}{N}\sum(E_{cal}-E_{Exp})^2\right]^{\frac{1}{2}}$ is used to compare between experimental results and calculations of energy levels (N is the number of levels).The (RMSD) for ^{158}Gd is found to be 0.044 in ground state band for seven levels and it is 0.0165 for five levels in γ-band. However, it is 0.0466 in β-band for four levels, and the (RMSD) for ^{160}Gd is found to be 0.0724 in ground state band for nine levels and for seven levels it is 0.05 in **γ**-band. However, it is 0.028 in β-band for two levels (there is no available experimental data).

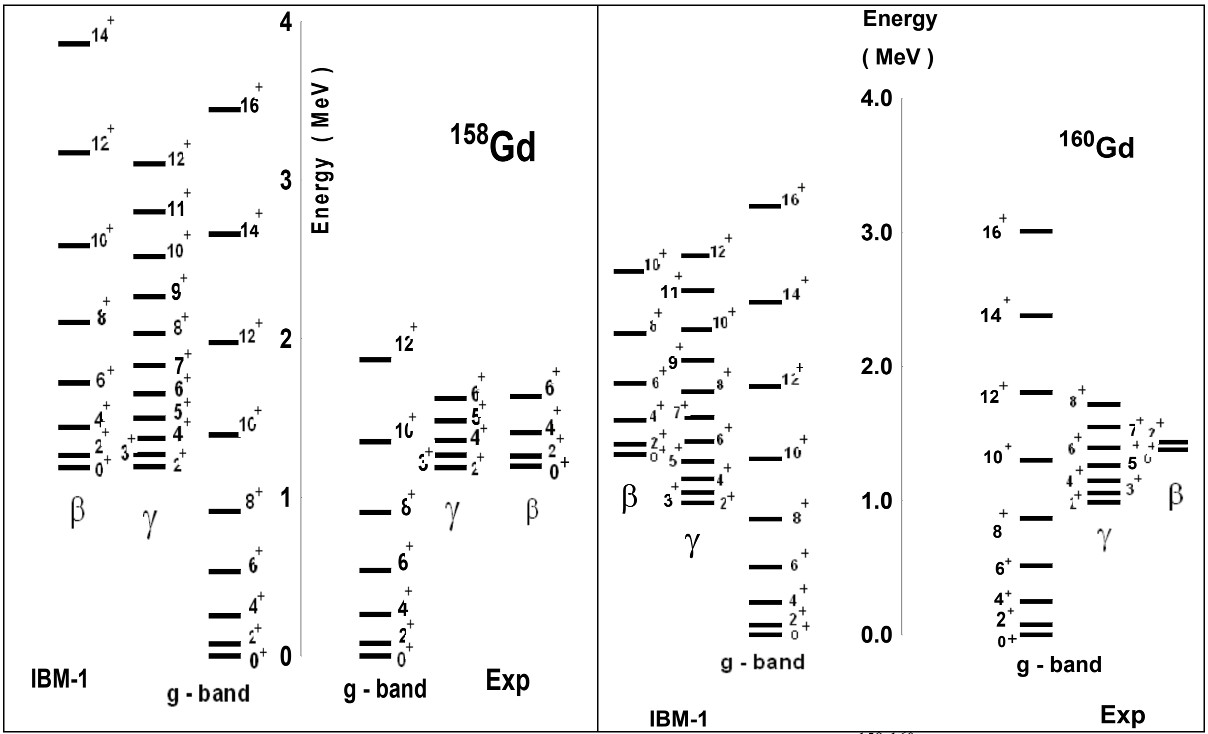

FIG. 1. The experimental [12,27] and calculated energy levels of "$^{158-160}$Gd isotopes".

A very good agreement has been obtained between the experimental and the calculated energy level variations with the angular momentum as depicted in Fig.2. This figure shows that the experimental and the theoretical energy levels of the ground state, β and γ bands in "$^{158-160}$Gd isotopes" increase with increasing angular momentum. A very good agreement was also found between the experimental and the theoretical energy levels of the ground state band and a good agreement was achieved in energy levels of β and γ bands. In general, the experimental and the IBM-1 calculations of energy levels in "$^{158-160}$Gd isotopes" increase with angular momentum as l(l+1) because these isotopes are rotational (deformed) nuclei. Comparison between the ground state, β and γ bands of the experimentally determined energy levels and the IBM-1 calculations is shown in Fig. 2.

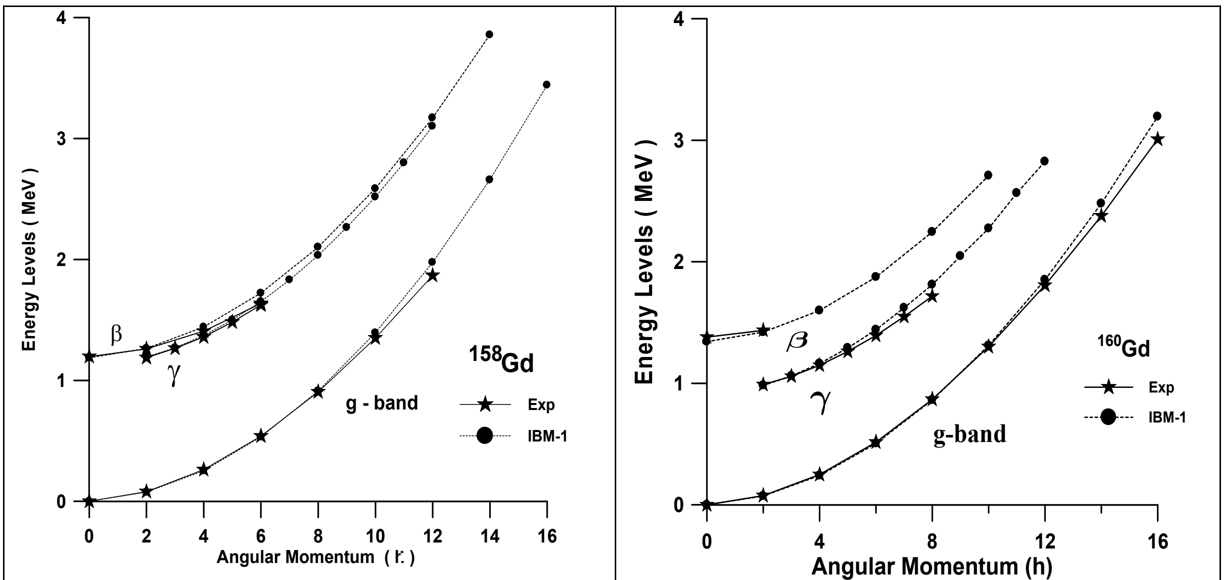

FIG. 2. Variation of energy levels with the angular momentum for"$^{158-160}$Gd isotopes".

The condition of energy ratios of the corresponding limits is shown in Table 1. It has been found from calculations of energy levels from IBM-1, for "$^{158-160}$Gd isotopes" that $E(\frac{4_1^+}{2_1^+})$ = 3.33, 3.326 and $E(\frac{6_1^+}{2_1^+})$ = 6.99, 6.964; therefore the "$^{158-160}$Gd isotopes" are considered as rotational (deformed) nuclei and the dynamical symmetry of these isotopes is SU(3) in the IBM-1.

Electric Transition Probabilities B(E2)

Once the wave functions have been fixed by fitting the energy levels, one could determine the B(E2) values between these levels. The calculated B(E2) values for transitions in "$^{158-160}$Gd isotopes" were obtained by employing Eq. 8. The used $T^{(E2)}$ parameters of "$^{158-160}$Gd isotopes" are given in Table 3.

TABLE 3. The used $T^{(E2)}$ parameters α_2 and β_2 for "$^{158-160}$Gd isotopes".

Isotopes	α_2 (eb)	β_2 (eb)
^{158}Gd	0.148	-0.0317
^{160}Gd	0.132	-0.033

A comparison between the experimentally determined values of B(E2) [12,16,27,28] and those calculated by the IBM-1 model is given in Table 4, this table show that the B(E2) transitions between γ-band to g-band and β-band to g-band are smaller than the B(E2) transitions between g-band to g-band. This table also shows that, in general, there is a good agreement between the experimentally reported and calculated B(E2) values in ground state bands in "$^{158-160}$Gd isotopes" except the transition 10_1^+ to 8_1^+ in "^{158}Gd isotope", where the experimental and calculated B(E2) values of this transition have weak agreement. The experimental and calculated B(E2) transitions between β-band to g-band and γ-band to g-band in general are in a very good agreement except the transitions 4_2^+ to 2_1^+, 2_3^+ to 2_1^+ and 0_2^+ to 2_1^+ in "^{158}Gd isotope" which have weak agreement. The weak agreement between the experimental and calculated values in some B(E2) in those isotopes can be explained by the fact that many small components of the initial and final wave functions contribute coherently to the value of this reduced E2 transition probability, since these small components are not stable enough against small changes in the model parameters [35]. There is no available experimental transition data to many transitions in Table 4, therefore, these data have been predicted by using IBM-1. Experimental and calculated B(E2) values are also compared with available theoretical values.

TABLE 4. Experimental and calculated B(E2) values of g-band to g-band, γ-band to g-band, β-band to g-band and β-band to γ-band in "$^{158-160}$Gd isotopes".

^{158}Gd isotope								
(g-band to g-band)		B(E2)(e^2b^2)		(γ-band to g-band)		B(E2)(e^2b^2)		
initial state	final state	Exp.	IBM-1	initial state	final state	Exp.	IBM-1	IBA[36]
2_1^+	0_1^+	1.005	1.063	2_2^+	0_1^+	0.0178	0,018	0.0170
4_1^+	2_1^+	1.467	1.499	2_2^+	2_1^+	0.03	0.029	0.0266
6_1^+	4_1^+	-	1.611	2_2^+	4_1^+	0.0014	0.0019	0.0018
8_1^+	6_1^+	1.65	1.626	4_2^+	2_1^+	0.0057	0.0091	-
10_1^+	8_1^+	1.69	1.589	4_2^+	4_1^+	0.037	0.034	-
12_1^+	10_1^+	1.55	1.516					
(β-band to g-band)		B(E2)(e^2b^2)		(β-band to γ-band)		B(E2)(e^2b^2)		
initial state	final state	Exp. IBM-1 IBA[36]		initial state	final state	Exp.	BM-1	
0_2^+	2_1^+	0.006 0.0097 -		0_2^+	2_2^+	-	0.051	
2_3^+	0_1^+	0.0016 0.0015 0.002		2_3^+	2_2^+	-	0.0099	
2_3^+	2_1^+	0.001 0.0025 0.0033		2_3^+	4_2^+	-	0.0165	
2_3^+	4_1^+	0.007 0.006 0.0076		4_3^+	4_2^+	-	0.0118	

^{160}Gd isotope			
(g-band to g-band) initial state final state	B(E2)(e^2b^2) Exp. IBM-1	(γ-band to g-band) initial state final state	B(E2)(e^2b^2) Exp. IBM-1 IBA[36]
2_1^+ 0_1^+ 4_1^+ 2_1^+ 6_1^+ 4_1^+ 8_1^+ 6_1^+ 10_1^+ 8_1^+ 12_1^+ 10_1^+	1.041 1.0417 - 1.473 - 1.593 - 1.622 - 1.603 - 1.5503	2_2^+ 0_1^+ 2_2^+ 2_1^+ 2_2^+ 4_1^+ 4_2^+ 2_1^+ 4_2^+ 4_1^+	0.0202 0.02027 0.0176 0.0366 0.0365 0.0275 0.0037 0.003 0.0018 - 0.0081 - - 0.0433 -
(β-band to g-band) initial state final state	B(E2)(e^2b^2) Exp. IBM-1	(β-band to γ-band) initial state final state	B(E2)(e^2b^2) Exp. IBM-1
0_2^+ 2_1^+ 2_3^+ 0_1^+ 2_3^+ 2_1^+ 2_3^+ 4_1^+	- 0.00022 - 0.00004 - 0.00003 - 0.00013	0_2^+ 2_2^+ 2_3^+ 2_2^+ 2_3^+ 4_2^+ 4_3^+ 4_2^+	- 0.0988 - 0.0165 - 0.0321 - 0.0189

Note: The relation between Weisskopf unit and electron barn unit is B(E2) (W.U) = 5.9435×10^{-6} $A^{4/3}$ (e^2b^2) [37].

In general, there is a good agreement between the experimentally reported B(E2) values and the theoretically calculated ones.

Comparison between the ground state bands of the experimentally determined B(E2) values and the IBM-1 calculations in ^{158}Gd isotope is shown in Fig.3.

According to the basic conditions of B(E2) ratios of corresponding limits in Table 1, the experimental and the calculated values of B(E2) ratios are given in Table 5.

A comparison between Table 5 and the basic conditions of B(E2) ratios in Table 1 shows that "$^{158-160}Gd$ isotopes" are considered as rotational (deformed) nuclei possessing dynamical symmetry SU(3) according to the IBM-1.

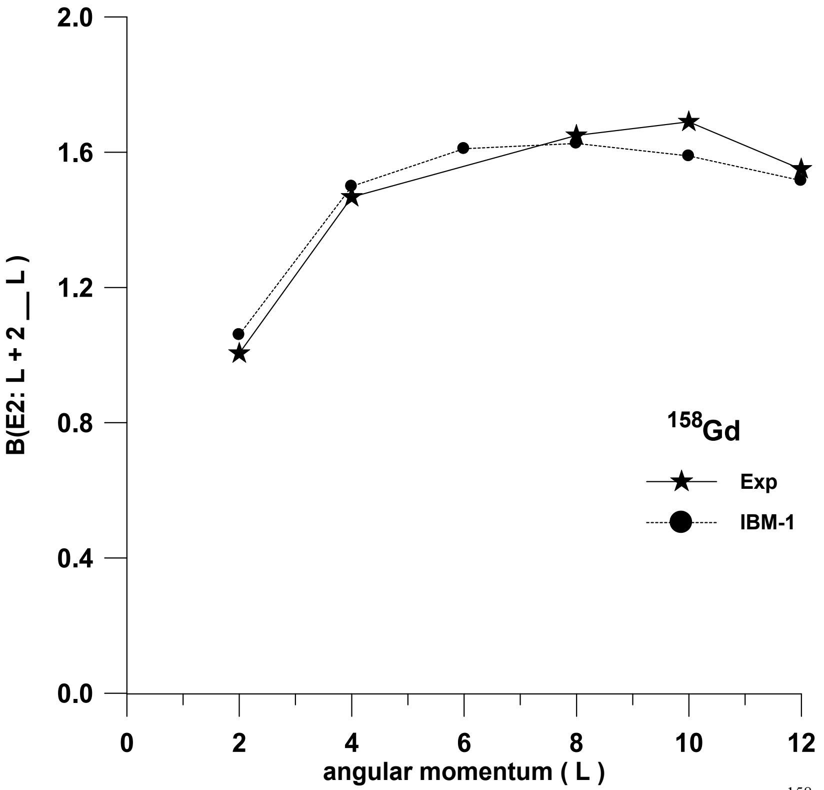

FIG. 3. Variation of B(E2) values with the angular momentum in g-band for ^{158}Gd.

TABLE 5. Experimental and calculated values of B(E2) ratios in " $^{158\text{-}160}$Gd isotopes".

Isotope	$\frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$		$\frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$		$\frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	
	Exp	IBM-1	Exp	IBM-1	Exp	IBM-1
^{158}Gd	1.459	1.41	0.0058	0.0091	0.03	0.027
^{160}Gd	-	1.414	-	0.0002	0.035	0.035

Conclusions

Theoretical calculations using IBM-1 model were performed for"$^{158\text{-}160}$Gd isotopes" with proton number 64. These isotopes have a total number of bosons of 13 and 14, respectively and considered as fully rotational (fully deformed) nuclei, and the dynamical symmetry of these isotopes is SU(3). The low-lying positive parity states (energy levels) and the theoretically obtained B(E2) values for these isotopes using IBM-1 model are compared with the experimentally reported values. A very good agreement was obvious. Therefore, it is possible to describe the energy levels of "$^{158\text{-}160}$Gd isotopes" by using IBM-1 model. Moreover, it is worth to mention that more experimental investigation on "$^{158\text{-}160}$Gd isotopes" B(E2) values is required in order to identify the strength of E2 transitions within the ground state band, from β-band to ground band and from γ-band to ground band and from β-band to γ-band.

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