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ARTICLE

Concerning the Existence of a Discrete Space

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Abstract: Loop quantum gravity is considered to be one of the two major candidates for a theory of quantum gravity. The most appealing aspect about this theory is that it predicts that space is not infinitely divisible, but that it has a granular structure. This paper illustrates a missed proof which validates the previous prediction of the theory. It does not prove or validate the model of quantized geometry of the spacetime which is predicted by the theory itself, but instead, it proves the necessity of space discreteness, and the existence of space quanta due to a simple fact or observation which is the existence of the origin position in a coordinates system. This is done in this paper by defining one of the major concepts in physics, which is the quantization concept, in a simple way, and applied directly to the observation. Although the area of quantum gravity – in general – requires a sophisticated level of mathematics, this paper was built on a simple mathematical level to make it accessible to any reader.

Keywords: Quantum gravity; Quantization concept; Discrete space.

Introduction

The problem of quantum gravity represents one of the major problems in physics today. Mainly, the problem does not arise from lack of working theories in this field, but arises –till now– from the absence of any direct experiment or observation to confirm any theory in this field. Loop quantum gravity is considered to be one of the two major candidates for a theory of quantum gravity. The most appealing aspect about loop quantum gravity is that it predicts that space is not infinitely divisible, but that it has a granular structure [1]. This paper illustrates a missed proof, which validates the previous prediction of the theory. It does not prove or validate the model of the quantized geometry of the spacetime which is predicted by the theory, but instead, it proves the necessity of space discreteness, and the existence of space quanta, due to a simple fact which is the existence of the origin position in a coordinates system, but before considering this fact, it is essentially important to consider the concept of quantization.

The Quantization Concept

Quantized quantities have a discrete spectrum and only certain values are allowed, take for example the energy of a one - dimensional quantum harmonic oscillator which is a quantized quantity, from Eq. 1:

$$E_n = \left(n + \frac{1}{2}\right) hv \text{ where } n = 0, 1, 2, 3, \dots \dots (1)$$

the spectrum is:

$$\frac{1}{2}hv, 1\frac{1}{2}hv, 2\frac{1}{2}hv, 3\frac{1}{2}hv, \dots \dots$$

where E is the oscillator's energy, h is Planck's constant and v is the oscillator's frequency.

Absence of any possible value in the spectrum between (½ hv) and (1½ hv) results in the existence of a ***gap*** in the spectrum between them, because values like (hv) or (1¼ hv) are not allowed, and therefore absent from the spectrum.

Also, the absence of any possible value in the spectrum between (1½ hv) and (2½ hv) results

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in the existence of a gap between them, because values like ($2h\nu$) or ($2¼\ h\nu$) are absent, and therefore are not allowed in the spectrum. The situation is the same between any other two successive values in the spectrum. Therefore, there will be a gap between any two successive values in the spectrum. As a result of this, the spectrum will be discrete.

The quantization of any physical variables arises from the boundary condition used in solving the equation of motion of the system, so any system has its own quantization, but a simple concept which is concluded from the example above is that gaps in the spectrum of a physical quantity in a specific physical system –as in the example above– result from the ***absence*** of any possible value between the successive values in the spectrum of that system, and the existence of these gaps makes the spectrum discrete and not continuous, which is the characteristic feature of quantized quantities regarding the system which is considered. This represents the quantization concept.

Existence of the Origin Position in a Coordinates System

The physical spacetime represents a geometrical model or a "system" that combines space and time into a single continuum. Mathematically, it is a manifold consisting of "events" which are described by some type of coordinates system. Typically, three spatial dimensions (length, width, height) and one temporal dimension (time) are required. The poof in this paper can only prove the discreteness of space without proving the discreteness of time. For this reason, the spacetime will be treated at specific moment for an observer located in a frame of reference, because at this situation, spacetime intervals can only refer to the spatial measurements, which represent what is required here. For illustration, if event one had occurred at (x_1, y_1, $z_{1,1}$), and event two had occurred at (x_2, y_2, z_2, t_1), for an observer located in the same frame of reference, the spacetime interval (Δs) that separates event one from event two will be:

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

which is only a spatial interval, since $\Delta t = 0$.

Initially, by considering one spatial dimension for simplicity, the space will be a line, therefore positions in this line are represented with one axis (x) and position spectrum is illustrated on the axis by labeling the axis with position units, as shown in Fig. 1.

Now, let us consider the following facts about this spectrum:

1-From Fig. 1, the positive part of the spectrum represents a continuum of positions directed in the positive direction with respect to the origin, and the negative part of the spectrum represents a continuum of positions directed in the opposite direction with respect to the origin.

2-The existence of the origin position in the spectrum which represents a position that is located outside the positive and the negative parts of the spectrum. Therefore, it is a neutral position (null vector).

In the spectrum above, the number of possible values (positions) between the positive part and the negative part of the spectrum is one, which is the origin position itself, but the number of possible values (positions) between the positive part of the spectrum and the origin is zero.

3- By recalling the quantization concept and by looking at the spectrum in Fig. 1, the ***absence*** of any possible value (position) between the origin and the positive part of the spectrum results in the existence of a gap between them. This happens because there is no any possible value allowed in the spectrum between them.

4- The gap that exists between the origin and the positive part of the spectrum will represent a gap between the origin and a first positive position, since the positive part of the spectrum is merely a continuum of positive positions. Therefore, the gap between the origin and the positive part of the spectrum represents a gap between the origin and a positive position. Call this position (x_1).

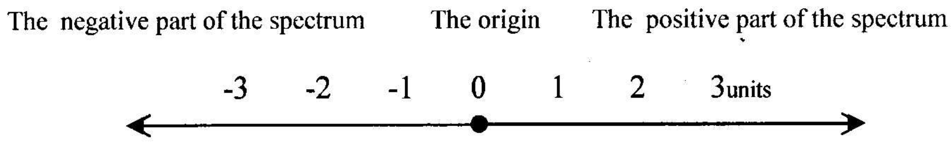

FIG. 1. Position spectrum is illustrated on the (x) axis.

Therefore, there will be a gap between the origin and the ***following*** position in the spectrum, since the origin position is a relative position and not an absolute one (Fig. 2). Position (x_1) can also be considered as an origin position, then there will be a gap between (x_1) and the following position in the spectrum. Call this position (x_2). Position (x_2) can also be considered as an origin position, since the origin is relative, therefore there will be a gap between (x_2) and the following position in the spectrum. Call this position (x_3), … and so on.

Therefore, the position spectrum in the positive direction will take the form:

$0, x_1, x_2, x_3, x_4 \ldots\ldots\ldots$

which is discrete and not continuous, and the number of possible positions in the axis interval (Δx) is ***limited*** and not infinite, but what does this mean?

Before answering the question above, it is important to note that the argument above cannot answer whether the successive positions are equally spaced or not, but it only shows that they are spaced.

FIG.2. The origin is a relative position. Take this example for illustration. For an observer located in position (1), the value of position (2), for example is 4 units to the right (+4 units), and for another observer located in position (2), the value of position (1) is 4 units to the left (−4 units). Both observers are right, since each one uses his own position as origin.

5- The "classical" definition of a line is that it represents a one - dimensional continuum of infinite numbers of points. This definition makes any given value of a line infinitely divisible. By considering this definition, since every point in the line refers to −or represents− a position in the space, the existence of an infinite number of points between two positions, take for example the interval (Δx) which is bounded by position A and position B as shown in Fig. 3.

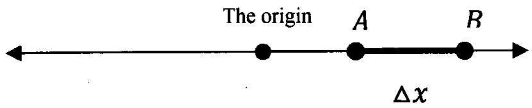

FIG. 3. (Δx) interval bounded by position (A) and position (B).

The existence of an infinite number of points between A and B results in the existence of an infinite number of positions between them, or in the interval (Δx), since every point refers to a position. This makes a contradiction with the conclusion in paragraph (4). By redefining a line as a continuum of quanta, one - dimensional quanta, instead of points, the spatial quantum represents an elementary value of length, hence it is not divisible, and therefore the observation of space below the quantum level or length value is not possible, because it results in the divisibility of the quantum itself. As a result of this, the quantum can only contain −or refer to− one possible position in space, which is at the level of the quantum itself. Therefore, the first quantum in the positive direction will refer to position (x_1), and the second quantum will refer to position (x_2), … and so on. Since the quantum has a non-zero value of length, the number of quanta between position A and position B in (Δx) interval is limited and not infinite. Therefore, the number of positions between A and B is also limited, and this is consistent with the conclusion in paragraph (4).

From the previous discussion, the existence of the origin position illustrates a discreteness in space structure, but it does not illustrate a specific or certain shape of microscopic geometry at the quantization length scale. This will represent a problem when trying to extend the previous conclusion in (4) to include two and three spatial dimensions. The problem is solved by using a large length scale relative to the scale of quantization, because at this large scale the microscopic quantized geometry is reduced to the classical macroscopic geometry as an approximation (just as the classical mechanics is used as an approximation of quantum mechanics at the macroscopic length scale).

Therefore, by choosing the large macroscopic length scale, the classical Cartesian coordinates system is used as an approximation, ***but*** it is important to bear in mind that the axes (x), (y) and (z) are quantized and not continuous, since they contain an origin position.

By considering areas, an additional spatial dimension (y) is required, and it is quantized just like (x), since it contains an origin position. The "classical" definition of area is that it represents a two - dimensional continuum of an infinite number of points, and this definition makes any given value of area infinitely divisible. This definition will make a contradiction with the conclusion in paragraph (4) as will be illustrated below.

Now, take a circle, for example, from the classical definition of area, it represents a continuum of an infinite number of points spreading in two dimensions. Since every point in the circle refers to –or represents– a position in space, the existence of an infinite number of points in the circle results in the existence of an infinite number of positions inside it. This results in the existence of an infinite number of positions in (Δx) and (Δy) intervals which bound this circle as illustrated in Fig. 4-A below.

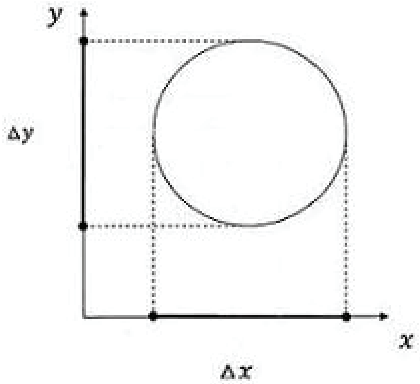

FIG. 4-A. A circle which is bounded by (Δx) and (Δy) intervals.

This happens because every position in the circle refers to a position in (x) and (y) axes, for example, position or point number (1) in the circle refers to position (x_1, y_1) in the axes, position or point number (2) in the circle refers to position (x_2, y_2), and point or position number (3) will refer to position (x_3, y_3) in the circle … and so on. Since the number of points inside the circle is infinite, this results in an infinite number of positions in (Δy) and (Δx) intervals, as illustrated in Fig. 4-B below.

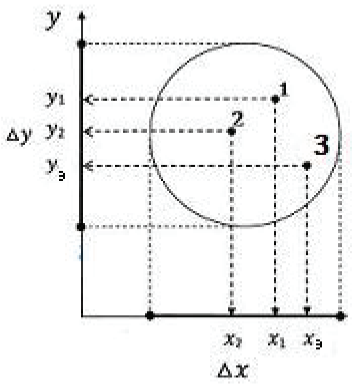

FIG.4-B.

The previous result makes a contradiction with the conclusion in paragraph (4), because the number of positions in (Δx) and (Δy) intervals will be infinite, and not limited as shown from the conclusion in paragraph (4).

By redefining area as a continuum of quanta instead of points, the spatial quantum represents an elementary value of area, therefore it is not divisible, and hence it cannot refer to more than one position in space, just as the case with the previous length quantum. Therefore, every quantum will refer to a single position inside the circle. Since the quantum has a non-zero value of area, the number of quanta, and therefore positions inside the circle, will be limited. Now, since the Cartesian coordinates are used as an approximation, every position in the circle is approximated to a position in (x) and (y) axes. Position number (1) in the circle is approximated to position (x_1, y_1) in the axes, position number (2) in the circle is approximated to position (x_2, y_2) … and so on, just like the idea from Fig. 4-B, because the number of positions inside the circle is limited. This results in a limited number of positions in (Δx) and (Δy) intervals which bound the circle's area, which is consistent with the conclusion in paragraph (4).

By considering volume, the same concept used in dealing with area holds here, but with an additional dimension (z) because volume is a three - dimensional quantity. This will lead to redefining volume as a continuum of three - dimensional quanta, instead of points, as illustrated below.

Take the cube in Fig. 5 as example. Classically, the volume inside the cube is defined as a continuum of an infinite number of points spreading in three dimensions. This definition makes any given value of volume infinitely divisible. The number of positions inside the cube will be infinite, since every point refers to a position, and the number of points is infinite. This results in an infinite number of positions in (Δx), (Δy) and (Δz) intervals, which bound the cube (the cube's edges), since every position in the cube refers to a position (x, y, z) in the axes. For illustration, just as the case with the circle in Fig. (4-B), but here with an additional dimension (z), point number (1) in the cube will refer to positions (x_1, y_1, z_1) in the axes intervals, and point number (2) will refer to positions (x_2, y_2, z_2) in the axes intervals, … and so on. Therefore, the existence of an infinite number of points inside the cube results in the existence of an infinite number of positions in (Δx), (Δy) and (Δz) intervals (the cube's edges). This makes a contradiction with the previous conclusion about quantized axes, with a limited number of positions inside the intervals (Δx), (Δy) and (Δz).

This contradiction is solved by redefining the volume as a continuum of quanta instead of points. The quantum has a volume of a non-zero value, and since it is elementary and not divisible, it will refer to a single position in space, just as the case dealing with area. Therefore, the number of the quanta inside the cube, and hence positions, will be limited. This results in a limited number of positions in (Δx), (Δy) and (Δz) intervals, since every position in the cube is approximated to a position in (x), (y) and (z) axes, in agreement with the conclusion in paragraph (4).

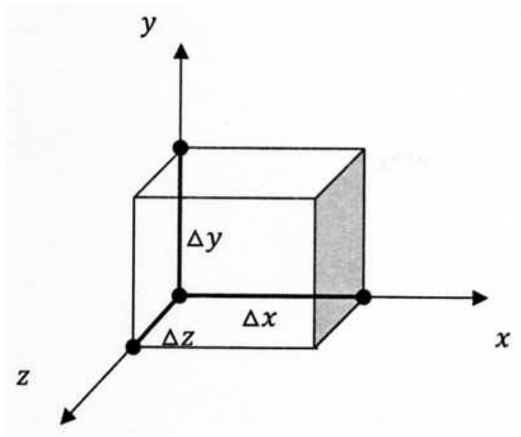

FIG.5.

Reference

[1] Rovelli, C., "*Quantum Gravity*", (Cambridge University Press, 2004).