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## ARTICLE

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### On the Gravitational Properties of Dark Matter

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**Abstract:** Analyzing ten known galactic clusters, we were able to identify a serious mass discrepancy when comparing masses calculated from the velocity dispersions of galaxies through virial method and the masses estimated through gravitational lensing. The masses obtained through virial theorem are more than twice those obtained from weak gravitational lensing observations. This might indicate that the equivalence of inertial and gravitational mass is violated in the case of dark matter.

**Keywords:** Dark matter; Gravitational mass; Inertial mass; Virial theorem.

## Introduction

Dark matter (DM) is a concept that was invented in order to explain the mass discrepancy which was calculated in studying the rotation of galaxies and their motion in clusters. The mass needed to explain the motion of stars in galaxies and the galaxies in galactic clusters is much larger than the observed mass [1]. The difference between the calculated and the observed mass was called *Dark Matter*, and it is named as such since it does not emit any form of known radiation.

Physicists can measure the mass of dark matter indirectly by observing its gravitational effects in a variety of ways: rotation of galaxies and their motions within the galactic clusters [2] and [3], gravitational lensing [4], and the Cosmic Microwave Background (CMB) analysis [5]. All these observations have confirmed the need for dark matter as an extra component for the total mass of the universe.

The nature of dark matter is far from being explained let alone be carefully understood. For example, some observations indicate that DM is collisionless [6, 7]. On the other hand, the application of known laws of gravitational physics, using the assumption of the equivalence of the inertial mass and the gravitational mass,

requires DM to get concentrated in centers of galaxies and galactic clusters, not in the halos as is usually indicated by observations, which is counter intuitive. Moreover, being non-emissive of any sort of radiation, DM must be either a dynamical effect or a new sort of mass that is composed of a weird sort of constituents.

Several proposals were put forward in order to explain DM. Among them was the dynamical effect proposal of Milogrim [8] to explain the rotation curves of the galaxies by assuming that Newton's second law of motion should be modified for systems experiencing extremely low accelerations. Other proposals include the expectations to find some massive objects in the galactic halos called MACHO, but particle physicists suggested that DM might be some sorts of exotic particles like Weakly-Interacting Massive Particles (WIMPS) or the supersymmetric components generated by the development of the standard model.

The other problem we face with DM is the estimations of the mass obtained through different observations and analyses. Here, we have different sorts of discrepancies in these estimations, some are attributed to inaccuracies in the measurement techniques and some others

are attributed to the physical conditions of the systems under measurement. Whereas the CMB analysis suggests that the DM component of the universe is about 6 times the baryonic mass [5], the analysis of mass estimates by gravitational lensing of several clusters indicates that the total mass of these clusters is about 2.4 times the baryonic mass [9]. This has motivated us to calculate the inertial masses of some galactic clusters and compare them with masses obtained from gravitational lensing. The inertial masses are obtained from the observations of the velocity dispersions in clusters using virial method for those clusters which satisfy the necessary stability condition required by the virial method. Our results show that the mass calculated via the virial method is more than two times larger than that obtained through estimates based on weak gravitational lensing observations for the same object. This will open prospects to discuss the nature of DM, and may answer several other questions; for example: Is DM a real mass or an effective mass?

We will start by presenting the basic formulation for the mass function derived from the virial theorem and then calculate the virial mass of some clusters and compare the results with the estimated mass from gravitational lensing. Then, we discuss the problem of DM distribution in galaxies and comment on the implications of having the gravitational potential of the DM being different from that of ordinary baryonic matter. Here, we try to give an explanation for the results obtained by Bidin and his collaborators [10] which suggest a polate distribution for DM in galaxies. Finally, we discuss the implications and possibilities of having DM enjoying different gravitational properties.

## The Virial Theorem

The first application of the virial theorem for the determination of the mass of a cluster was conducted by Zwicky who applied the method to the Coma cluster and predicted the existence of DM [3]. We can use the virial theorem to estimate the total mass of an object such as a galaxy or a cluster of galaxies from the movement of its individual members. Suppose you have a finite collection of point particles interacting gravitationally via classical mechanics, and suppose that:

1) The time averages of the total kinetic energy and the total potential energy are well defined.

2) The positions and velocities of the particles are bounded for all time.

Then, the average gravitational potential energy of the constituents is twice their average kinetic energy  $\langle T \rangle = -\frac{1}{2} \langle V \rangle$  [11].

The virial theorem applies to systems of stars that have reached a steady equilibrium state. It can be used for many galaxies, but can also be used for other systems such as some star clusters. The virial theorem cannot be used for clusters of galaxies that are still forming.

Now, assume that the system is stationary, then we can apply the virial theorem. Suppose that the radius vector from a fixed point in the cluster to the mass  $m_i$  is  $\vec{r}_i$ , then we have:

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = \vec{F}_i,$$

Multiplying by  $\vec{r}_i$  gives:

$$\frac{1}{2} \frac{d^2 (m_i r_i^2)}{dt^2} = \vec{r}_i \cdot \vec{F}_i + m_i \left( \frac{d \vec{r}_i}{dt} \right)^2, \quad (1)$$

Assume that the cluster -on the average- is uniformly distributed inside a sphere of radius  $R$ , and we have  $G = -2 \langle T \rangle$ ,

$\rho = \frac{3}{4} \frac{M}{\pi R^3}$ , In this case:

$$\left. \begin{aligned} \vec{G} &= \vec{r}_i \cdot \vec{F}_i = \frac{-GMm_i \vec{r}_i}{R^3} \\ &= -2 \langle T \rangle = -m_i \langle \vec{v}_i^2 \rangle \\ \langle \vec{v}_i^2 \rangle &= \frac{GM \langle r_i^2 \rangle}{R^3}, \end{aligned} \right\} \quad (2)$$

where  $\vec{v}_i$  is the velocity vector. To find the average of  $r^2$ , we use the standard statistical procedure:

$$\left. \begin{aligned} \langle r^2 \rangle &= \frac{\int r^4 dr d\Omega}{\int r^2 dr d\Omega} = \frac{3}{4\pi R^3} \int_0^R r^4 dr \\ &= \frac{3}{5} R^2, \end{aligned} \right\} \quad (3)$$

Then, the total average mass of the cluster can be written as:

$$M = \langle v_i^2 \rangle R^3 \frac{1}{G \langle r_i^2 \rangle},$$

This means that:

$$M = \frac{5}{3} \frac{R}{G} \langle v_i^2 \rangle. \quad (4)$$

Clearly, the total mass of the cluster depends on the radius of the cluster and on the average velocity. This velocity is a measurable value which can be determined from red-shift phenomena, because only the velocity components ( $v_s$ ) along the line sight from the observer are known from the observed spectra. Then, we have for a velocity distribution of spherical symmetry:

$$\langle\langle v^2 \rangle\rangle = 3 \langle\langle v_s^2 \rangle\rangle, \quad (5)$$

where double brackets mean double average with respect to time and mass.

The velocity dispersion comes from the fact that the rotation is random, with as many galaxies orbiting in one direction as others orbiting in another (approximately equal numbers of galaxies orbiting in all directions). The line of sight velocity distribution of the galaxies in cluster (the broadening function) is Gaussian.

Average velocity is the speed of the center of mass. The standard deviation from the mean is called (velocity dispersion), then:

$$\langle\langle v^2 \rangle\rangle = 3 \langle\langle v_s^2 \rangle\rangle = 3\sigma^2,$$

where  $\sigma$  is the velocity dispersion for isotropic motion (nothing special about the directions), then we have:

$$M = 5 \frac{R}{G} \sigma^2. \quad (6)$$

This is the basic formula that will be used to calculate the inertial mass, because the measurable value is the velocity dispersion (which is a kinematical property), and since

astrophysical objects move with very low accelerations, then we can take the mass obtained by the virial theorem as representing the inertial mass.

## Comparison of Mass Estimates from Gravitational Lensing and Virial Theorem

There are two known methods for calculating masses of galaxies in clusters. These are:

- 1) The mass estimated via weak lensing. The experimental methods to calculate the mass are discussed in several papers and reviews (see [12-15]). Here, we have enough data gathered from observations, and therefore can be taken to be a reliable reference.
- 2) The mass calculated according to the virial theorem, the formula for this mass is shown in Eq. (6).

The mass obtained from lensing is taken to represent the gravitational mass, but the mass obtained from the virial theorem is considered to be representing the inertial mass. Accordingly, the comparison between these two masses is expected to show whether dark matter has different gravitational properties or not. A compilation of the observational data for masses via weak lensing, velocity dispersion and the radius of each cluster under study is done, then we calculate the virial mass from Eq. (6). Table (1) shows the results of these calculations and shows the difference between masses calculated from gravitational lensing and the virial method. The data have been selected in light of the agreement of clusters radii, the radius of equilibrium which is necessary for the application of the virial theorem, and the radius of the measurement of weak lensing. The observational errors have been marked in the table wherever was available in the original sources.

TABLE 1.

Cluster	$z_d$	$R$ (Mpc)	$\sigma(km/s)$	$M_L(10^{14} M_\odot)$	$M_V(10^{14} M_\odot)$	$\frac{M_V}{M_L}$
A1689[15]	0.17	3	1989	89	138	1.55
A2163[16]	0.201	0.9	1680	$13 \pm 7$	29.54	$2.27^{+2.6}_{-0.77}$
A2218[17]	0.175	0.8	$1370^{+160}_{-210}$	$7.8 \pm 1.4$	$17.46^{+4.3}_{-4.9}$	$2.24^{+0.6}_{-0.6}$
A209[18]	0.206	0.39	$898^{+92}_{-102}$	1.25	$3.66^{+0.78}_{-0.79}$	$2.93^{+0.6}_{-0.6}$
C10024[19]	0.391	3	1300	40	58.96	1.5
MS1224[20]	0.327	0.96	802	7	7.18	1.03
MS1008[21]	0.306	0.741	$1024 \pm 110$	3.72	$9^{+1.88}_{-1.80}$	$2.42^{+0.5}_{-0.5}$
MS1455[22]	0.2568	1.617	$964^{+87}_{-95}$	$4.521^{+3.085}_{-2.507}$	$17.47^{+3.3}_{-3.3}$	$3.865^{+2}_{+0.13}$
RXJ1347[21]	0.451	1.408	$1400 \pm 130$	23.94	$32.09^{+6.2}_{-5.7}$	$1.34^{+0.26}_{-0.24}$
1ES065[23]	0.269	0.25	$1400 \pm 100$	$2.8 \pm 0.2$	$5.7^{+0.84}_{-0.79}$	$2.04^{+0.03}_{-0.3}$

In the above table,  $R$  is the radius of the cluster,  $z_d$  is the red shift,  $\sigma$  is the velocity dispersion,  $M_L$  is the lensing mass (obtained from weak lensing),  $M_V$  is the virial mass, and last column is the ratio of virial mass to lensing mass.

Clearly, all virial (inertial) masses are larger than lensing (gravitational) masses. The last column has the mean:

$$\mu = \frac{\sum \xi_i}{n} = 2.12, \quad (7)$$

where

$$\xi_i = \left( \frac{M_V}{M_L} \right),$$

and the standard deviation is

TABLE (2)

Cluster	$M_L(10^{14} M_\odot)$	$M_V(10^{14} M_\odot)$	$\frac{M_V - M_L}{M_L}$
A1689[16]	89	138	0.55
A2163[17]	$13 \pm 7$	29.54	$1.27^{+2}_{-0.8}$
A2218[18]	$7.8 \pm 1.4$	$17.46^{+4.3}_{-4.9}$	$1.238^{+0.5}_{-0.3}$
A209[19]	1.25	$3.66^{+0.78}_{-0.79}$	1.928
C10024[20]	40	58.96	0.474
MS1224[21]	7	7.18	0.0257
MS1008[22]	3.72	$9^{+1.88}_{-1.80}$	1.42
MS1455[23]	$4.521^{+3.085}_{-2.507}$	$17.47^{+3.3}_{-3.3}$	$2.866^{+4.2}_{-1.2}$
RXJ1347[22]	23.94	$32.09^{+6.2}_{-5.7}$	0.34
1ES065[24]	$2.8 \pm 0.2$	$5.7^{+0.84}_{-0.79}$	$1.036^{+0.15}_{-0.14}$

$$s = \sqrt{\frac{\sum (\xi_i - \mu)^2}{n-1}} = 0.84. \quad (8)$$

We can see that the ratios of masses have a mean value in the middle of mass ratio values, with small standard deviation. We have five values larger than the mean and four values smaller than it, and 0.8 of data is situated on one standard deviation ( $1s$ ) from the mean, and *all values* are situated on two standard deviations ( $2s$ ) from the mean.

These results show that the inertial contribution of the DM is higher than gravitational effects, the inertial mass is about twice the gravitational mass. This interesting result may indicate that DM has gravitational properties which are different from those of ordinary matter. Table (2) shows the ratio of difference between the two masses to lensing mass.

The mean of these values is

$$\left\langle \frac{\Delta M}{M} \right\rangle = 1.12, \quad (9)$$

and the standard deviation is

$$s = 0.84. \quad (10)$$

The difference between the two masses is partially attributed to the gravitational properties of the DM. According to this understanding, this result reflects one important aspect of the DM behavior.

## The Galactic Disc and DM Discrepancy

The first indication for the possible presence of dark matter came from the dynamical study of our Galaxy. In 1922, the British astronomer James Jeans [25] re-analyzed vertical motions of stars near the plane of the Galaxy that were studied earlier by the Dutch astronomer Jacobus Kapteyn [26]. Both astronomers calculated from these data the density of matter near the Sun. They also estimated the density due to all stars near the galactic plane. Kapteyn found that the spatial density of known stars is sufficient to explain the vertical motions. In contrast, Jeans indicated the presence of two *dark stars* to each bright star.

More modern attempts were carried out in the 1980s by Bahcall and by Kuijken and Gilmore [27] which conclude that there is no evidence for a significant amount of DM in the disk. There has been considerable debate about the interpretation of the results. Early studies claimed evidence of dark matter in the Galactic disc, but more recently some consensus has developed that there is little DM in the disc itself [10]. This indicates that DM distribution does not follow baryonic matter distribution closely on a small scale. A very recent analysis by Moni Bidin et al. [10] suggests that the DM distribution takes the shape of a polate halo. This will be analyzed below and we will try to give our interpretation to these results in the light of our findings in this paper. Earlier, Shaw et al. [28] have shown that cluster halos have polate morphology becoming more so with increasing mass.

## The Surface Mass Density of Galactic Disc

The Jeans Equations can be applied to our Galaxy to measure the surface mass density of the galactic disc at the solar distance from the center using observations of the velocities of stars along the line of sight lying some distance above or below the galactic plane. This analysis is important because it allows the quantity of DM in the disc to be estimated. Determining whether there is DM in the galactic disc or not is a very important constraint on its nature.

The second Jeans equation in a cylindrical coordinate system centered on the Galaxy, with  $z=0$  in the plane, and  $R=0$  at the galactic center for the  $z$ -direction, is given by [30]:

$$\left. \begin{aligned} & \frac{\partial}{\partial t}(n\langle v_r \rangle) + \frac{\partial}{\partial r}(n\langle v_r^2 \rangle) \\ & + \frac{\partial}{\partial z}(n\langle v_r v_z \rangle) + \frac{n}{r}(\langle v_r^2 \rangle - \langle v_\phi^2 \rangle) \\ & = -n \frac{\partial \Phi}{\partial r}, \end{aligned} \right\}$$

where  $n$  is the star number density,  $v_r$  and  $v_z$  are the velocity components in the  $r$  and  $z$ -directions,  $\Phi(R, z, t)$  is the galactic gravitational potential, and  $t$  is time. The galaxy is in a steady state, so  $n$  does not change with time. Therefore, the first term is  $\frac{\partial}{\partial t}(n\langle v_z \rangle) = 0$ .

As it is to be expected, observations show that [27]:

$$\frac{\partial}{\partial r}(n\langle v_r v_z \rangle) = \frac{n}{r}\langle v_r v_z \rangle \approx 0, \quad (11)$$

this is because of the cancelling of positive and negative terms of the  $z$ -components of the velocity. Therefore,

$$\frac{\partial}{\partial z}(n\langle v_z^2 \rangle) = -n \frac{\partial \Phi}{\partial z}, \quad (12)$$

where  $\langle v_z^2 \rangle$  is the mean square velocity in the direction perpendicular to the Galactic plane. The Poisson equation in cylindrical coordinates for this system is given by:

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G\rho, \quad (13)$$

(If we observe stars directly above and below the galactic plane, all at the same radius  $r$ , we can neglect the  $\frac{\partial \Phi}{\partial r}$  and  $\frac{\partial^2 \Phi}{\partial \phi^2}$  terms).

Then, substituting in Eq. (12), we get:

$$\frac{\partial}{\partial z} \left[ \frac{-1}{n} \frac{\partial}{\partial z} (n \langle v_z^2 \rangle) \right] = 4\pi G \rho.$$

Integrating on the axis perpendicular to the galactic plane from  $-z$  to  $z$ , the surface mass density within a distance  $z$  of the plane at a galactocentric radius  $r$  is:

$$\left. \begin{aligned} \Sigma(r, z) &= \int_{-z}^z \rho dz \\ &= \frac{1}{4\pi G} \int_{-z}^z \frac{\partial}{\partial z} \left( \frac{-1}{n} \frac{\partial}{\partial z} (n \langle v_z^2 \rangle) \right) dz \\ &= \frac{-1}{2\pi G n} \frac{\partial}{\partial z} (n \langle v_z^2 \rangle) \Big|_z, \end{aligned} \right\} \quad (14)$$

where we are assuming symmetry about  $z=0$ . Therefore, the surface mass density within a distance  $z$  of the plane at the solar galactocentric radius  $R$  is:

$$\frac{\partial \Phi}{\partial z} = \Sigma(R, z) = \frac{-1}{2\pi G n} \frac{\partial}{\partial z} (n \langle v_z^2 \rangle) \Big|_z. \quad (15)$$

If the star number density  $n$  can be measured as a function of height  $z$  from the plane, and if the  $z$ -component of the velocities  $v_z$  can be measured as spectroscopic radial velocities, then we can solve for  $\Sigma(R, z)$  as a function of  $z$ . This gives, after modeling the contribution from the dark matter halo, the mass density of the galactic disc. The recent work by Moni Bidin and collaborators [10] shows that there is little dark matter in the Milky Way disk. This was interpreted to mean that the dark matter forms a highly polate halo for the galaxy. But this interpretation is not consistent with the gravitational behavior of the massive halo if the gravitational properties of DM are the same as those of baryonic matter.

### The Gravity of Dark Matter

The previous interpretation assumes that DM behaves gravitationally like ordinary matter, and has explained the discrepancy by assuming that the DM distribution is very extended in the  $r$ -direction, not in the  $z$ -direction. This interpretation is based on the common idea that the distribution of mass in a cluster (including

both the visible and dark matter) determines the gravitational potential.

Normally, the potential is a function of space and time  $\Phi(\vec{r}, t)$ , where  $\vec{r}$  is the position vector of a point at time  $t$ . If the cluster has reached a steady-state, then  $\Phi = \Phi(\vec{r})$  only. Accordingly, the potential at any point is related to the local density by Poisson's equation,  $\nabla^2 \Phi = 4\pi G \rho$ . The gravitational potential can be taken at any point as the sum of the potential of the dark and visible matter,  $\Phi = \Phi_{DM} + \Phi_{VS}$ . It is easy to see that the dynamical density can therefore be expressed as the sum of two contribution,  $\rho = \rho_{DM} + \rho_{VS}$ . Splitting the gravitational potential in this way safeguards the possibility that the gravitational coupling between the dark and ordinary baryonic matter is not the same as that of the standard gravitational coupling of matter. Basically, there is no strong reason for assuming that  $\Phi_{DM}$  has same gravitational effects as those of  $\Phi_{VS}$ . However, the test of this assumption will follow through the subsequent analysis of the kinematics of the systems under consideration.

The gravitational field-Observationally-depends on velocity component in the same direction of field, namely Eq. (30). So, if  $\Phi = \Phi_{DM} + \Phi_{VS}$ , then we have:

$$\left. \begin{aligned} -\frac{\partial}{\partial z} (\Phi_{DM} + \Phi_{VS}) \Big|_z \\ = \Sigma(R, z) \\ = \frac{1}{2\pi G n} \frac{\partial}{\partial z} (n \langle v_z^2 \rangle) \Big|_z \end{aligned} \right\}. \quad (16)$$

Now, according to the separation assumption, there are two possibilities:

1. If  $\Phi_{VS}$  is effectively dominant, then we can assume that the potential of visible matter is driving the stars to move with velocities that we can measure, and then the surface mass density can be calculated from this motion. On the other hand, having DM with weaker gravitational coupling than ordinary matter will result in resisting the motion since the inertia is large.
2. If the two gravitational coupling constants are different, then the total gravitational potential is inhomogeneous. This leads to distortion in the distribution of matter, and we expect that

the matter will not be distributed spherically. This second possibility may explain the Bidin result and is in agreement with his interpretation about the DM distribution. Therefore, we consider the results obtained by Bidin as an observational evidence for the claimed difference between the gravitational coupling of DM and ordinary matter.

## Discussion and Conclusions

The original assumption of DM first appeared when the motion of galaxies within the Coma cluster was observed to be inconsistent with the estimated mass of the cluster obtained from direct observations. Motion of galaxies within the cluster suggested that more mass existed within it and the missing mass was called dark matter. On the other hand, galactic rotation curves show that the outer parts of the galaxies are moving faster than expected by Kepler's law. This again suggested that galaxies contain more mass than observed and that the hidden mass is distributed within the galactic halo. Besides, the investigations of the CMB suggested that the average matter density in the universe is larger than the observed matter density by about 5-6 times. All these three evidences assured cosmologists and astrophysicists that DM exists. However, no one could figure out much of the properties of DM. According to the observations from the rotation curves of the galaxies, DM is thought to reside in the galactic halo, which is counter-intuitive by all means since the large amount of mass should reside at the center of the galaxy and not in the galactic halo.

In this paper, we have investigated the gravitational and the inertial properties of the DM by comparing the masses of several clusters calculated via the virial theorem with the masses

calculated from observations obtained by weak gravitational lensing. We infer that the mass obtained from the virial calculations is the inertial mass and the mass obtained from gravitational lensing is the gravitational mass. Our study shows that the average inertial mass of the clusters is about 2.4 times the average gravitational mass. This suggests that the gravitational coupling of DM is weaker than that of the ordinary baryonic matter.

We believe that this result of our study provides some basis to explain the recent results published on the distribution of DM in the galaxy [10] which show that the distribution of DM is not spherically symmetric and that it takes a polate shape. This was interpreted according to the results of our work here as being due to the two-component potential and the dominance of DM in the galaxy. If DM has a weaker gravitational coupling than ordinary matter then this will cause some asymmetric distribution of matter within the galaxy. However, this is only a qualitative suggestion, the full fledged calculations of the morphology of the galaxy need more accurate specification of the coupling strength and the amounts of both components. This question is beyond the scope of this work.

There remain several other questions on this topic. The one is: how would a gravitationally weak DM contribute to the expansion of the universe? The CMB observations suggest a flat universe with critical density on the other hand observations of the super nova type Ia suggested that the universe is accelerating [31]. The question then arises as: how would a gravitationally weakly coupled DM affect the acceleration of the universe?

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