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ARTICLE

Various Properties of Heavy Quarkonia from Flavor-Independent Coulomb Plus Quadratic Potential

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Abstract: We examine heavy quarkonia ($c\bar{c}$ and $b\bar{b}$) characteristic properties in the general framework of non-relativistic potential model consisting of a Coulomb plus quadratic potential. The potential parameters are determined by simultaneous fit of the $1S$ states of both $c\bar{c}$ and $b\bar{b}$ some known experimental values and with the help of Virial theorem, so that the potential is flavor-independent. The obtained results are compared with the corresponding results from Cornell potential and with the available experimental data. The overall agreement with the experimental data is quite good, particularly for the mass spectra.
Keywords: Heavy quarkonia; Mass spectra; Phenomenological potential, QQ-onia package.

Introduction

Since their discoveries, investigation of heavy quarkonium systems provides us with great tools for quantitative tests of quantum chromodynamics (QCD). For a detailed review of recent progress in heavy quarkonium physics, see e.g. [1, 2]. Because of the heavy masses of the constituent quarks, a good description of many features of these systems can be obtained using non-relativistic models, where one assumes that the motion of constituent quarks is non-relativistic, so that the quark-antiquark strong interaction is described by a phenomenological potential [1, 2]. There are many potential models that are commonly used to study heavy quarkonium spectra; for instance, Martin, logarithmic and Cornell potentials [1, 3, 4, 5]. Any of these potentials should take into account the two distinctive features of the strong interaction; namely, asymptotic freedom and confinement. It is known that exact analytical solutions of Schrödinger equation are only possible for certain spherical potentials, particularly when the centrifugal potential is included. Therefore, approximation methods were developed such as supersymmetry and $1/N$ expansion, and some numerical methods and packages were developed, such as the QQ-onia package that we will use for the purpose of this work [6 - 9].

There are lots of experimental and theoretical previous results in the context of potential model. Most recent experimental findings, such as the discovery of new states of $(c\bar{c})$and $(b\bar{b})$ systems and the determination of various leptonic and E_1 decay widths, motivate researchers to revisit the potential models for these systems. In this work, we will investigate various properties of the $(c\bar{c})$ and $(b\bar{b})$ systems, using the QQ-onia package described in [6]. To be specific, the following important physical properties of these systems will be extracted: the mass spectrum M, which is considered the first indication for testing the goodness of the potential model under consideration; the mean

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square radius of the state $\sqrt{\langle r^2 \rangle}$, which is important in the determination of hadronic transition widths [10]; the wavefunction at the origin $|R(0)|^2$, which is an important quantity in the evaluation of the production and decay amplitudes within the framework of effective field theory; the average heavy-quark velocity $\langle v^2 \rangle$, which is important for the estimation of relativistic corrections, and is beneficial for the non-relativisitc qunatum chromdynamics (NRQCD) formalism, as well as in the estimation of the quarkonium production rate [10], the mass level splitting $n(^3P_j)$ and $n(^1P_1)$ due to spin-orbital and spin-spin interactions; and the E_1 transition rate $2(^3S_1) \to 1(^3P_j) + \gamma$, which is important in determining the strength of the electric dipole transitions and the energy level schemes. The detailed definitions of such terms can be found in [6] and the references therein. Comparison with the available experimental findings and other theoretical calculations will be presented.

The Model

Heavy quarkonium systems can be analyzed phenomenologically within the framework of a potential model using the non-relativistic Schrödinger equation. For the purpose of our work, we will use a potential composed of Coulomb plus quardatic or oscillator potential of the form:

$$V(r) = -\frac{a}{r} + br^2, \tag{1}$$

where the potential parameters a and b are usually fixed by fitting the experimentally measured mass spectra of these systems. It has been used by many authors; see for instance [11, 12]. It is considered an applicable candidate to other phenomenological potentials if it can be used to extract the various physical properties of these systems, such as mass spectra, the square of the wavefunction at the origin $|R(0)|^2$ and the mean square radius $\sqrt{\langle r^2 \rangle}$ … etc., which are of great importance for a better understanding of QCD physics. The important part of choosing the form of the potential is that it takes into account the two distinctive features of the strong interaction; namely, asymptotic freedom and confinement. The Coulomb part of Eq. (1) represents the short distances ($r \leq 0.1$ fm) interaction, which is dominated by the one-gluon exchange, while the second quadratic part represents the confinement.

The potential parameters are determined using the $1S$ available experimental data for the $c\bar{c}$ and $b\bar{b}$ simultaneously and the Virial theorem. Using the same fit parameters for $c\bar{c}$ and $b\bar{b}$ indicates the flavor-independence of the strong interaction and is evidence for QCD and the standard model, and thus the potential model becomes universal. The expectation value of the energy is:

$$\langle E \rangle = \langle T \rangle + \langle V \rangle, \tag{2}$$

and from Virial theorem [13] we have:

$$\langle T \rangle = \frac{1}{2}\left\langle r\frac{dV}{dr}\right\rangle, \tag{3}$$

then, we obtain:

$$\langle T \rangle = \frac{1}{2}\frac{a}{\langle r \rangle} + b\langle r^2 \rangle \ . \tag{4}$$

Hence, Eq. (2) becomes:

$$\langle E \rangle = \frac{-a}{2\langle r \rangle} + 2b\langle r^2 \rangle \ . \tag{5}$$

Here, we adopted the method used in [14] for the expectation value of linear momentum $\langle p \rangle \sim \frac{1}{\langle r \rangle}$. This can also be understood from Bohr's quantization condition $pr = n\hbar = 1$

(for 1S state $n = 1$ and using natural units $\hbar \equiv 1$). Thus, we may, as an approximation, take $\langle p \rangle \langle r \rangle \sim 1$. This recipe gives the correct level spacing for Coulomb and harmonic oscillator potentials as also illustrated in pp (163-164) of [15]. By substituting the above equation in the mass equation [3]:

$$M_{q\bar{q}} = 2m_q + \langle E \rangle, \tag{6}$$

we obtain:

$$M_{q\bar{q}} = 2m_q - \frac{a}{2\langle r \rangle} + 2b\langle r^2 \rangle. \tag{7}$$

Applying the above equation to the $1S$ ground states of $(c\bar{c})$ and $(b\bar{b})$, we obtain:

$$M_{c\bar{c}}^{exp}(1S) = 2m_c - \frac{a}{2\langle r_c \rangle_{1S}} + 2b\langle r_c^2 \rangle_{1S} \tag{8}$$

and,

$$M_{b\bar{b}}^{exp}(1S) = 2m_b - \frac{a}{2\langle r_b \rangle_{1S}} + 2b\langle r_b^2 \rangle_{1S}. \tag{9}$$

The experimental values of various quantities appearing in the above two equations are tabulated in Table 1. The numerical values for a and b obtained by solving simultaneously the above two equations are, respectively, approximately 0.279 and 1.722 GeV^3. These parameters are now used for both $c\bar{c}$ and $b\bar{b}$

states, and thus the potential model with these parameters is flavor-independent, as expected from QCD [16].

TABLE 1. Values of the parameters used in our calculations [17].

Parameter	Value ($c\bar{c}$)	Value ($b\bar{b}$)
m_q	1.50 GeV	5.18 GeV
$M^{exp}(1S)$	3.096 GeV	9.460 GeV
$\langle r\rangle_{1S}$	0.232 fm	0.131 fm
$\langle r^2\rangle_{1S}$	0.2025 fm	0.0484 fm

Results and Discussion

In this work, a global Coulomb plus quadratic potential is used to calculate the spin-averaged mass spectra, the square of the wave function at the origins, $|R^l(0)|^2$ (in GeV$^{(3+2l)}$), the mean square radius, $\sqrt{\langle r^2\rangle}$, the heavy-quark velocity, $n(^3P_j)$ and $n(^1P_1)$, mass, and $E_1[2(^3S_1) \to 1(^3P_j)$ using QQ-onia package. The potential parameters were determined using the available $1S$ state experimental data for both $c\bar{c}$ and $b\bar{b}$. We list our results in Tables 2 to 7, and comparisons with the results from Cornell potential and from the available experimental data are shown in these tables.

According to the results, we can conclude that the used model can produce the various properties of $c\bar{c}$ and $b\bar{b}$ with good agreement. Some of the extracted properties from Cornell potential are in better agreement with the corresponding experimental data, while some other properties or values are obtained with better value from the used model. We believe that for better model of these systems, one should develop a model which is composed from a Coulomb plus a polynomial potential $V(r) = -\frac{a}{r} + \sum_{n=0}^{2} b_n r^n$, which will be of our future investigation.

TABLE 2. The spin-averaged mass spectrum for $c\bar{c}$ and $b\bar{b}$ (in GeV). Experimental data are taken from [17].

State	Mass $c\bar{c}$			Mass $b\bar{b}$		
nL	Cornell	This Work	Exp.	Cornell	This Work	Exp.
1S	3.095	3.096	3.096	9.460	9.640	9.460
1P	4.216	4.116	-	9.958	9.980	9.900
2S	3.662	3.662	3.686	10.027	10.027	10.023
1D	3.843	3.843	3.770	10.208	10.208	10.162
2P	3.948	3.948	3.927	10.262	10.310	10.262
3S	4.032	4.032	4.040	10.399	10.399	10.355
4S	4.309	4.309	4.263	10.679	11.40	10.579

TABLE 3. $|R^l(0)|^2$ (in GeV$^{(3+2l)}$) for $c\bar{c}$ and $b\bar{b}$.

State	$c\bar{c}$			$b\bar{b}$		
nL	Cornell	This Work	From [18]	Cornell	This Work	From [18]
1S	1.649	1.240	1.454	14.090	14.310	14.05
1P	-	-	-	2.06	-	-
2S	1.423	1.135	0.927	5.940	6.260	5.668
1D	-	-	-	0.837	-	-
2P	-	-	0.131	2.440	0	2.067
3S	1.197	0.865	0.791	4.276	0	4.271
4S	-	-	-	3.67	-	3.663

TABLE 4. The mean square radius $\sqrt{\langle r^2\rangle}$ (in fm).

State	$\sqrt{\langle r^2\rangle}$ for $c\bar{c}$			$\sqrt{\langle r^2\rangle}$ for $b\bar{b}$		
nL	Cornell	This Work	From [19]	Cornell	This Work	From [19]
1S	0.21	0.26	0.47	0.20	0.22	0.20
1P	0.36	0.51	0.74	0.38	0.36	0.39
2S	0.27	0.37	0.96	0.46	0.38	0.48
1D	0.37	0.41	1.00	0.52	0.46	0.53
2P	0.33	0.45	-	0.63	-	0.64
3S	0.32	0.50	1.30	0.71	-	0.72
4S	-	-	1.70	0.91	-	0.92

TABLE 5. The heavy-quark velocity $\langle v^2 \rangle$.

State	$\langle v^2 \rangle$ for $c\bar{c}$			$\langle v^2 \rangle$ for $b\bar{b}$		
nL	Cornell	This Work	From [19]	Cornell	This Work	From [19]
1S	0.261	0.263	0.20	0.096	0.073	0.096
1P	0.285	0.335	0.20	0.065	0.081	0.065
2S	0.271	0.104	0.24	0.078	0.091	0.076
1D	0.289	0.296	0.23	0.067	0.092	0.067
2P	0.273	0.146	-	0.075	-	0.076
3S	0.238	0.336	0.30	0.085	-	0.085
4S	-	-	0.35	0.097	-	0.097

TABLE 6. $n(^3P_j)$ and $n(^1P_1)$ Mass (in GeV). Experimental data are taken from [17].

State	$c\bar{c}$			$b\bar{b}$		
nL	Cornell	This Work	Exp.	Cornell	This Work	Exp.
1^3P_0	3.614	3.540	3.484	9.867	9.871	9.860
1^3P_1	3.491	3.537	3.508	9.893	9.895	9.893
1^1P_1	3.250	3.485	3.533	9.900	9.900	9.948
1^3P_2	3.533	3.533	3.557	9.911	9.908	9.913
2^3P_0	4.007	3.938	3.880	10.236	10.262	10.232
2^3P_1	3.872	3.916	3.900	10.256	10.262	10.255
2^1P_1	3.614	3.828	3.919	10.262	10.262	10.338
2^3P_2	3.919	3.919	3.939	10.271	10.262	10.269

TABLE 7. $E_1[2(^3S_1) \rightarrow 1(^3P_j)]$ (in keV). Experimental data are taken from [17].

Final	Width for $c\bar{c}$				Width for $b\bar{b}$			
J	E_γ	Cornell	This Work	Exp.	E_γ	Cornell	This Work	Exp.
0	162.48	0.32	1.08	-	-	1.47	1.05	1.22
1	129.63	0.49	1.64	-	-	2.25	1.61	2.21
2	110.44	0.51	1.69	-	-	2.23	1.66	2.29

Conclusion

In this work, various properties of heavy quarkonia ($c\bar{c}$ and $b\bar{b}$) were calculated using a proposed flavor-independent Coulomb plus quadratic potential; i.e., with the same potential parameters. The potential parameters were approximated by simultaneous fit of the masses of the 1S state of $c\bar{c}$ and $b\bar{b}$ using the available experimental data of M, $\langle r \rangle$ and $\langle r^2 \rangle$. The numerical solution of Schrödinger equation to calculate the various properties with the obtained potential model was carried out using the QQ-onia package described in [6]. The results were compared with the corresponding values for the case of Cornell potential and with the available experimental data. From the obtained results, we may infer that this model can predict the mass spectra, and the results are close to the results from Cornell potential. However, the other properties vary and agree with some and disagree with some others. We think that we must extend our phenomological model as a combined model from Cornell and quadratic potential for better generation of the data. In this case, more potential parameters will be included and thus more experimental data will be used. This will be of our future investigation.

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