Volume 9, Number 2, 2016. pp. 95-102

Jordan Journal of Physics

ARTICLE

The Change in the Properties of ^{130}Xe - ^{130}Nd Isobar States

Imad Mamdouh Ahmed

Department of Physics, College of Education for Pure Sciences, University of Mosul, Mosul, Iraq.

Received on: 3/3/2016; Accepted on: 20/6/2016

Abstract: The properties of the ground and negative parity state bands of ^{130}Xe - ^{130}Nd isobars have been studied. The ratio E_γ / I has been calculated as a function of the spin (I) to determine the ground-state evolution. The ratio between the energies of the (I+2) and (I) states as a function of the spin (I) has been drawn to determine the property of the ground-state band. The odd-even staggering has been drawn to determine the difference of the energies of ground state band and negative parity band. The energy levels for the ground state band of ^{130}Xe - ^{130}Nd have been calculated using Bohr-Mottelson Model (BM), Interacting Boson Model (IBM-1), Interacting Vector Boson Model (IVBM) and Doma-El-Gendy (D-G) relation. The energy levels of the negative parity band have been calculated using BM and IVBM models. The calculated energy levels in comparison with the experimental data indicated the quality of the fitness presented in this work.

Keywords: Ground- state band; Negative parity band; E-GOS; The ratio between the energies of I+2 and I states; IBM-1; IVBM; D-G relation; Bohr and Mottelson.

PACS numbers: 21.60.Ev, 21.60.Fw, 21.10.Re.

Introduction

The different even-even nuclei properties vary with the number of constituent nucleons, which is associated with the corresponding changes in the nuclear excitation spectrum and in the decay properties of the excited states [1,2]. Even-even nuclei energy levels could be grouped into Ground State Band (GSB) with $I^\pi = 0^+, 2^+, 4^+, ...$ and Negative Parity Band (NPB) with $I^\pi = 1^-, 3^-, 5^-, ...$ [3]. GSB and NPB become interwoven after the first few values of angular momentum *I*, forming a single octupole band with $I^\pi = 0^+, 1^-, 2^+, 3^-, ...$ [4-7]. Primary information on the property of the nucleus could be obtained from the position of the first excited state $(E2_1^+)$ which is approximately equal 100, 300 and 500 keV and the ratio of the second excited state to the first excited state $(R = E4_1^+ / E2_1^+)$ which obeys $3 < R \leq 3.3$, $2.4 < R \leq 3$ *and* $2 \leq R \leq 2.4$ for rotational, γ-soft and vibration nuclei, respectively [8]. The relationship between the gamma energy over spin E_γ / I as a function of the spin *I* (E-GOS) has indicated good information on the evolution that occurs in the yrast line of the nuclei. This was introduced by Regan *et al.* [9]. They have studied many nuclei around A=110 regions and observed the evolution in their yrast states [9]. The ratio between the energies of *I+2* and *I* states gives a good indication of the property of the nucleus [10,11], when being drawn *versus I* and then compared with the standard values of the vibration U(5), rotational SU(3) and γ-soft O(6) nuclei. A relation of the rotational energy *E* of an axially symmetric nucleus as a function of *I(I+1)* has been introduced by Bohr and Mottelson (BM) [3]. The complicated level scheme properties were well approximated in terms of the U(6) unitary group in interacting

Corresponding Author: Imad Mamdouh Ahmed **Email**: imad_mamdouh07@yahoo.com

boson model (IBM-1) studies. The group reduction scheme of the U(6) produces three limits that terminate in the O(2) group. The three limits are: the vibrational U(5), rotational SU(3) and γ-soft O(6) [12]. Nuclei might have an intermediate structure of the U(5) - SU(3), U(5)-O(6) and SU(3) - O(6) limits which is another phenomenological study [13]. The interacting vector boson model (IVBM) is based on two kinds of vector bosons, the proton *p* and the neutron *n* bosons that constitute the collective excitations in the nucleus. The IVBM developed by Ganev *et al.* [14] is used to describe the ground and octupole bands of the nucleus. Doma and El-Gendy (D-G) [15] applied the collective model to calculate the rotational and vibrational energies of some even-even nuclei. They introduced a new equation which depends on the moment of inertia as well as on the spin of the nucleus [15].

Many studies have been concerned with nuclei of mass $A \approx 130$ region. One of these studies tested the O(6) symmetry to ^{130}Xe,^{130}Ba and observed some deviation from a pure O(6) character. So, the authors added an SU(3) perturbation to the Hamiltonian [16]. P. Von *et al.* tested the O(6)-U(5) character of nuclei with $A = 130$ region [17]. L. Bettemana *et al.* compared the O(6) and O(6)-U(5) calculations with experimental and other calculation results of ^{130}Xe, ^{130}Ba and ^{130}Ce. They found that the O(6)-U(5) calculations are more reliable [18]. Salah A. Eid and M. Diab considered the ^{130}Xe nucleus that has the E(5) symmetry, where they used IBM-1 to calculate its energy levels and compared them with the measured values [19]. Since there are many opinions on these regions, in the present work, the E-GOS, the ratio of the energy states and the staggering methods are introduced to insure the property of $^{130}Xe - {}^{130}Nd$ isobars. After indicating the property of each isobar, the energy levels of the GSB are calculated using the following models; BM, IBM-1, IVBM and D-G relation and the energy levels of NPB are also calculated using BM and IVBM models, for ^{130}Xe - ^{130}Nd isobars, then the results are compared with their experimental counterparts.

Methods of Calculation

The E-GOS method gives good information on the evolution in the excited states of several even-even nuclei when the plot of E_γ / I *versus* *I* is studied. A vibrational nucleus in the E-GOS curve drops quickly from its highest value (≈250keV) at $(I = 2_1^+)$ to its lower value (0) at (I→∞). For γ-soft nuclei, the curve drops slowly from its highest value (≈150keV) at $(I = 2_1^+)$ to the quartered first excited state $E2_1^+/4$ at (*I*→∞), but for rotational nuclei the curve increases slowly from its smallest value (≈50keV) at $(I = 2_1^+)$ to its highest value which is $4\hbar^2/2\vartheta$ at (I→∞) [9]. The relationships between gamma energy over spin $R = E_\gamma / I$ and the spin *I* for the three limit cases are given by [20,21]:

$$\text{Vibrator: } R = \frac{\hbar\omega}{I} \to 0 \text{ when } I\to\infty \; . \tag{1}$$

$$\text{Rotor: } R = \frac{\hbar^2}{2\vartheta}(4 - \frac{2}{I}) \to 4\frac{\hbar^2}{2\vartheta} \text{ when } I\to\infty \; . \tag{2}$$

$$\gamma\text{-soft: } R = \frac{E2_1^+}{4}(1 + \frac{2}{I}) \to \frac{E2_1^+}{4} \text{ when } I\to\infty . \tag{3}$$

The ratio between the experimental energy values of I+2 and I states as a function of I has been constructed to define the symmetry for the excited states of even-even nuclei [10, 11]:

$$r\left(\frac{I+2}{I}\right) = \left.\begin{matrix}\left[\left(\frac{R(I+2)}{I}\right)_{\text{exp.}} - \frac{(I+2)}{I}\right] \\ \times \frac{I(I+1)}{2(I+2)}\end{matrix}\right\} ; \tag{4}$$

where $\left(\frac{R(I+2)}{I}\right)_{\text{exp.}}$ is the experimental energy values ratio between I+2 and I states. The value of (r) has changed from 0.1 to 1.0 for yrast bands of even-even nuclei. The ratio (r) for vibrational, rotational and transitional nuclei is given by:

$0.1 \le r \le 0.35$ for vibrational nuclei.

$0.4 \le r \le 0.6$ for transitional nuclei.

$0.6 \le r \le 1.0$ for rotational nuclei.

In BM model, for small I, the energy E(I) can be expanded in power of I(I+1). The GSB and NPB levels are given by [3,6]:

$$E(I) = AI(I+1) - BI^2(I+1)^2 + CI^3(I+1)^3 ; \tag{5}$$

$$E(I) = E_0 + AI(I+1) - BI^2(I+1)^2 + CI^3(I+1)^3 ; \tag{6}$$

where E_0 is the band head energy of the NPB. The values of the coefficients A, B and C can be determined from a fitting to the available measured energy levels of the band.

The general Hamiltonian for IBM-1 is given by [22]:

$$H = \sum_{i=1}^{N} \varepsilon_i + \sum_{i<j}^{N} V_{ij} \quad ; \tag{7}$$

where ε is the intrinsic boson energy and V_{ij} is the interaction strength between bosons i and j. The multipole form the Hamiltonian is [23]:

$$\left. \begin{aligned} H &= \varepsilon \hat{n}_d + a_0 \hat{P}^+ \hat{P} + a_1 \hat{L}.\hat{L} \\ &+ a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \end{aligned} \right\} ; \tag{8}$$

where n_d is the number of d-bosons; $P\cdot P$, $L\cdot L$, $Q\cdot Q$, $T3\cdot T3$ and $T4\cdot T4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons, while a_0, a_1, a_2, a_3 and a_4 are strengths of interaction of each term, respectively. The Hamiltonian used in terms of multipole expansion tends to reduce three forms of it to meet the requirements of the three symmetry limits; the vibrational U(5), rotational SU(3) and γ-soft O(6). In the U(5) limit, the effective parameter is ε. In the O(6) limit, the dominating parameter is a_o. In the SU(3) limit, the effective parameter is a_2. The eigenvalues for these three limits are given by [24]:

U(5):

$$\left. \begin{aligned} E(\varepsilon, n_d, \nu, L) &= \varepsilon \hat{n}_d + K_1 n_d (n_d + 4) \\ &+ K_4 \nu(\nu+3) + K_5 L(L+1) \end{aligned} \right\} ; \tag{9}$$

O(6):

$$\left. \begin{aligned} E(\sigma, \tau, L) &= K_3[N(N+4) - \sigma(\sigma+4)] \\ &+ K_4 \tau(\tau+3) + K_5 L(L+1) \end{aligned} \right\} ; \tag{10}$$

SU(3):

$$\left. \begin{aligned} E(\lambda, \mu, L) &= K_2(\lambda^2 + \mu^2 + 3(\lambda+\mu) \\ &+ \lambda\mu) + K_5 L(L+1) \end{aligned} \right\} ; \tag{11}$$

where, K_1, K_2, K_3, K_4 and K_5 are other forms of the strength of parameters. ν is the number of bosons not coupled to zero angular momentum in U(5) symmetry, L is the angular momentum of the nucleus, N is the number of bosons in the nucleus, σ is the number of bosons not coupled to zero angular momentum in O(6) symmetry, τ is the number of d bosons s not coupled to zero and (λ, μ) are the angular momentum of bosons and its third component, respectively. Many nuclei have a transition property between two or three of the above limits and their eigenvalues for the yrast states are given by [23]:

U(5)-O(6):

$$\left. \begin{aligned} E(\varepsilon, n_d, \tau, L) &= \varepsilon \hat{n}_d + K_1 n_d (n_d + 4) \\ &+ K_4 \tau(\tau+3) + K_5 L(L+1) \end{aligned} \right\} ; \tag{12}$$

U(5)-SU(3):

$$\left. \begin{aligned} E(\varepsilon, \lambda, L) &= \varepsilon \hat{n}_d + K_2(\lambda^2 + 3(\lambda+\mu)) \\ &+ K_5 L(L+1) \end{aligned} \right\} ; \tag{13}$$

O(6)-SU(3):

$$\left. \begin{aligned} E(\tau, \lambda, L) &= K_2(\lambda^2 + 3(\lambda+\mu)) \\ &+ K_4 \tau(\tau+3) + K_5 L(L+1) \end{aligned} \right\} . \tag{14}$$

The eigenvalues for the GSB and NPB states in IVBM are given by [7]:

$$E(I) = \beta I(I+1) + \gamma I \; . \tag{15}$$

$$E(I) = \beta I(I+1) + (\gamma+\eta) I + \zeta \; . \tag{16}$$

The values of β and γ can be determined from a fitness to the positive GSB, while η and ζ are estimated from the fitness to NPB. Analyzing Eqs. (15) and (16), it can be seen that the eigenstates of the GSB and NPB states consist of rotational $I(I+1)$ and vibrational I modes. Doma and El-Gendy have derived a new formula of the rotational energy levels, that depends upon the spin I and the nuclear moment of inertia ϑ, by analyzing the well-known experimental rotational energy levels of the even-even deformed nuclei in the high mass region in the following simple form [15]:

$$E(I) = \frac{A I(I+1)}{\left[1 + \dfrac{D I(I+1)}{1 - C I(I+1)}\right]} \; ; \tag{17}$$

where A is the reciprocal moment of inertia of the nucleus, $A = \dfrac{\hbar^2}{2\vartheta}$. The values of D and C can be determined from a fit to the GSB.

The odd-even staggering (or $\Delta I=1$ staggering) can be measured by the quantity [6]:

$$\Delta E_{1,\gamma}(I)=\frac{1}{16}\left\{\begin{pmatrix}6E_{1,\gamma}(I)-4E_{1,\gamma}(I-1)\\ -4E_{1,\gamma}(I+1)\\ +E_{1,\gamma}(I-2)+E_{1,\gamma}(I+2)\end{pmatrix}\right\}; \quad (18)$$

where the transition energies are:

$$E_{1,\gamma}(I) = E(I+1) - E(I). \quad (19)$$

The quantity $\Delta E_{1,\gamma}(I)$ exhibits values of alternating sign over an extended region of the angular momentum. Odd-even staggering starts from relatively high values and then decreases with increasing angular momentum. Reaching a vanishing value ($\Delta E_{1,\gamma}(I)=0$), after staggering starts raising and then dropping again, it gives an overall picture of beats. When the staggering reaches a vanishing value, a phase change takes place [6].

Results and Discussion

Useful information on the shape transition of even-even nuclei can be obtained from the first excited state, the ratio of the second to the first excited states R(4/2), E-GOS curve and the ratio $r\left(\frac{(I+2)}{I}\right)$. Experimental energy levels of the ground and negative parity state band of $^{130}Xe-^{130}Nd$ isobars were taken from Ref. [25]. Table 1 shows the values of first excited states and the ratios R(4/2) for these isobars, which primarily indicate the vibrational property of ^{130}Xe nucleus, the γ-soft property of ^{130}Ba and ^{130}Ce nuclei and the rotational property of ^{130}Nd nucleus.

TABLE 1. Experimental values [25] of $E2_1^+$ and the ratio $R(4/2)=\frac{E4_1^+}{E2_1^+}$ for ^{130}Xe - ^{130}Nd isobars

Isobar	^{130}Xe	^{130}Ba	^{130}Ce	^{130}Nd
$E2_1^+$(keV)	536.07	357.38	253.85	159.05
R4/2	2.247	2.523	2.798	3.052

Fig. 1 shows the E-GOS curves of the ground state band of ^{130}Xe - ^{130}Nd isobars. A comparison of these curves with the ideal limits of vibrational, rotational and γ-soft nuclei gives good information of the property of the states of the isobars. The high drop of the curve of ^{130}Xe from the first excited state 2_1^+ to 10_1^+ state gives this isobar the vibrational property, but the behavior of the curve changes after the 10_1^+ state. The slow drop of the curve of ^{130}Ba gives it the γ-soft property. The behavior of the curve of ^{130}Ce is the slow drop of values up to 10_1^+ state, giving it the γ-soft property, and the behavior changes after this state. The curve of ^{130}Nd primarily raises slowly, then drops slowly which gives it the rotational- γ-soft property.

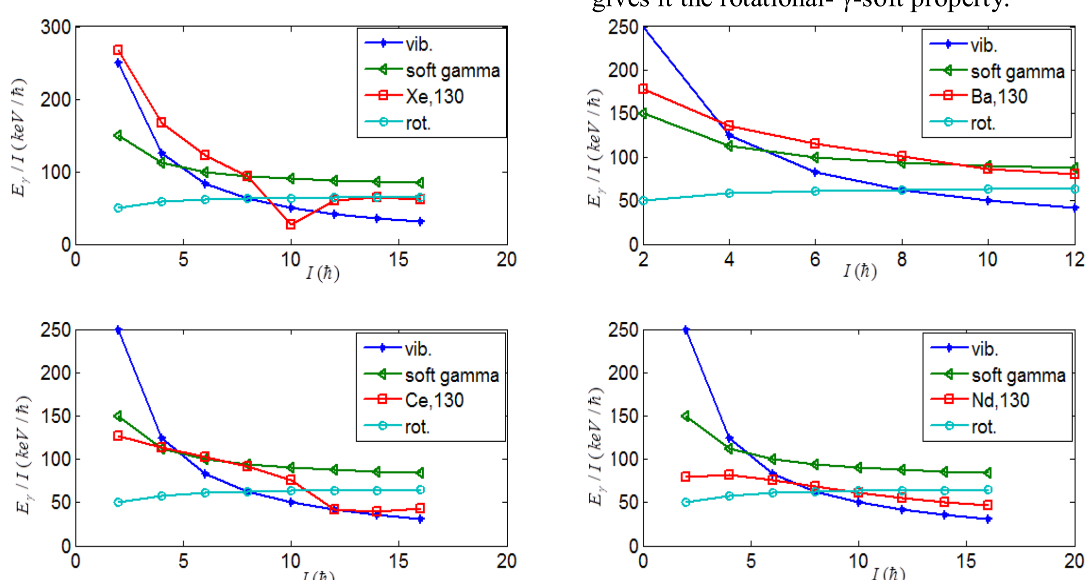

FIG. 1. E-GOS of the ground-state band for ^{130}Ba-^{130}Xe isobars.

Neither the first of two excited states nor the E-GOS curve gives the exact property of each isobar. For this reason, the relation between $r\left(\frac{I+2}{I}\right)$ and I is plotted and compared with the behavior of vibrational U(5), rotational SU(3) and γ-soft O(6) nuclei. Fig. 2 shows these curves, where the ratio for all values of I in ^{130}Xe is smaller than 0.5 passing limit values of the vibrational limit which gives this nucleus the U(5)-O(6) character. This ratio for ^{130}Ba is alternative within the limits of O(6) nuclei, which gives this property for this nucleus. However, the ratio $r\left(\frac{I+2}{I}\right)$ of ^{130}Ce is changed from the O(6) character to the 10_1^+ state and the character has been changed to the SU(3) after this state, while the ratio of ^{130}Nd nucleus is smaller than 0.8 and greater than 0.46, which gives an SU(3)-O(6) property to this nucleus.

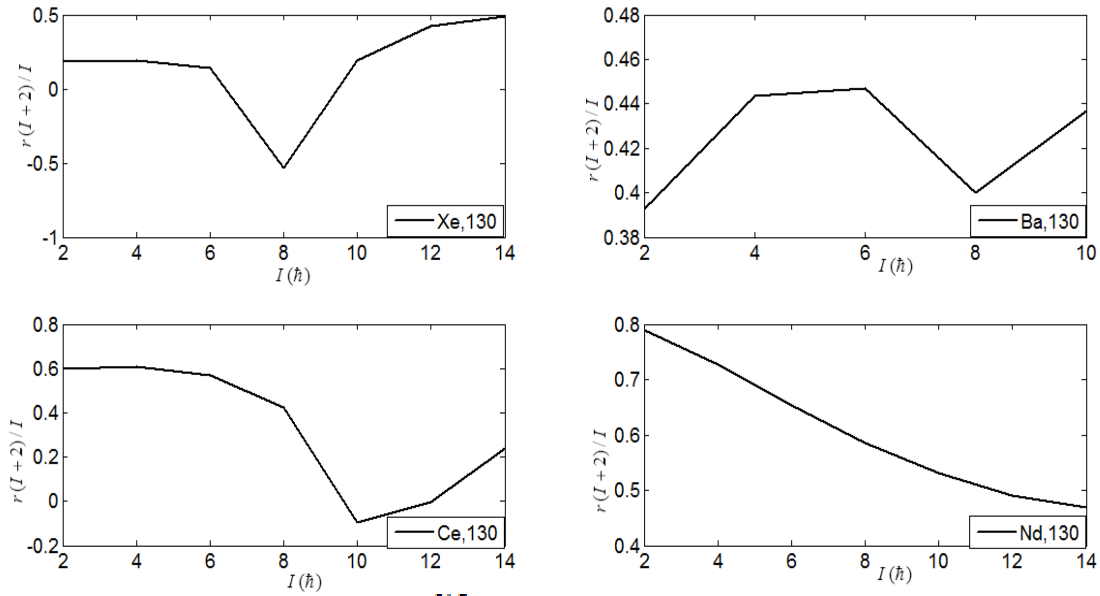

FIG. 2. The ratio $r\left(\frac{I+2}{I}\right)$ as a function of I for ^{130}Ba-Xe isobars.

Fig. 3 shows the application of Eq. (18) to find out the experimental staggering factor $\Delta E_{1,\gamma}(I)$ of ^{130}Xe-^{130}Nd isobars. In all cases, one obtains a clearly pronounced staggering pattern, that is a zigzagging behavior of the quantity $\Delta E_{1,\gamma}(I)$ as a function of angular momentum I. The vanishing value of the staggering $\Delta E_{1,\gamma}(I)=0$ has not been reached, which means that each isobar has a saturation in its property along with its states; that is ^{130}Xe has a pure U(5) character, ^{130}Ba and ^{130}Ce have an O(6) character and ^{130}Nd stays in the SU(3)-O(6) property.

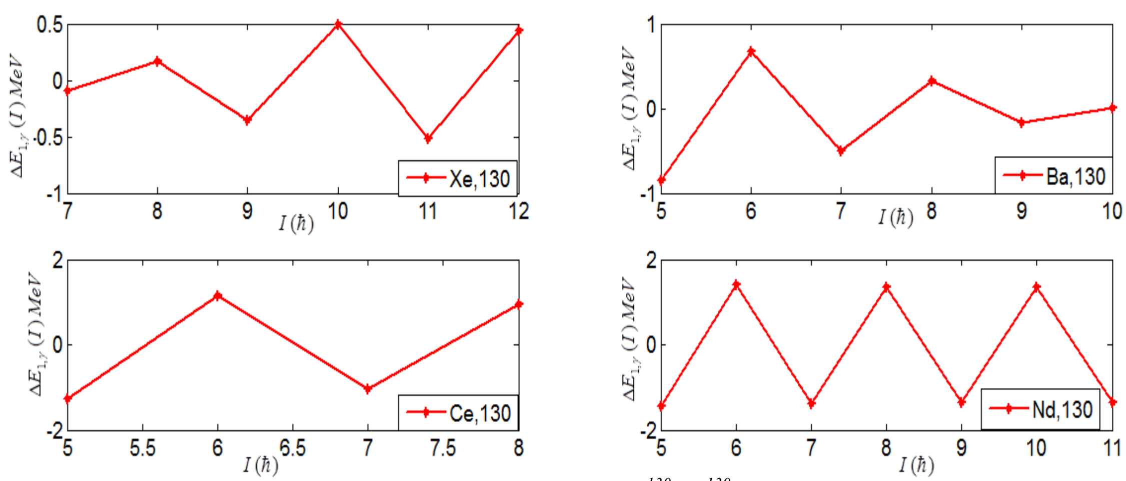

FIG. 3. Experimental staggering of ^{130}Xe-^{130}Nd isobars.

MATLAB 6.5 software is used to calculate the energy levels of the GSB and the NPB for ^{130}Xe-^{130}Nd isobars using BM, IBM-1, IVBM and D-G methods. The number of bosons and the best values of the parameters which give the best fitting between theoretical and experimental energy levels of the above isobars are represented in Table 2, into which the number of bosons has been calculated from the sum of proton bosons of the closed shell with magic number (50) and the neutron bosons of the closed shell with magic number (82). Table 2 shows the values of parameter A for the D-G formula which is chosen to be $\frac{\hbar^2}{2\vartheta}=E2_1^+/6$ according to the rotational energy relation, where $E(I)=\frac{\hbar^2}{2\vartheta}I(I+1)$, while its values according to BM are close to $\frac{\hbar^2}{2\vartheta}=E2_1^+/6$ only for ^{130}Nd, since it is a rotational nucleus. The other two parameters B and C of BM and C and D of D-G are determined by fitting each of the equations with the measured energy levels. The parameter ε which occurs only for ^{130}Xe is close to $E2_1^+=536\,keV$ due to the vibration characteristic of this nucleus. The parameter K_4 for ^{130}Ba has the biggest value in comparison with the other nuclei, which means that this nucleus is a more $\gamma-soft$ nucleus than any other nuclei. The value of γ parameter of IVBM is a vibrational one and one can observe that this value of the vibrational nucleus ^{130}Xe is the highest one, while the smallest value is for the rotational nucleus ^{130}Nd. The smallest value of the rotational β parameter is for ^{130}Xe and one expects that the highest value is for ^{130}Nd. However, this will not happen, because of the fitting of the measured value of energy with the IVBM equation. The best fitting parameters of the NPB of BM and IVBM are shown in Table 3. The calculated and experimental energies of GSB and NPB of ^{130}Xe-^{130}Nd are shown in Table 4. It is obvious that the calculated energy levels are in good agreement with the experimental ones for all isobars and for all states and the calculation of IBM-1 and IVBM is more reliable than that of BM and D-G for the calculation of the first two excited states, especially for ^{130}Xe and ^{130}Ba nuclei, with a deviation of calculation from IVBM for the first excited state of ^{130}Nd. So, the IBM-1 calculation is the most suitable one in this work.

TABLE 2. BM, IBM-1, IVBM and D-G parameters of GSB in keV for ^{130}Xe-^{130}Nd isobars

Isobar	BM			IBM-1						IVBM		D-G		
	A	B	C* 10^{-3}	No. of bosons	ε	*K1*	*K2*	*K4*	*K5*	β	γ	A	C* 10^{-2}	D* 10^{-1}
^{130}Xe	53.9	0.28	0.6	5	487.9	11.7	---	8.4	-1.2	3.8	274.9	89.4	-0.5	0.3
^{130}Ba	47.9	0.25	0.8	7	---	---	---	126.8	-16.6	15.06	158.5	59.6	-1.9	0.52
^{130}Ce	38.9	0.19	0.7	9	---	---	---	79.6	-3.1	16.8	99.5	42.3	-4.5	1.32
^{130}Nd	24.2	0.05	0.1	11	---	---	-0.37	110.7	-19.5	11.6	77.4	26.6	-2.8	1.47

TABLE 3. BM and IVBM parameters of NPB in keV for ^{130}Xe-^{130}Nd isobars

Isobar	BM			IVBM	
	A	*B*10*$^{-2}$	*C*10*$^{-2}$	η	ξ
^{130}Xe	5.9	-6.5	-0.01	-34.77	633.9
^{130}Ba	5.65	-10.08	-0.03	-171.7	1806.3
^{130}Ce	11.3	-5.07	-0.01	-101.3	1806.4
^{130}Nd	10.97	-4.5	-0.01	-3.0	1418.4

TABLE 4. The experimental [25] and calculated energy levels in keV of GSB and NPB for ^{130}Xe-^{130}Nd isobars

Isobar		I_1^π	2_1^+	3_1^-	4_1^+	5_1^-	6_1^+	7_1^-	8_1^+	9_1^-	10_1^+	11_1^-	12_1^+	13_1^-	14_1^+
^{130}Xe	$E_{exp.}$		536	---	1205	2060	1944	2375	2697	3072	2972	3893	3693	4540	4591
	$E_{cal.}$	BM	313	---	969	2292	1808	2571	2633	3017	3292	3648	3772	4433	4334
		IBM-1	573	---	1176	---	1809	---	2473	---	3167	---	3892	---	4647
		IVBM	573	---	1167	1949	1809	2528	2473	3137	3167	3777	3892	4447	4647
		D-G	456	---	1157	---	1840	---	2489	---	3148	---	3851	---	4622
^{130}Ba	$E_{exp.}$		357	1949	902	2168	1593	2568	2395	3067	3260	3659	4222	4354	---
	$E_{cal.}$	BM	279	2030	864	2200	1623	2524	2425	3035	3230	3699	4229	4344	---
		IBM-1	407	---	935	---	1584	---	2352	---	3242	---	4251	---	---
		IVBM	407	1948	935	2192	1584	2557	2352	3043	3242	3649	4251	4376	---
		D-G	409	---	974	---	1600	---	2338	---	3221	---	4265	---	---
^{130}Ce	$E_{exp.}$		254	1955	710	2313	1324	2761	2053	3320	2809	4027	…	…	…
	$E_{cal.}$	BM	227	2098	706	2337	1342	2724	2044	3287	2811	4026	---	---	---
		IBM-1	300	---	734	---	1302	---	2005	---	2842	---	---	---	---
		IVBM	300	2003	734	2301	1302	2734	2005	3302	2842	4004	---	---	---
		D-G	331	---	749	---	1287	---	1985	---	2858	---	---	---	---
^{130}Nd	$E_{exp.}$		159	1825	486	2144	940	2587	1487	3133	2100	3765	2764	4463	3468
	$E_{cal.}$	BM	143	1963	463	2191	927	2558	1493	3082	2117	3753	2769	4509	3451
		IBM-1	124	---	516	---	972	---	1494	---	2082	---	2735	---	3454
		IVBM	224.3	1802	541.3	2163	951	2608	1454	3136	2049	3749	2737	4446	3517
		D-G	306	---	621	---	982	---	1433	---	1991	---	2659	---	3440

4. Conclusions

The measured values of the first excited states $E2_1^+$ and the ratios of the second to the first excited states $E4_1^+/E2_1^+$ as well as the E-GOS, the ratio $r\left(\frac{(I+2)}{I}\right)$ and the staggering curves have been applied for describing GSB and NPB for ^{130}Xe-^{130}Nd isobars. These studies insured that ^{130}Xe nucleus is lined up along the U(5) side, ^{130}Ba and ^{130}Ce have an O(6) property, whereas ^{130}Nd has an O(6)-SU(3) property. It has been demonstrated that the GSB and NPB in the studied isobars exhibit $\Delta I = 1$ staggering. The vanishing value of the staggering $\Delta E_{1,\gamma}(I)=0$ has not been reached, which means that each nucleus is stable in its property. The BM, IBM-1, IVBM and D-G formula have been applied for calculating energy levels of GSB and NPB using the BM and IVBM models. The calculated energy levels in comparison with the experimental data indicate the quality of the fit presented in this work. It can be concluded that the IBM-1 calculation is the most suitable one in this work.

References

[1] Iachello, F. and Talmi, I., Rev. Mod. Phys., 59 (1987) 339.

[2] Greiner, W. and Maruhn, J.A., “Nuclear Models”, (Springer Verlag Heidelberg, 1996) 375.

[3] Bohr, A. and Mottelson, B.R., “Nuclear Structure”, (World Scientific Publishing), II (1998) 748.

[4] Raymond, K.S. and Sood, P.C., Phys. Rev. C, 34 (1986) 2362.

[5] Phillips, W.R. *et. al.*, Phys. Rev. Lett., 57 (1986) 3257.

[6] Bonatsos, D. *et al.*, Phys. Rev. C, 62 (2000) 024301-12.

[7] Ganev, H. *et al.*, Phys. Rev. C, 69 (2004) 014305-7.

[8] Krane, K.S., “Introductory Nuclear Physics”, (John Wiley and Sons, 1987).

[9] Regan, P.H. *et al.*, Phys. Rev. Lett., 90 (2003) 152502.

[10] Bonatsos, D. and Skoures, L.D., Phys. Rev. C, 43 (1991) 952R.

[11] Khalaf, A.M. and Ismail, A.M., Prog. Phys., 2 (2013) 98.

[12] Iachello, F. and Arima, A., “The Interacting Boson Model”, (Cambridge, 1987).

[13] Iachello, F., Phys. Rev. Lett., 85 (2000) 17.

[14] Ganev, H., Garistov, V.P. and Georgieva, A.I., Phys. Rev. C, 69 (2004) 014305- 7.

[15] Doma, S.B. and El-Gendy, H.S., Inter. J. Mod. Phys. E, 21 (2012) 1250077.

[16] Servin, A., Heyde, K. and Jolie, J., Phys. Rev. C, 36(6) (1987).

[17] Von Brentano, P., Gelberg, A., Harissopulos, S. and Casten, R.F., Phys. Rev. C, 38(5) (1988).

[18] Bettemann, L., Fransen, C., Heinze, S., Jolie, J., Linnmann, A., Mucher, D., Rother, W., Ahn, T., Castin, A., Pietralla, N. and Luo, Y., Phys. Rev. C, 79 (2009) 034315.

[19] Eid, S.A. and Diab, S.M., Progress in Phys., 1 (2012).

[20] Scharff-Goldhaber, G. and Weneser, J., Phys. Rev., 98 (1955) 212.

[21] Bohr, A. and Mottelson, B.R., Mat. Phys. Med., 27 (1953).

[22] Scholten, O. *et al.*, Ann. Phys., 115 (1978) 325.

[23] Iachello, F., Abrahams, K., Allart, K. and Dieperink, A.E., “Nuclear Structure”, (Plenum Press, New York, 1981).

[24] Iachello, F. and Aima, A., “The Interacting Boson Model”, (Cambridge University Press, Cambridge, 1987).

[25] ENSDF, http:// www.nndc.bnl.gov/ensdf (Nuclear data sheet) (2009).