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ARTICLE

The Study of Zero-Spin Isotopes with the Modified Manning-Rosen Potential by Relativistic Cluster Models

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Abstract: In this paper, we studied the zero-spin isotopes with N=Z and A=4n in relativistic cluster models as a system which can be considered to be composed of α-particles. For interaction between the clusters, we use modified Manning-Rosen potentials and solve the relativistic Klein-Gordon (KG) equation using the Nikiforov-Uvarov (NU) method to calculate the energy spectrum. We found the ground state energy and the first excited energy. Finally, the calculated results are compared with the experimental data for light nuclei, such as ^{8}Be, ^{12}C and ^{16}O. The results show that the modified Manning-Rosen (MR) potential is adaptable for cluster interactions.
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1. Introduction

One of the fundamental models of nuclear structure is the cluster model which has a long history [1]. Cluster interpretation is a suitable model to describe nuclear states and has been successful in reproducing energy spectra and other nuclear properties, such as electro-magnetic properties, α-emission widths and α-particle elastic scattering data in nuclei near the double closed shell. In 1936, when Bethe predicted that nuclei are made of alpha particles and proposed a geometrical arrangement of alpha particles inside nuclei, the cluster model was introduced [2]. In 1937, Wheeler [3] extended this work, and similar models were suggested concurrently by Wefelmeier [4], Weizsacker [5] and Fano [6]. Freer and Merchant, in 1997, studied the role of clustering and cluster models in nuclear reactions and examined the evidence for α-cluster chain configurations in the light even - even nuclei from ^{8}Be to ^{28}Si [7]. Recently, several systems of even-even nuclei were studied with a cluster model and their results were reasonably compared to the experimental spectra for ground state and some excited states [7-12]. The alpha particle, ^{4}He nucleus, is the most common cluster which is exceptionally stable (its first excited state is 20.2 MeV [13]), and its binding energy is 7.07 MeV/nucleon. It is also very compact (a charge radius of 1.673fm [14]), and has zero-spin for all quantum numbers (spin and isospin), which makes it easy to combine into larger systems. Also, it is the first doubly magic nucleus with the first closed shell $1S_{1/2}$, which accounts for its exceptional stability. Also, in nuclear and high energy physics, particles are in a strong potential field, and for studying the internal structure of any quantum mechanical systems, such as nuclei, the relativistic effect must be considered. In relativistic quantum mechanics, one can apply the KN equation to the treatment of a zero-spin particle, such as the alpha particle and the Dirac equation for the spin half particle. The KN equation is frequently used to describe the particle dynamics, and in recent years, many studies have been carried out to explore the relativistic energy Eigen-values and

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corresponding wave functions of the KN equation [15-18]. Hence, in this article, we study the light nuclei in the relativistic cluster model and solve the KN equation for the isotopes with N=Z and A=4n (n=2, 3, 4 …) as a system which can be considered to be composed of α-particles.

Modeling the effective interaction among clusters is very important. The cluster-core interaction leads to the identification of clustering in the nuclear matter and the description of clustering phenomenon in various nuclei. In 1960s, Ali and Bodmer used the experimental data on alpha-alpha scattering and obtained potentials which were fitted to the scattering phase shifts [19, 20]. During the past decade, the modified phenomenological Saxon-Woods plus Cubic Saxon-Woods cluster potential has successfully described various phenomena related to alpha clustering in light as well as even-even heavy nuclei [21, 22]. Prior to the development of the Saxon-Woods plus Cubic Saxon-Woods potential form, such a microscopic interaction has been employed in various forms to describe cluster bound states in light nuclei [23] and exotic decays in heavy nuclei [22]. Despite its success, the modified Saxon-Woods potential model tells us very little about the microscopic nature of clustering in closed shell nuclei.

The MR potential is one of the most useful and convenient models for studying the energy Eigen-values. It also gives an excellent description of the interaction between the two atoms in a diatomic molecule and constitutes a convenient model for other physical situations in term of their bound states and scattering properties. So, in our work, we offer the modified Manning-Rosen potential as a reasonable potential to study nuclear structure in cluster model, due to some similarity of multi-atomic molecules to multi-alpha-cluster nuclei. The short range Manning-Rosen potential is given by [24-29]:

$$V(r)=\frac{-A\hbar^2}{2\mu b^2}\frac{e^{-r/b}}{1-e^{-r/b}}+\frac{\alpha(\alpha-1)\hbar^2}{2\mu b}\left[\frac{e^{-r/b}}{1-e^{-r/b}}\right]^2, \quad (1)$$

where A and α are dimensionless parameters, while b is the screening parameter which has the dimension of length. Our modified Manning-Rosen potential is:

$$V_{M.}(r)=\left.\begin{aligned}&\frac{-A\hbar^2}{2\mu b^2}\frac{e^{-r/b}}{1-e^{-r/b}}\\&+\frac{\alpha(\alpha-1)\hbar^2}{2\mu b}\left[\frac{e^{-r/b}}{1-e^{-r/b}}\right]^2+\frac{k}{r^2}\end{aligned}\right\}, \quad (2)$$

where k is the coulomb repulsion potential coefficient between the clusters.

In section two, we solve the KN equation to find the Eigen-values and Eigen-functions with the modified Manning-Rosen Potential. Then, in section three, we examine the results for some isotopes. At the end, conclusions are given in section four.

2. The Eigen-values and Eigen-Functions with the Manning-Rosen Potential for Nα–Body System

The many-body forces are more easily introduced and treated within the hyperspherical harmonics formalism. Now, we consider a system of identical α particles. The D-dimensional time-independent arbitrary l-state radial KN equation with scalar and vector potentials S(r) and V (r), respectively, where r = |r| describing a spinless particle, such as α-particle, takes the general form [30-32]:

$$\left.\begin{aligned}&\nabla_D^2\psi_{l_1\ldots l_{D-2}}^{(l_{D-1}=l)}(r)\\&+\frac{1}{\hbar^2c^2}\left\{\left[E_{nl}-V(r)\right]^2-\left[Mc^2+S(r)\right]^2\right\}\psi_{l_1\ldots l_{D-2}}^{(l_{D-1}=l)}(r)\\&=0,\end{aligned}\right\} \quad (3)$$

where E_{nl}, M and ∇_D^2 denote the KG energy, the mass and the D-dimensional Laplacian, respectively. If the scalar and vector potentials S (r_{ij}) and V (r_{ij}) are a two-body potential of interaction, so we can expand them in the hyperspherical harmonics formalism. We define a set of the Jacobi coordinates for r_{ij}, where $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$ [34]:

$$x^2=\sum_{i=1}^{N-1}(r_i-R)^2=\frac{2}{N-1}\sum_{k,l>k}r_{kl}^2,\ R=\frac{1}{N}\sum_i^N r_i. \quad (4)$$

The center of mass R eliminates using the Jacobi coordinates. In addition, x is a D-dimensional position vector in Jacobi

coordinates. Therefore, by choosing a common ansatz for the wave function in the form:

$$R_l(x) = x^{-(D-1)/2} u_l(x), \tag{5}$$

Eq. (3) reduces to the form shown below and KG equation turns into a Schrödinger-like equation. Thus, the bound state solutions are very easily obtained with the NU method [35-37]:

$$\left.\begin{aligned} &\frac{d^2 u_l(x)}{dx^2} + \\ &\frac{1}{\hbar^2 c^2}\left\{\begin{aligned} &\left[E_{nl} - V(x)\right]^2 - \\ &\left[Mc^2 + S(x)\right]^2 - \\ &\frac{(D+2l-1)(D+2l-3)}{4x^2} \end{aligned}\right\} u_l(x) \\ &= 0. \end{aligned}\right\} \tag{6}$$

By replacing C = A and $D = -A - \alpha(\alpha - 1)$, the modified MR potential can be written in the following simple form:

$$V_{M.}(x) = -\frac{Ce^{-x/b} + De^{-2x/b}}{(1 - e^{-x/b})^2} + \frac{k}{x^2} \ . \tag{7}$$

Under the equally mixed potentials *S(r)* = *V(r)* and using Eq. (7), we obtain:

$$\left.\begin{aligned} &\frac{d^2 u_l(x)}{dx^2} + \\ &\frac{1}{\hbar^2 c^2}\left(\begin{aligned} &E_{nl}^2 - m^2 c^4 - \\ &2(E_{nl} - mc^2)\left(-\frac{\hbar^2}{2\mu b^2}\left(\frac{Ce^{-x/b} + De^{-2x/b}}{(1 - e^{-x/b})^2}\right)\right) \\ &-\frac{(D+2l-1)(D+2l-3) - 4k}{4x^2} \end{aligned}\right) u_l(x) \\ &= 0. \end{aligned}\right\} \tag{8}$$

The good approximation for the kinetic energy term $((D+2l-1)(D+2l-3) - 4k)/4x^2$ in the centrifugal barrier is taken as [37-39]:

$$\frac{1}{x^2} \approx \frac{1}{b^2}\frac{e^{-x/b}}{(1 - e^{-x/b})^2} \ . \tag{9}$$

To obtain the following hypergeometric equation, we substitute Eq. (9) into Eq. (8) and make change of the variables $x \to z$, $z = e^{-x/b}$, through the mapping function $x = f(z)$:

$$\left.\begin{aligned} &\frac{d^2 R(z)}{dz^2} + \\ &\frac{(1-z)}{z(1-z)}\frac{d}{dz} + \\ &\frac{b^2}{\hbar^2 c^2}\left[\begin{aligned} &\left\{\begin{aligned} &(E_{nl}^2 - m^2 c^4)(1 - z^2) \\ &-2(E_{nl} - mc^2)(Cz + Dz^2) \end{aligned}\right\} \\ &\frac{}{z^2(1-z)^2} \end{aligned}\right] R(z) \\ &-\frac{1}{4\hbar^2 c^2}\left[\frac{(D+2l-1)(D+2l-3) - 4k}{z^2(1-z)^2}\right] R(z) \\ &= 0. \end{aligned}\right\} \tag{10}$$

By using Nikiforov-Uvarov method, we obtain the Eigen-values and Eigen-functions as follows [35-37]:

$$\left.\begin{aligned} &n + (2n+1)\left(\begin{aligned} &0.5 + \sqrt{\xi_3} + \\ &\sqrt{1/4 + \xi_1 + \xi_3 - \xi_2} \end{aligned}\right) \\ &+ n(n-1) - \xi_2 + 2\xi_3 \\ &+ 2\sqrt{\xi_3(1/4 + \xi_1 + \xi_3 - \xi_2)} \\ &= 0, \end{aligned}\right\} \tag{11}$$

and:

$$\left.\begin{aligned} &R(z) = \\ &z^{\sqrt{\xi_3}} \times (1-z)^{0.5 + \sqrt{1/4 + \xi_1 + \xi_3 - \xi_2}} \times \\ &P_n^{(2\sqrt{\xi_3},\, 1 + \sqrt{\xi_3} + \sqrt{1/4 + \xi_1 + \xi_3 - \xi_2})} \times (1 - 2z) \end{aligned}\right\}, \tag{12}$$

where:

$$\xi_1 = \frac{b^2}{\hbar^2 c^2}\left[m^2 c^4 - E_{nl}^2 + 2(mc^2 - E_{nl})D\right] \tag{13}$$

$$\xi_2 = \frac{-b^2}{\hbar^2 c^2}\left[\begin{aligned} &2(E_{nl}^2 - m^2 c^4) \\ &+2(mc^2 - E_{nl})C \\ &-\frac{1}{4b^2}[(D+2l-1)(D+2l-3) \\ &-4k] \end{aligned}\right] \tag{14}$$

$$\xi_3 = \frac{b^2}{\hbar^2 c^2}(m^2c^4 - E_{nl}^2). \quad (15)$$

The wave function that is obtained by Eq.12 is according to the boundary conditions for all the isotopes in the alpha cluster model. In the next section, we study ground state and first excited state energy for various combinations of light α- cluster nuclei.

3. Examples of Clustering

3.1. Example ^{8}Be

In our study, the simplest case is that of the two α-particle system ^{8}Be which would have a dumbbell shape and be a two-body system. Ikeda predicted that cluster structures are most obvious at an excitation which coincides with a particular decay threshold [40]. Experiments show that alpha + alpha cluster structure is found in the ground state of ^{8}Be, because it has a lifetime of ~ 10^{-16} s. The binding energy of ^{8}Be is -57.75 MeV and its ground state is unbound to 2α decay by 92 keV. It has a first excited 2^+ state at -53.27 MeV with a width of 1.51 MeV, as well as a 4^+ state at -46.6 MeV with a width of 3.5 MeV. These three states have an energy separation which is consistent with a rotational behavior given by $\hbar^2 j(j+1)/2\boldsymbol{I}$, where $\boldsymbol{I}$ is the moment of inertia. The calculated value for the moment of inertia is consistent with the picture of two touching α–particles; an essentially super-deformed nucleus [9-41].

With the picture of two touching α-particles and the use of the modified Manning-Rosen potential between them, we reproduce the spectrum of the ground state and the first excited state by Eq.11. Results are shown in Table 1 and compared with experimental data. It appears that our result have good agreement with the experimental results.

TABLE 1. The spectrum of the energy levels in ^{8}Be.

Levels	E_{cal}(MeV)	E_{exp}(MeV)
Ground state	-57.75	-56.50
First excited state	-53.27	-53.47

3.2. Example ^{12}C

The structure of "Hoyle" state; the first excited 0^+ state at -84.51 MeV in ^{12}C isotope is influenced by clustering or by symmetries thereof. So, the system can be constructed from a variety of geometric arrangements of three-alpha particles. It might be expected that the compact equilateral-triangle arrangement is the lowest energy configuration [8-42].

In the case of ^{12}C, the structure of the ground state is influenced by clustering or by symmetries thereof. So, the system can be constructed from a variety of geometric arrangements of three-alpha particles. It might be expected that the compact equilateral-triangle arrangement is the lowest energy configuration [43]. Hence, three identical body forces of the internal particle motion are described in terms of the Jacobi relative coordinates ρ, λ and R; center of mass. In the theory of many-particle systems, Jacobi coordinates often are used to simplify the mathematical formulation. Now, we can introduce the hyper-radius quantity x and the hyper-angle ξ as follows [33- 43]:

$$x = \sqrt{\rho^2 + \lambda^2} \quad \xi = \tan\left(\frac{\rho}{\lambda}\right), \quad (16)$$

where:

$$\rho = \frac{\vec{r_1} - \vec{r_2}}{\sqrt{2}}, \ \lambda = \frac{\vec{r_1} + \vec{r_2} - 2\vec{r_3}}{\sqrt{6}}, \ R = \frac{\vec{r_1} + \vec{r_2} + \vec{r_3}}{\sqrt{3}}. \quad (17)$$

$\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ are the relative positions of the three particles. We solved the KN equation in new coordinates, similar to the previous ones. We found the best values of **b**, **α**, **A** and **k** by fitting to the experimental data at Hoyle state that has the cluster structure. Then, we calculated the approximate ground state energy and the first excited energy (in MeV). The results are shown in Table 2 for the ^{12}C isotope.

TABLE 2. The spectrum of the energy levels in ^{12}C.

Levels	E_{cal}(MeV)	E_{exp}(MeV)
Ground state	-92.61	-92.16
First excited state	-88.01	-87.72

The binding energy of ^{12}C is -92.16 MeV and in our model, we obtain a value of -92.61MeV. For the first excited state, our result is -88.01 MeV which is near to the experimental data.

3.3. Example ^{16}O

The ^{16}O isotope possesses the second closed shell $1P_{1/2}$, but not quite the degree of inertness of the α- particle. The Ikeda diagram suggests that ^{16}O has a ^{12}C + alpha structure at an excitation energy of around -120.46 MeV and a four-alpha particle structure at an excitation

energy of around -113.18 MeV [40]. The experimental evidence supports the Ikeda model too. The experimental moments of inertia can be related back to shapes and alpha cluster configurations suggested by the moment of inertia calculations. These calculations do not rule out the possibility that the nucleus maintains a homogeneous composition throughout the deformed shape. However, the subsequent decay into two ^{8}Be, believed to have a 2α–cluster structure, and from there into four ^{4}He, supports the idea of four-alpha clusters [44].

TABLE 3. The spectrum of the energy levels ^{16}O

Levels	E_{cal}(MeV)	E_{exp}(MeV)
Ground state	-127.89	-127.62
First excited state	-121.84	-121.57

Now, we give in Table 3 a summary of our results for cluster states in ^{16}O nuclei obtained using our potential of Eq.6 in Jacobi coordinates.

The values of the ground state energy and the first excited energy are -127.62 MeV and -121.57 MeV, respectively, while we calculated them to be -127.89 MeV and -121.84 MeV, respectively.

4. Conclusions

In the present paper, we studied the light – α cluster (N=Z) nucleus. We selected the modified Manning – Rosen potential between the alpha clusters regardless of the internal structure of them. By solving the Klein Gordon equation in D-dimensions space using the Jacobi coordinates and NU method for our potential, we found the Eigen-values and Eigen-functions, generally. Then, we examined the results for ^{8}Be, ^{12}C and ^{16}O. Results for the ground state and the first excited state in the studied isotopes showed good agreement with the experimental data.

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