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ARTICLE

Coupling of Upper Hybrid Surface Plasmon Modes in Magnetoplasma Waveguide Structure

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Abstract: We investigate the spectra of high-frequency electrostatic surface electron plasmon oscillations propagating normal to a dc-magnetic field. These oscillations are supported by two identical magnetoplasma slabs separated by a vacuum slab. Propagation characteristics of surface magnetoplasma oscillations and their coupling are studied by simultaneously solving the homogeneous system of equations obtained by matching the electrostatic fields at the interfaces together with the warm plasma dielectric function of upper hybrid waves. We demonstrate the existence of two propagating magnetoplasma electrostatic surface modes (backward and forward modes). The backward mode emerges at frequency $\omega = \omega_{\mathrm{u}h} = \sqrt{\omega_{\mathrm{p}e}^2 + \omega_{ce}^2}$, where $\omega_{\mathrm{p}e}$ and ω_{ce} are the electron plasma frequency and the electron cyclotron frequency, respectivily, and the forward propagating mode emerges at a lower frequency $\omega = \omega_{\mathrm{u}h} - \omega_{\mathrm{p}e}$. The forward and backward surface modes become coupled and form a single mode at upper hybrid resonance quasi-static value $\omega = \omega_{\mathrm{u}h}/\sqrt{2}$.

Keywords: Upper hybrid modes, Plasma slab waveguide, Coupled plasmon surface modes.

1. Introduction

The study of electron plasma wave interaction in the presence of magnetic fields is of importance for plasma heating and plasma diagnostics [1-6]. In the presence of a dc-magnetic field, the bulk electron plasma frequency $\omega_{\mathrm{p}e}$ transforms into the upper hybrid (UH) frequency $\omega_{\mathrm{u}h}$. Understanding the mechanisms of excitation of UH waves is crucial for explaining certain features of the stimulated emission of electromagnetic waves observed in ionospheric heating experiments. In planetary magnetospheres and in the Earth's ionosphere, a large amplitude upper hybrid wave can nonlinearly excite a slow electromagnetic wave of X or Z type [7, 8]. In space plasmas, UH waves can be generated either by mode coupling or by electron beams, which then can decay into electromagnetic and lower hybrid waves [9].

Electrostatic UH waves can be induced when an obliquely incident electromagnetic wave is converted into an electrostatic wave of UH type due to density irregularities. Effects of mass motion on the evolution of electrostatic waves and instabilities have been studied analytically by Mohanty and Naik [10]. It was found that streaming motion enhances the wave frequency and diminishes the terms related to growth or damping of electrostatic instabilities.

Due to the fact that plasma can have a negative dielectric constant in certain frequency domains, propagation of true surface waves along a plasma-dielectric interface is possible [11]. There is a great interest in studying the propagation of waves on the plasma boundary, as most plasmas in laboratory and space

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applications involve boundaries. It is well known that the propagation of a pure surface wave (i.e., a wave whose field decreases exponentially away from the interface) between two media is possible only when the permittivities of the two media have opposite signs [12-20].

This article studies high-frequency electron surface plasma modes of oscillation propagating perpendicular to an undisturbed magnetic field. These oscillations are supported by two identical, parallel and homogeneous plasma slabs separated by a vacuum slab. In Sec. 2, we present the model equations of the high-frequency electrostatic modes of magnetoplasma slabs. In Sec. 3, numerical examples of vacuum-plasma-vacuum-plasma-vacuum geometry are presented. Finally, we discuss the results and present the main conclusions of the paper in Sec. 4.

2. Model Equations

We consider a dielectric slab of thickness d and permittivity ϵ_d extending infinitely in the yz-plane between two infinite parallel plasma slabs of equal thicknesses ℓ and dielectric permittivities $\epsilon_{\mathrm{p}} = \epsilon_0 \varepsilon_{\mathrm{p}}$, where ε_{p} is the plasma dielectric function. The rest of the space is taken to be vacuum, as shown in Fig. 1. For high-frequency waves, the dynamics of the ions can be neglected, which allows for treating the ions as a fixed uniform background of positive charges. The motion of the electrons is governed by the following closed system of equations [21, 22],

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \vec{\nabla} \cdot (n_{\mathrm{e}} \vec{v}_{\mathrm{e}}) = 0, \quad (1)$$

$$m_{\mathrm{e}} n_{\mathrm{e}} \frac{d\vec{v}_{\mathrm{e}}}{dt} = -e n_{\mathrm{e}} \vec{E} - e n_{\mathrm{e}} \vec{v}_{\mathrm{e}} \times \vec{B} - m_{\mathrm{e}} n_{\mathrm{e}} \nu \vec{v}_{\mathrm{e}} - \gamma_{\mathrm{e}} k_B T \, \vec{\nabla} n_{\mathrm{e}}, \quad (2)$$

$$\vec{\nabla} \cdot \vec{E} = -\frac{e}{\epsilon_0} (Z n_{0i} - n_{\mathrm{e}}), \quad (3)$$

where m_{e} is the electron mass, e is the magnitude of electronic charge, ϵ_0 is the permittivity of free-space, γ_{e} is the ratio of specific heats, k_{B} is Boltzmann constant, T_{e} is the temperature of electrons, n_{e} is the electron density, ν is the electron collision frequency with neutrals, n_{0i} is the equilibrium ion density and Z is the charge state. The coupled equations (1–3) can be linearized with:

$$n_e = n_{0e} + n_{1e}, \vec{v}_e = \vec{v}_{0e} + \vec{v}_{1e}, \vec{E} = \vec{E}_0 + \vec{E}_1, \vec{B} = B_0 \, \hat{z}, \quad (4)$$

where the quantities with subscript 0 express the state of the magnetoplasma in the absence of oscillations. Perturbation terms of corresponding quantities are denoted by subscript 1. In the absence of an initial electron drift ($\vec{v}_{0e} = 0$), this procedure results in the following dielectric permittivity of the electrostatic electron plasma waves in a homogeneous magnetoplasma [22-24]:

$$\varepsilon_{\mathrm{p}} = 1 - \frac{\omega_{\mathrm{p}e}^2}{\omega(\omega + i\nu) - \omega_{\mathrm{c}e}^2 - 3\, v_{\mathrm{th},e}^2 k^2}, \quad (5)$$

where $\omega_{\mathrm{p}e}$ is the electron plasma frequency, ω_{ce} is the electron cyclotron frequency, ν is an effective collision frequency of electrons with neutrals, and $v_{\mathrm{th},e} = \sqrt{k_B T_{\mathrm{e}}/m_{\mathrm{e}}}$ is the electron thermal speed. The natural modes of Eq. (5) are the electrostatic upper hybrid waves across the dc-magnetic field in warm, collisional, and magnetized plasma.

In the electrostatic limit $\omega\sqrt{\mu_0 \epsilon} \ll 1$, the magnetic field component of the electromagnetic wave can be neglected. Accordingly, in a source-free nonconducting medium, the electric field obeys Laplace's equation. Without loss of generality, we will consider the surface wave modes that propagate along the interface in the y-direction such that:

$$E_{\mathrm{y}}(x, y) = u(x)\, e^{iky}, \text{and } E_{\mathrm{x}}(x, y) = -\frac{i}{k} \frac{dE_{\mathrm{y}}(x,y)}{dx}, \quad (6)$$

where $u(x)$ is a function that accounts for the electric field variations with x. Consequently, the function $u(x)$ obeys the equation $u''(x) - k^2\, u(x) = 0$. For the waves guided by any planar structure, the propagation constant k must be the same in all regions of the guiding structure. This is a necessary condition for the existence of guided mode. Thus, in the electrostatic limit under consideration, the wave vector k is the same over all the regions of the structure.

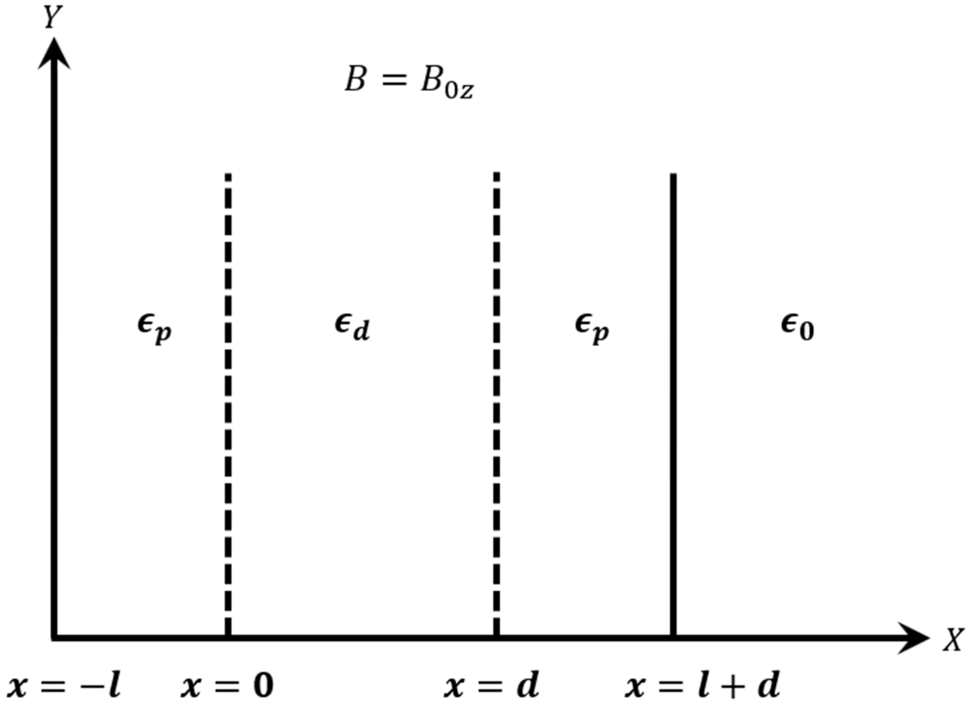

FIG. 1. Geometry of the problem.

Then, the electric field $E_{\mathrm{y}}(x, y)$ in each slab of the waveguide geometry of Fig. 1 can be written as:

$$E_{\mathrm{y}}^{(1)} = A_1\, e^{k(iy+x)} \qquad -\infty < x \leq -\ell \quad (7)$$

$$E_{\mathrm{y}}^{(2)} = A_2\, e^{k(iy+x)} + A_3\, e^{k(iy-x)} \qquad -\ell \leq x \leq 0 \quad (8)$$

$$E_{\mathrm{y}}^{(3)} = A_4\, e^{k(iy+x)} + A_5\, e^{k(iy-x)} \qquad 0 \leq x \leq d \quad (9)$$

$$E_{\mathrm{y}}^{(4)} = A_6\, e^{k(iy+x)} + A_7\, e^{k(iy-x)} \qquad d \leq x \leq d + \ell \quad (10)$$

$$E_{\mathrm{y}}^{(5)} = A_8\, e^{k(iy-x)} \qquad d + \ell \leq x < \infty \quad (11)$$

In the electrostatic limit, applying the boundary conditions returns the continuity of the tangential component of the electric field and the normal component of the electric displacement vector $\vec{D} = \varepsilon\vec{E}$. Accordingly, matching the tangential electric field E_{y} and the normal electric displacement D_{x} components at all interfaces leads to the following 8×8 homogeneous system of equations:

$$A_1 e^{-k\ell} - A_2 e^{-k\ell} - A_3 e^{k\ell} = 0 \quad (12)$$

$$\frac{A_1}{\varepsilon_p} e^{-k\ell} - A_2 e^{-k\ell} + A_3 e^{k\ell} = 0 \quad (13)$$

$$A_2 + A_3 - A_4 - A_5 = 0 \quad (14)$$

$$\frac{\varepsilon_p}{\varepsilon_d} A_2 - \frac{\varepsilon_p}{\varepsilon_d} A_3 - A_4 + A_5 = 0 \quad (15)$$

$$A_4 e^{kd} + A_5 e^{-kd} - A_6 e^{kd} - A_7 e^{-kd} = 0 \quad (16)$$

$$A_4 e^{kd} - A_5 e^{kd} - \frac{\varepsilon_p}{\varepsilon_d} A_6 + \frac{\varepsilon_p}{\varepsilon_d} A_7 = 0 \quad (17)$$

$$A_6 e^{k(d+\ell)} + A_7 e^{-k(d+\ell)} - A_8 e^{-k(d+\ell)} = 0 \quad (18)$$

$$A_6 e^{k(d+\ell)} - A_7 e^{-k(d+\ell)} + \frac{A_8}{\varepsilon_p} e^{-k(d+\ell)} = 0 \quad (19)$$

3. Numerical Analysis and Example

In Figs. 2 and 3, we use Eqs. (12-19) together with Eq. (5) to plot the normalized mode frequency $\omega/\omega_{\mathrm{pe}}$ versus $k\lambda_{\mathrm{D}}$ for the case of vacuum as a central region between the plasma slabs with $\varepsilon_d = 1$. Here, $\lambda_{\mathrm{D}} = v_{th,e}/\omega_{pe}$ is the electron Debye wavelength. The values of $k\lambda_{\mathrm{D}} \ll 1$ are within the validity of the warm plasma approximation (long wavelength limit) [21-25].

The curves in Fig. 2 show the normalized mode frequencies for different values of cyclotron frequency $\omega_{ce}/\omega_{\mathrm{pe}}$. For $\omega_{ce} = 0$, we observe two well known electrostatic modes, which emerge from $\omega = 0$ and $\omega = \omega_{\mathrm{pe}}$. Both modes become coupled as $k\lambda_{\mathrm{D}}$ increases and eventually degenerate into a single mode at the quasi-static frequency $\omega = \frac{\omega_{pe}}{\sqrt{2}}$ of the plasma-vacuum interface [13, 15, 25]. By increasing the magnetic field, the mode frequencies shift up and become coupled at higher quasi-static surface wave frequency $\omega = \omega_{uh}/\sqrt{2}$. The shift is of the order of $\Delta\omega = \omega_{uh} - \omega_{pe}$. The bulk plasma mode at $\omega = \omega_{pe}$ for $\omega_{ce} = 0$ shifts upward toward the upper hybrid frequency $\omega_{uh} = \sqrt{\omega_{pe}^2 + \omega_{ce}^2}$. For $\omega_{ce} = \omega_{pe}$, for example, the upper hybrid frequency is $1.4\omega_{pe}$ with an upward shift of $\Delta\omega = 0.4\omega_{pe}$.

Curves of Fig. 3 show the normalized mode frequencies for different values of vacuum slab width to plasma slab width d/ℓ. To find out the effect of the width of the central vacuum slab on the coupling of the electrostatic surface modes in a magnetoplasma, we consider the fixed value $\omega_{ce} = \omega_{pe}$ as a representative case. In the absence of the central slab (i.e., $d = 0$), the magnetoplasma slab has a width of 2ℓ and is surrounded by semi-infinite vacuum regions. In this case, we observe two uncoupled modes; namely, $\omega = \omega_{uh} = 1.4\omega_{pe}$ and a second mode, which emerges from the bulk plasma mode and approaches the quasi-static value $\omega \approx \omega_{uh}/\sqrt{2} = 1.22\omega_{pe}$. For $d \neq 0$, the bulk upper hybrid mode at $\omega = \omega_{uh} = 1.4\omega_{pe}$ and the bulk plasma mode at $\omega = \omega_{pe}$ become coupled and the coupled mode frequency approaches the upper hybrid quasi-static value $\omega = \omega_{uh}/\sqrt{2} = 1.22\omega_{pe}$.

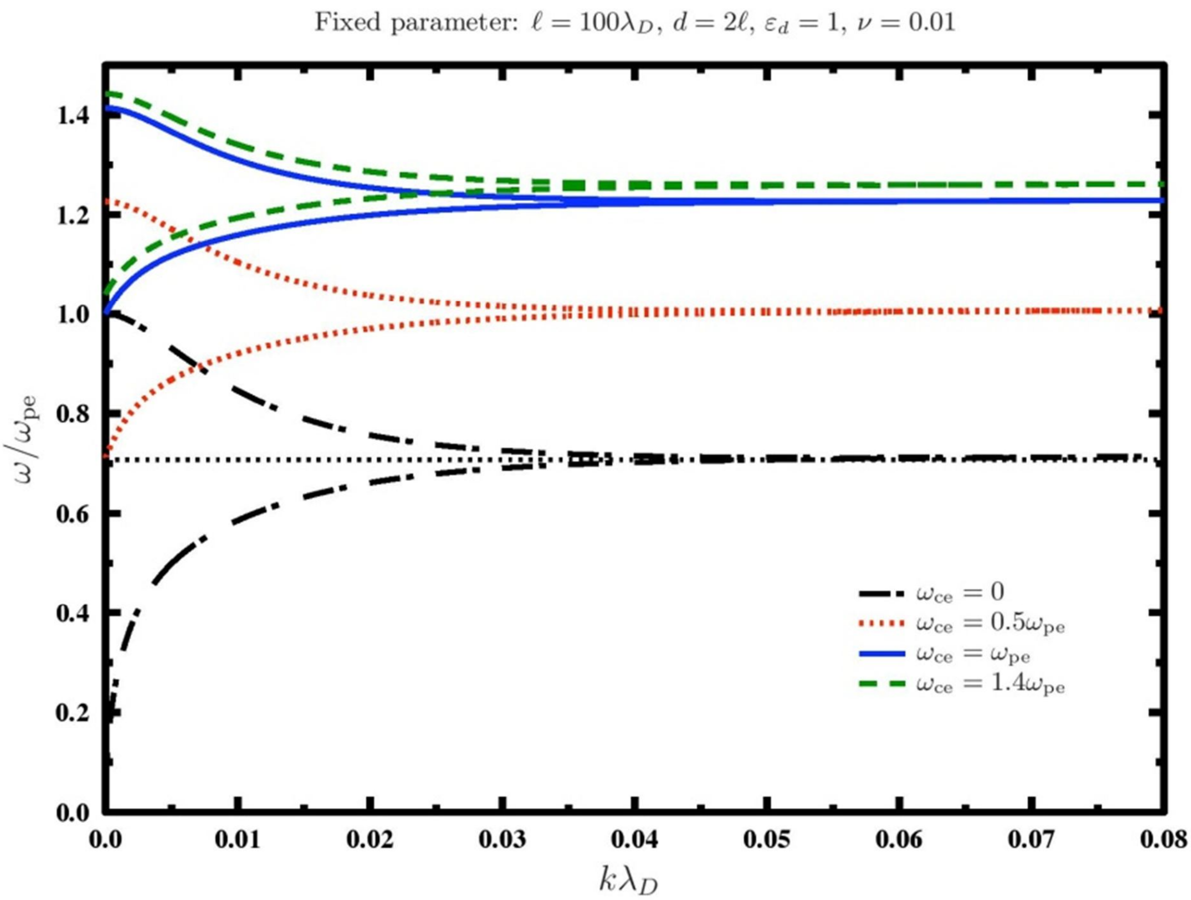

FIG. 2. Coupled plasma modes for different magnetic fields.

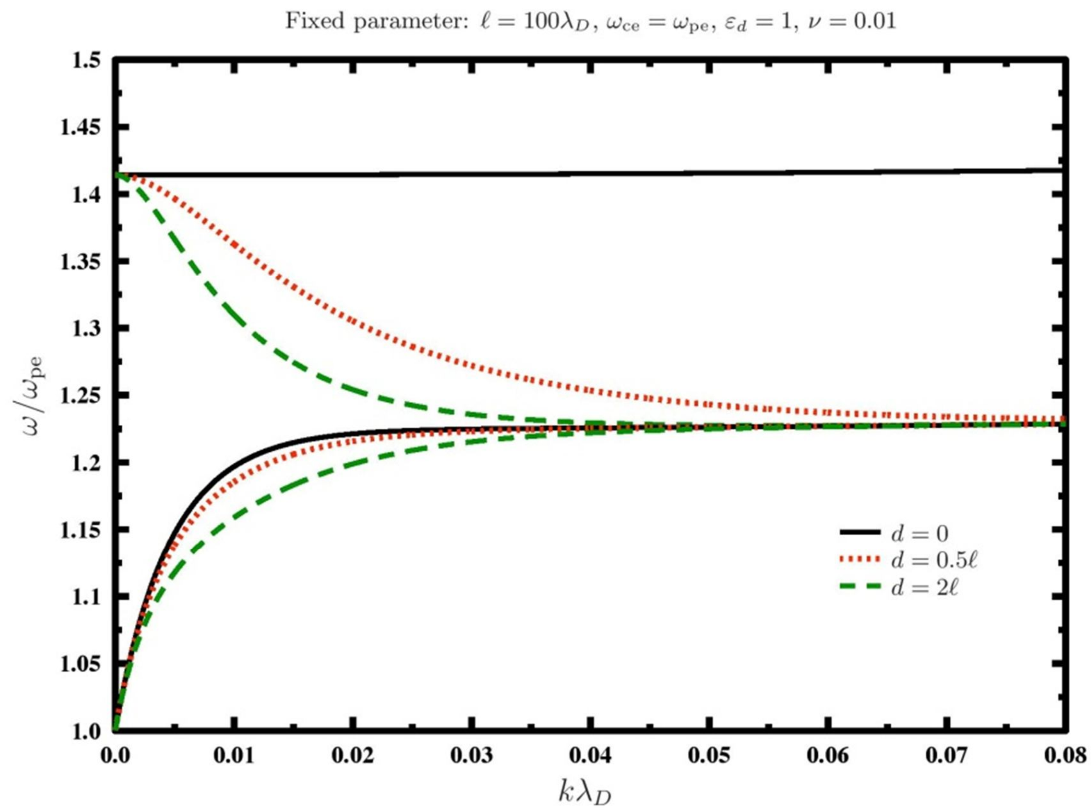

FIG. 3. Coupled plasma modes for different central slab thicknesses.

The existence of these upper hybrid frequencies has been verified experimentally by studying the microwave transmission across a magnetic field. As the plasma density is varied, the transmission through the plasma exhibits a dip at the density value that makes ω_{uh} equal to the applied frequency: upper hybrid oscillations are excited, and energy is absorbed from the beam.

4. Discussion and Conclusions

This work has been devoted to the investigation of the coupling of electrostatic surface magnetoplasma modes supported by two identical parallel plasma slabs separated by a vacuum slab. The general characteristics for the surface magnetoplasma modes are obtained by simultaneously solving the homogeneous system of Eqs. (12-19) together with Eq. (5). The normalized mode frequencies $\omega/\omega_{\mathrm{pe}}$ have been plotted versus $k\lambda_{\mathrm{D}}$ in the range of the validity of the warm plasma approximation $k\lambda_{\mathrm{D}} \ll 1$. A numerical value of the collision frequency with neutral $\nu = 0.01$ keeps the magnetoplasma slabs non-collisional. It is well known in literature that the effect of collisions is to down shift the surface wave spectra and the corresponding quasi-static resonance frequency [15, 25].

The curves in Figs. 2 show the existence of backward and forward propagating electrostatic modes for $\omega_{ce} = 0$ or $\omega_{ce} \neq 0$. The backward mode emerges at $\omega = \omega_{\mathrm{u}h} = \sqrt{\omega_{\mathrm{p}e}^2 + \omega_{ce}^2}$, which reduces to $\omega = \omega_{\mathrm{p}e}$ for unmagnetized plasma. The second mode is a forward propagating mode, which emerges at a lower value $\omega = \omega_{\mathrm{u}h} - \omega_{\mathrm{p}e}$ and becomes $\omega = 0$ in the absence of the dc-magnetic field [13, 15].

Backward and forward electrostatic magnetoplasma surface modes for different values of d/ℓ at fixed value $\omega_{\mathrm{ce}} = \omega_{\mathrm{pe}}$ are shown in Fig.3. When the central vacuum slab is absent ($d = 0$), the magnetoplasma slab supports two uncoupled modes at $\omega = \omega_{\mathrm{u}h} = 1.4\omega_{\mathrm{p}e}$ and $\omega = \omega_{\mathrm{pe}}$. For $d \neq 0$, the bulk upper hybrid mode at $\omega = \omega_{\mathrm{u}h} = 1.4\omega_{\mathrm{p}e}$ and the bulk plasma mode at $\omega = \omega_{pe}$ become coupled and approach the upper hybrid quasi-static resonance value $\omega = \omega_{\mathrm{u}h}/\sqrt{2} = 1.22\omega_{\mathrm{pe}}$.

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