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ARTICLE

Estimating Fundamental Parameters of Celestial Objects Using Some Observational Parameters

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Abstract: Far away objects seem to appear smaller than the original size. There is confusion about the concept of the perceived size of objects based on distance. In this work, new mathematical formulae are given for the apparent size of objects in both isoscele view and scalene view in terms of absolute distance. The new formulae for the temperature of black bodies and the true diameter of luminous objects are obtained from the apparent size relations. The present study will highlight the diameter of luminous objects and the expression for the temperature of black bodies.

Keywords: Apparent size, Appearance, Isoscele view, Scalene view.

1. Introduction

The apparent size of an object changes with the distance and orientation of the object. The parallel lines seem to be converged at some point. Boring (1964) [1] gave only a theoretical explanation of the concept of converging parallel lines. A few years ago, many experiments were conducted on the perceived size of objects based on distance. Holway and Boring (1941) developed the concept of perceived size of objects based on perceived distance. According to them, the perceived size is a function of perceived distance. In many studies, there is evidence that the relationship between perceived size and distance is unclear [2]. The constancy in the apparent size of the object is commonly known as size constancy and is also based on the perceptual distance from the observer [1]. The application of Titius-Bode law [3] to study the exoplanetary systems results showed that it can be applied to predict the positions of exoplanets and to study their deviations. The Schwarzschild spacetime [4] is applied to determine the relation between the observed flux and the strength of gravitational field of a neutron star and the emission angle. Thouless (1931) concluded that the apparent size of the object agrees with the absolute size constancy with the angle subtended by the object. The accurate values of the apparent size of the objects couldn’t be given and the useful applications were not developed from the relationships of perceived size. The concept of apparent size of the object with the absolute distance is not analyzed. So, in this study, new formulae are obtained based on the absolute distance to objects from the observer. The formulae given in this work can be used to study the perceived size of the object and the applications introduced from the apparent size relationships. The derived relationships from apparent size formulae are the true diameter expression for the luminous body and the temperature of black-body expression. The temperature of the blac-kbody is expressed only in terms of angular subtense and the apparent intensity of that black-body. The applications derived in this work can play a dominant role in astronomy.

In this work, we are considering a unique view that can range from $0° \, to \, 180°$ by keeping the apparent size and perceived size terms to be

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the same. Consider an isoscele view of visual angle θ towards length **s** of two points A and B.

Let d be the absolute distance from the observer O to the midpoint of the length s. The apparent length of AB decreases with the distance d from the observer due to the decrease in visual angle. At $\theta = 0, \dot{Ş} = 0,$ where, $\dot{Ş}$ = apparent length and *s* = absolute length.

2. Law of Appearance

The visible length $\dot{Ş}$ to the observer is directly proportional to the square of the absolute length s of the object and inversely proportional to the square of the absolute distance *d* from the observer O to the midpoint of the length of the object AB (Fig. 1).

$$\dot{Ş} \propto \frac{s^2}{d^2}$$

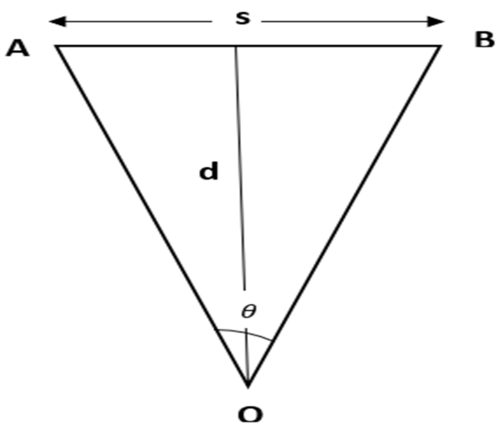


FIG. 1. Isoscele view from O towards length s.

So, by removing the proportionality limit, we get:

$$\dot{Ş} = \frac{ks^2}{d^2} \tag{1}$$

where, d = distance to the midpoint of the length of an object from the observer (in isoscele view), s = absolute length of the object. Here, the constant k is relative. It depends on the person or instrument like a telescope. We can deduce the law of size constancy from the required appearance law (1).

For the same apparent size of images, $\dot{Ş}_1 = \dot{Ş}_2$

$$\frac{s_1}{d_1} = \frac{s_2}{d_2}\,. \tag{2}$$

This is the law of size constancy in terms of absolute distance. From this result, Emmert's law holds only for the same apparent size of objects.

3. Apparent Size of the Object in Terms of Angular Diameter

Consider the isoscele view of visual angle θ towards length s of two points A and B. Let d be the absolute distance from the observer to the midpoint of the length s. Due to isoscele view, the distance between point A and the observer o and the distance between point B and the observer o are equal. Let us call this distance OA = r. By cosine law, $s^2 = r^2 + r^2 - 2r^2 \cos\theta$, then we obtain $d = \frac{s(1+\cos\theta)}{2\sin\theta}$ and again simplifying this, we will get $d = \frac{s\cot(\frac{\theta}{2})}{2}$. We can use this in (1) to get:

$$\dot{Ş} = 4k\tan^2\frac{\theta}{2}\,. \tag{3}$$

This is the relation to the apparent size of the object in terms of angular arc. If we know the angular subtense of the object, we can determine the apparent size of the object without having the true size of the object.

Here, If d = s, then $\dot{Ş}$ = k. Hence, (3) is based on this condition, $\dot{Ş}$ = k, then we get $\theta = \pi$ and $\theta = 53.13010235°$. The value of $\theta = \pi$ can be ruled out due to that the apparent size of the object is undetermined at the visual angle *180*°. So, at the value of $\theta = 53.13010235°$, the apparent length is equal to the constant k if and only if $d = s$.

4. Constant for Appearance Formula

The constant k is relative. If we consider the scale of the sun for apparent size, the sun appears as a basketball-size of 24.2 cm in diameter. The distance between the sun and earth $d_s = 149597871\ km$ and the diameter of the sun $s_s = 1.3927 \times 10^6\ km$. We can obtain the value of a constant by using (1); that is:

$$k = 2.79222904678658\ km\,. \tag{4}$$

5. Apparent Size of Moon

We calculate the apparent size of the moon to support the appearance law (1). So, the distance between earth and moon $d_m = 384{,}400\ km$ and the diameter of the moon $s_m = 3474.2\ km$. By substituting these values in (1), we can get the apparent size of the moon $\dot{Ş} = 22.80\ cm$. Therefore, the apparent sizes of the sun and moon are having closer values due to closer values of their angular diameters.

6. Apparent Size of the Venus

The closest distance between earth and venus $d_v = 38 \times 10^6\ km$. The diameter of venus $s_v = 12104\ km$. By substituting these values in (1), we can get the apparent size of venus $\dot{S} = 0.2832968\ mm$. Due to more reflection from the surface of venus, we observe that the size of venus is a little more than that.

In the real world, we don't stick only to the isoscele view. Often, we also encounter the scalene view.

7. View in a Scalene Triangle Form

If the line is aligned in the view so that the visual angle is 0°, then the apparent size of the line is zero. So, if we consider a scalene view of length AB (Fig. 1), this can be projected into the half part AC of the isoscele view and the view having 0° visual angle. Let $\angle ABC = \alpha$ and by applying sine rule to ΔOAB, we get $s \times (\sin\alpha) = r_1 \sin\theta_1$. If we use this in (1), we get $\dot{S}' = k\tan^2\theta_1$. This is the apparent size of the projected scalene view into the isoscele view. Also, by dividing the required $\dot{S}'$ with $\sin\alpha$ to get the apparent size $\dot{S}$ of scalene view AB, we get:

$$\dot{S} = \frac{ks\ tan^2\theta_1}{r_1 \sin\theta_1}. \tag{5}$$

If $\theta_1 = \theta$, then (5) tends to isoscele view (Fig. 2). Then,

$$\dot{S} = \frac{kstan^2\,\theta}{\sqrt{\left(\frac{s}{2}\right)^2 + d^2 sin\,\theta}}. \tag{6}$$

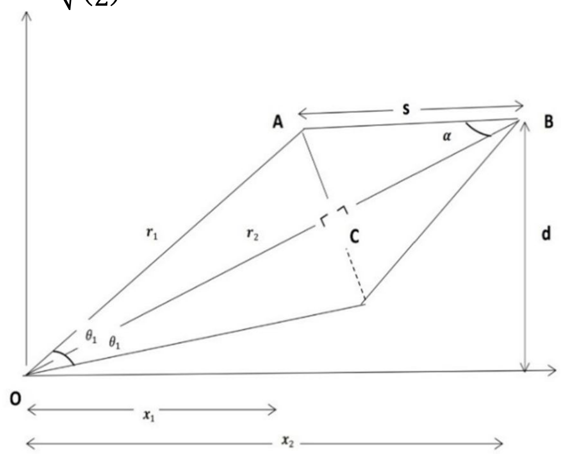


FIG. 2. View in the form of scalene triangle from observer O.

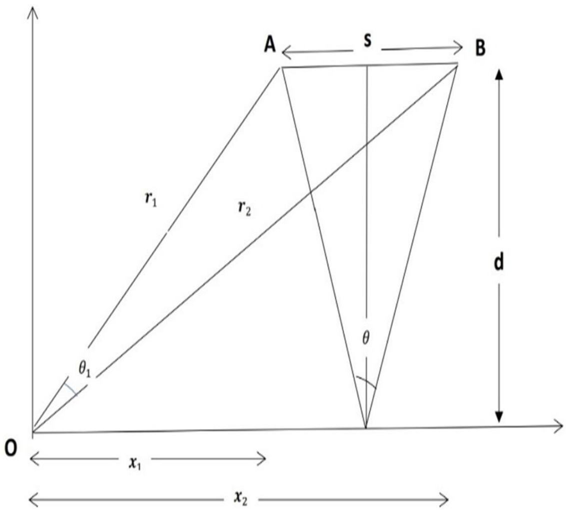


FIG. 3. Scalene view reduced to isoscele view with angular size **θ.**

This is the isoscele view relation obtained from the scalene view in Eq. (5). By arranging Eq. (6) in terms of only angular size, substituting $d = \frac{s\cot(\frac{\theta}{2})}{2}$, we will get:

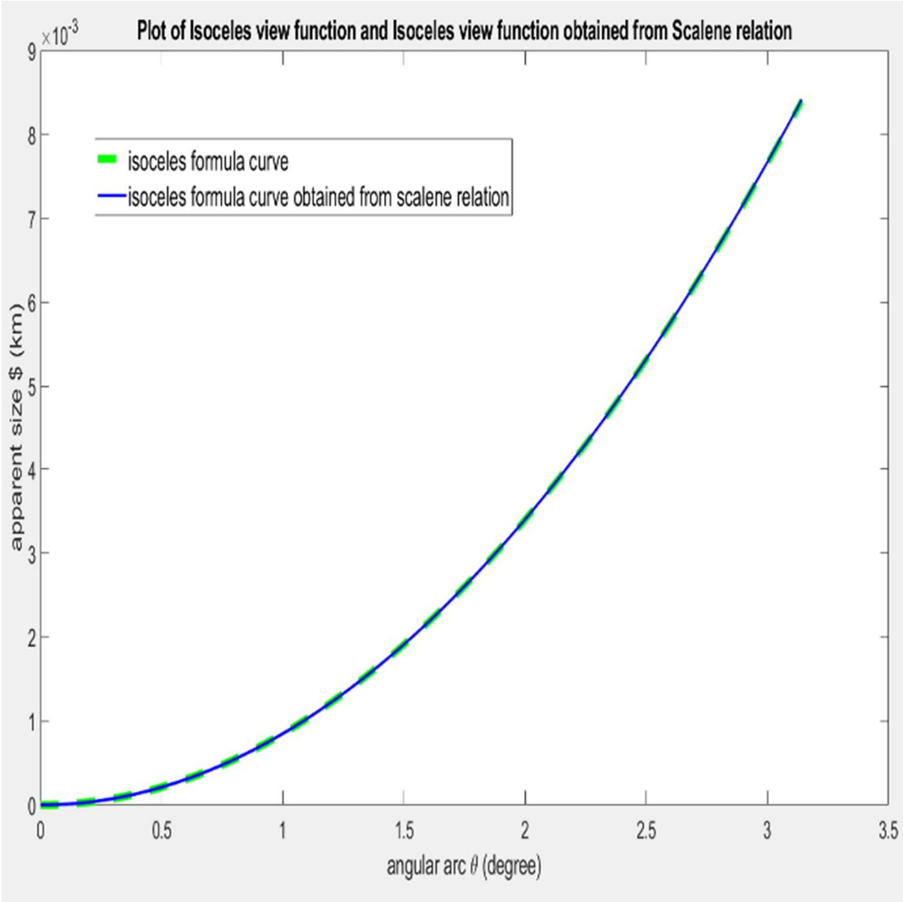
$$\dot{S} = \frac{ktan^2\,\theta}{\cos(\frac{\theta}{2})}. \tag{7}$$

If we plot the relations of the isoscele view of Eqs. (3) & (7) in the same graph (Graph 1), we will see that both curves are coinciding with each other at smaller angular arcs of the objects.

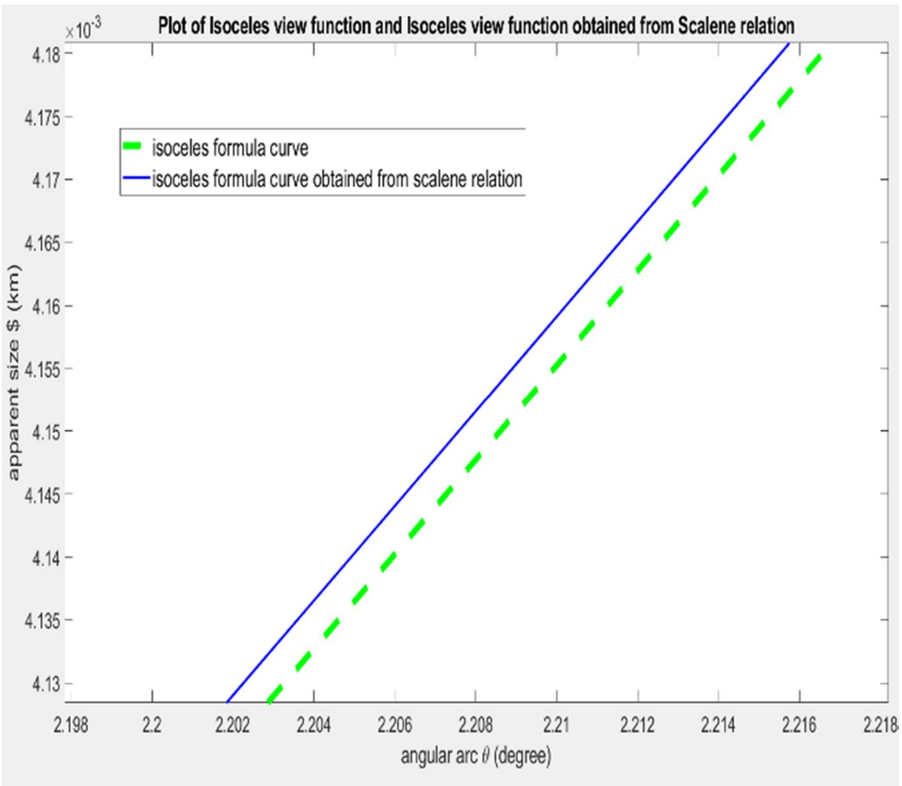
But, when we zoom Graph 1 vigorously, we will see that the curves do not perfectly coincide with each other (Graph 2).

We can find an error in isoscele view Eq. (7) obtained from scalene relation by subtracting isoscele Eq. (3). So, we can plot the error curve by the difference of Eqs. (7) & (3) using MATLAB.

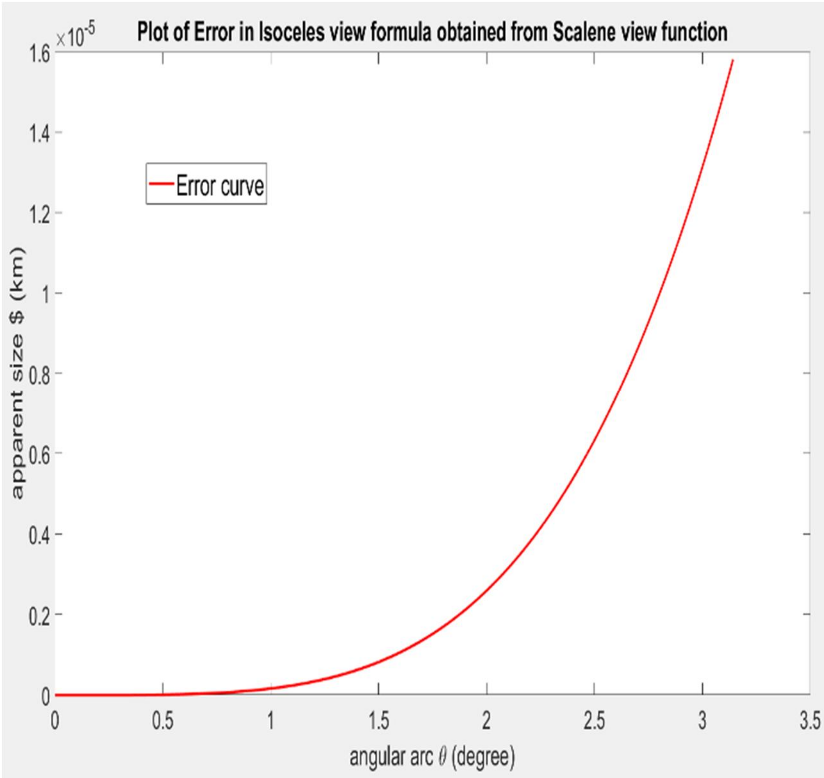
The scalene relation is a good approximation for only small angular sizes of objects or far away objects. The error in scalene relation increases if the angular size of the object increases.



GRAPH 1. Curves of Eqs. (3) & (7).



GRAPH 2. Plot of isoscele view function obtained from scalene function.



GRAPH 3. Error curve obtained by the difference of the functions from Eqs. (7) and (3).

8. Apparent Size of the Moon by Scalene View Relation

The apparent size of the moon can also be found through the isoscele view obtained from the scalene view relation to support the required Eq. (6). So, the distance between moon and earth $d_m = 384{,}400\ km$, the diameter of the moon $s_m = 3474.2\ km$ and the angular arc of the moon $\theta_m = 0.52°$. If we find the apparent size of the moon by substituting these values in (6), we get the apparent size of the moon, $\dot{\c{S}}_m = 22.90\ cm$.

The apparent size of the moon calculated by using the isoscele view Eq. (1) is 22.80 cm. It is almost similar to the apparent size calculation of the moon by using scalene view Eq. (6).

9. Diameter of the Luminous Object

We perceive that the intensity of far away luminous bodies is smaller than the absolute intensity. So, the apparent intensity is given as:

$$I = {}^{L}\!/_{4\pi d^2} \tag{8}$$

where, L = luminosity of the body, d = distance measured from the body to our view. Substituting the value of distance d obtained from apparent intensity Eq. (8) in the appearance formula (1) gives $\dot{\c{S}} = {}^{4\pi k s^2 I}\!/_{L}$. We can use this in the isoscele view relation in terms of angular size (8),

$$s = \sqrt{\frac{L}{\pi I}}\tan\frac{\theta}{2} \tag{9}$$

which is the relation to the diameter of the luminous object. From this, we can derive the temperature of the black body using Stefan-Boltzmann's law.

10. Temperature of the Black Body

The temperature of black bodies will be derived with the help of (9). By Stefan Boltzmann Law for black bodies [5], luminosity is given as

$$L = 4\pi\sigma T^4 R^2 \tag{10}$$

where σ = Stefan Boltzmann constant = 5.67×10^{-8} W/m^2K^4, R = radius of the black body and T = temperature in kelvin. We can substitute the Stefan-Boltzmann law (10) and S = 2R directly in Eq. (9), where, R = radius of the black body and S = diameter of that black body. So, we get:

$$S^2 = \frac{4\Pi\sigma T^4 R^2}{\Pi I} tan^2\frac{\theta}{2} \tag{11}$$

If we simplify Eq. (11) further, we will get:

$$T = \left[\frac{I}{\sigma \tan^2\frac{\theta}{2}}\right]^{\frac{1}{4}} \qquad (12)$$

where I represents the apparent intensity of the black body. So, the temperature of black bodies can be found if the angular arc and apparent intensity are known, which can be measured directly.

11. Diameter of the Sun

The diameter of the sun can be easily determined to support Eq. (9) by substituting the required values in that equation. So, the angular arc of the sun, $\theta = 0.5334°$, the apparent intensity of the sun, $I_{sun} = 1360.8\, W/m^2$ and the luminosity of the sun, $L = 3.827 \times 10^{26}\, W$. We can use these values in (9) to obtain the diameter of the sun, s = 1.39 × 10^6 km

12. Temperature of the Sun

Similarly, the temperature of the sun can be determined by the temperature Eq. (12), because all stars are black bodies. The apparent Intensity of the sun, $I = 1370 W/m^2$ and the angular diameter of the sun, $\theta = 0.5334°$. We got, $T = \left[\frac{I}{\sigma \tan^2\frac{\theta}{2}}\right]^{\frac{1}{4}}$, then using this, we will get the temperature of the sun,$T = 5778.73\, K$.

13. Conclusions

We have derived the relationships for apparent sizes in both isoscele and scalene views. Also, we derived the relative constant to verify the appearance law in calculating the apparent sizes of objects. Applications, such as the expression for the diameter of luminous objects and the expression for the temperature of black bodies, are also developed from the apparent size relationships.

Appendix I

In this appendix, we calculate the diameter of the Sirius star from $s = \sqrt{\frac{L}{\pi I}} \tan\frac{\theta}{2}$ to support the above derivation for the diameter relation. So, the luminosity of Sirius, $L = 25.4\, L⊙$. So, $\mathrm{L} = 25.4 \times 3.827 \times 10^{26}\, W$ and the apparent intensity of Sirius, $I = 10^{-7} \frac{W}{m^2}$. The new accurate angular diameter measurement was recently found [6]. So, the angular arc of Sirius, $\mathrm{S} = 5.63 \pm 0.08\ arc\, ms = 1.5638888 \times 10^{-6}\,°$, then we get the diameter of Sirius, S = 2.4 × 10^6 km.

Appendix II

In this appendix, we calculate the temperature to support the derivation of the temperature relation, $T = \left[\frac{I}{\sigma \tan^2\frac{\theta}{2}}\right]^{\frac{1}{4}}$. So, the angular arc of Sirius, $\theta = 1.5638888 \times 10^{-6}\,°$ and the apparent intensity of Sirius, $I = 10^{-7} W/m^2$. From there, we get the temperature of the Sirius star, T = 9864.56 K. Here, the approximate value of the apparent intensity of Sirius is taken. So, that is why the calculated value is approximately close to the absolute temperature.

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