

LRS Bianchi Type-II Cosmological Model with New Holographic Dark Energy in General Relativity with Quintessence

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Abstract: In this paper, we investigate the LRS Bianchi Type-II universe filled with pressureless cold dark matter and non-interacting new holographic dark energy in the framework of general relativity. To obtain the deterministic solution of Einstein's field equations, we assume the scalar of expansion to be proportional to the eigenvalue of the shear tensor. The expressions for some parameters of cosmological importance are obtained and physical and geometrical properties of the model are discussed. The correspondence between the new holographic dark energy and quintessence scalar field model is also established by comparing their equations of state and densities. Our results are consistent with the recent cosmological observations.

Keywords: Bianchi type-II, Holographic principle, Dark energy, Quintessence.

1. Introduction

In the last part of the twentieth century, two teams of astronomers, one led by A. G. Riess [1] and the other by S. Perlmutter [2, 3], independently reported that our universe is currently passing through a phase of accelerated expansion. Since then several cosmological observations like Cosmic Microwave Background (CMB) [4, 5], Large Scale Structure (LSS) [6-9], Planck collaboration results [10], Baryon Acoustic Oscillations (BAO) [11] as well as improved measurements of supernovae have confirmed cosmic acceleration. In the framework of standard cosmology, an exotic component that exerts large negative pressure, dubbed dark energy (DE), is needed to explain this acceleration. The most natural candidate for dark energy is found to be the cosmological constant Λ with an equation of state $\omega = -1$. But from a theoretical point of view [12-16], the cosmological constant faces the fine-tuning and cosmic coincidence problems. Therefore, various dynamical dark energy scenarios such as quintessence [17-19], phantom [20, 21], tachyon

[22, 23], k-essence [24], dilatonic ghost condensate [25], braneworld models [26], etc. have been proposed in the literature.

The holographic principle, a quantum gravitational principle, has provided an alternative perspective on the issue of dark energy. Within this framework, a new significant contender for dark energy has emerged, known as holographic dark energy. The holographic principle was first put forward by G. 't Hooft [27] to explain the thermodynamics of black hole physics. It states that the number of degrees of freedom directly related to entropy of a system scales with the enclosing surface area of the system and not with its volume. Fischler and Susskind [28] applied this principle to the cosmological context and stated that the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past lightcone of that surface. Since then, a number of authors [29-33] have proposed various choices of IR

cutoff or second-order geometrical invariants, which led to new problems in physics. Granda and Oliveros [34] proposed a new holographic dark energy density of the form $\rho_{NHDE} \approx \alpha H^2 + \beta \dot{H}$, where H is the Hubble parameter and α and β are constants to be determined so as to satisfy the current observational data. Granda and Oliveros [35] also established a correspondence between this new holographic dark energy with quintessence, tachyon, k-essence, and dilaton dark energy models. In recent times, Santhi *et al.* [36] have studied Bianchi type VI₀ space-time with anisotropic modified holographic Ricci dark energy in the Brans–Dicke theory of gravity. Rao and Prasanthi [37] have studied Bianchi type-III and LRS Bianchi type-I models filled with matter and modified holographic Ricci dark energy in the framework of the Saez-Ballester scalar-tensor theory of gravitation. Katore and Kapse [38] have studied the anisotropic and homogeneous Bianchi type-VI₀ universe filled with dark matter and holographic dark energy in the framework of general relativity and Lyra's geometry. Saridakis [39] has presented a model of holographic dark energy in which the infrared cutoff is determined by both the Ricci and the Gauss-Bonnet invariants. This model offers the advantage of having a holographic dark energy density that remains unaffected by the future or past evolution of the universe, relying solely on its current features. Ghaffari [40] has studied the cosmological dynamics of holographic dark energy in a DGP braneworld in which the holographic parameter c^2 is slowly varying with time. In this model, it is shown that for the two famous IR cutoffs, namely the Hubble radius and the Granda-Oliveros cutoff, there is a transition from a deceleration phase to an accelerated one for the universe. Srivastava *et al.* [41] have investigated holographic dark energy with a new infrared cutoff of Granda and Oliveros in Bianchi type-III anisotropic model with the dark matter and established a correspondence between k-essence scalar field and their new holographic dark energy model. By considering the time-varying deceleration parameter, Dixit *et al.* [42] have investigated Tsallis holographic dark energy in the framework of the FRW universe. Bhattacharjee [43] has studied the dynamics of Tsallis holographic dark energy and Rényi holographic dark energy prescribed by a non-linear interaction in the FRW space-time and for

a scale factor evolving with a composite power law-exponential form.

In this paper, we consider a locally rotationally symmetric Bianchi Type-II space-time filled with a mixture of cold dark matter and non-interacting new holographic dark energy. The paper is organized as follows: In Sec. 2, we derive the field equations for the Bianchi Type-II metric within the framework of general relativity. In Sec. 3, cosmological solutions of the field equations are obtained by considering the scalar of expansion to be proportional to the eigenvalue of the shear tensor, which in turn, gives a relation between the directional scale factors. In Sec. 4, we study some physical and geometrical properties of the model. The correspondence between the new holographic dark energy and quintessence scalar field model is established in Sec. 5 and the paper is concluded with a brief discussion in Sec. 6.

2. Metric and Field Equations

The LRS Bianchi Type-II space-time can be described by the metric [44]:

$$ds^2 = -dt^2 + A^2 (dx - zdy)^2 + B^2 (dy^2 + dz^2) \quad (1)$$

where A and B are directional scale factors and are functions of the cosmic time t alone.

We assume the space-time to be filled with a mixture of cold dark matter and non-interacting new holographic dark energy (NHDE) of density ρ_{NHDE} proposed by Granda and Oliveros [34]

$$\rho_{NHDE} = 3(\delta H^2 + \beta \dot{H}) \quad (2)$$

where δ and β are constants.

Overall energy-momentum tensor can therefore be considered to consist of two different components T_{ij} and \bar{T}_{ij} so that in natural units ($8\pi G = 1, c = 1$), the Einstein field equations may be taken in the form:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} - \bar{T}_{ij} \quad (3)$$

where R_{ij} is the Ricci tensor, R , the Ricci scalar curvature, and T_{ij} and \bar{T}_{ij} are the energy-momentum tensors of pressureless cold dark matter. Thus, the new holographic dark energy is given by:

$$T_{ij} = \rho_m u_i u_j \quad (4)$$

$$\bar{T}_{ij} = (\rho_{NHDE} + p_{NHDE})u_i u_j + g_{ij}p_{NHDE} \quad (5)$$

Here, ρ_m is the energy density of cold dark matter, and ρ_{NHDE} and p_{NHDE} are respectively the energy density and pressure of new holographic dark energy.

In comoving coordinates, the Einstein field Eq. (3) for the metric in Eq. (1) is obtained as:

$$2\frac{\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{1}{4}\frac{A^2}{B^4} = \rho_m + \rho_{NHDE} \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{A^2}{B^4} = -p_{NHDE} \quad (7)$$

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{3}{4}\frac{A^2}{B^4} = -p_{NHDE} \quad (8)$$

where an over dot denotes differentiation with respect to t .

The vanishing of the divergence of Einstein tensor yields:

$$T_{j,i}^i + \bar{T}_{j,i}^i = 0 \quad (9)$$

which gives the continuity equation

$$\dot{\rho}_m + \dot{\rho}_{NHDE} + (\rho_m + \rho_{NHDE} + p_{NHDE})\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0 \quad (10)$$

For the metric in Eq. (1), some parameters of cosmological importance such as the Hubble parameter H , the deceleration parameter q , the expansion scalar θ , the mean anisotropy parameter A_m , the shear scalar σ and the spatial volume V are given by:

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \quad (11)$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \quad (12)$$

$$\theta = 3H = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \quad (13)$$

$$A_m = \frac{1}{3}\sum_{i=1}^3 \frac{[H_i - H]^2}{H^2} = \frac{2}{9H^2}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \quad (14)$$

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^3 H_i^2 - \frac{1}{3}\theta^2\right] = \frac{3}{2}A_m H^2 \quad (15)$$

$$V = AB^2 \quad (16)$$

3. Cosmological Solutions of the Field Equations

We have four equations, namely Eqs (2) and (6) - (8), with five unknowns: A , B , ρ_m , ρ_{NHDE} and p_{NHDE} . So, one more physical condition relating the unknowns is required to obtain an exact solution of the field equations. To construct the fifth equation, we assume the scalar of expansion θ to be proportional to the

eigenvalue σ_2^2 of the shear tensor σ_i^j so that we may consider the relation between the directional scale factors as:

$$A = lB^m \quad (17)$$

where l and m are positive constants.

From Eqs. (7) and (8), we have:

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \left(\frac{\dot{B}}{B}\right)^2 + \frac{A^2}{B^4} = 0$$

Using Eq. (17), we obtain:

$$(m-1)\frac{\ddot{B}}{B} + (m-1)(m+1)\left(\frac{\dot{B}}{B}\right)^2 + l^2 B^{2(m-2)} = 0 \quad (18)$$

which shows that $m \neq 1$

In order to get a solution to this differential equation, we consider \dot{B} as a function of B , and then solving (18), we get:

$$\dot{B} = \sqrt{\frac{l^2}{2m(1-m)}B^{2m-2} + k_1 B^{-2m-2}} \quad (19)$$

where k_1 is a constant of integration.

From (19), we have:

$$dt = \frac{dB}{\sqrt{\frac{l^2}{2m(1-m)}B^{2m-2} + k_1 B^{-2m-2}}}$$

Now, using the transformation $B = T$ suitably, we obtain the solution in quadrature form as:

$$ds^2 = -\frac{dT^2}{\frac{l^2}{2m(1-m)}T^{2m-2} + k_1 T^{-2m-2}} + l^2 T^{2m} (dx - zdy)^2 + T^2 (dy^2 + dz^2) \quad (20)$$

From Eq. (11), the Hubble parameter for this model is obtained as:

$$H = \frac{m+2}{3} \left[\frac{l^2}{2m(1-m)} T^{2m-4} + k_1 T^{-2m-4} \right]^{\frac{1}{2}} \quad (21)$$

Other cosmological parameters are obtained as:

$$q = -1 - \frac{3 \left[\frac{l^2(m-2)}{m(1-m)} T^{4m-2} - 2k_1(m+2) \right]}{2(m+2) \left[\frac{l^2}{2m(1-m)} T^{4m} + k_1 \right]} \quad (22)$$

$$\theta = (m+2) \left[\frac{l^2}{2m(1-m)} T^{2m-4} + k_1 T^{-2m-4} \right]^{\frac{1}{2}} \quad (23)$$

$$A_m = \frac{2(m-1)^2}{(m+2)^2} \quad (24)$$

$$\sigma^2 = \frac{(m-1)^2}{3} \left[\frac{l^2}{2m(1-m)} T^{2m-4} + k_1 T^{-2m-4} \right] \quad (25)$$

$$V = lT^{m+2} \quad (26)$$

Using (21) in (2), we obtain the new holographic dark energy density as:

$$\rho_{NHDE} = k_2 T^{2m-4} + k_3 T^{-2m-4} \quad (27)$$

where, $k_2 = \left[\frac{\delta(m+2)}{3} - \beta(m-2) \right] \frac{l^2(m+2)}{2m(1-m)}$ and $k_3 = \left(\frac{\delta}{3} - \beta \right) k_1 (m+2)^2$.

Using (27) in (6), we get:

$$\rho_m = \left[\frac{(m+1)(m+2)l^2}{4m(1-m)} - k_2 \right] T^{2m-4} + [k_1(2m+1) - k_3] T^{-2m-4} \quad (28)$$

From Eq. (8), we get:

$$p_{NHDE} = \frac{(2-3m)(m+1)}{4m(1-m)} l^2 T^{2m-4} + (2m+1)k_1 T^{-2m-4} \quad (29)$$

Hence, the equation of state (EoS) parameter of the NHDE is obtained as:

$$\omega_{NHDE} = \frac{p_{NHDE}}{\rho_{NHDE}} = \frac{\frac{(2-3m)(m+1)}{4m(1-m)} l^2 T^{4m} + (2m+1)k_1}{k_2 T^{4m} + k_3} \quad (30)$$

The NHDE and cold dark matter density parameters are obtained as:

$$\Omega_{NHDE} = \frac{\rho_{NHDE}}{3H^2} = \frac{k_2 T^{2m-4} + k_3 T^{-2m-4}}{\frac{(m+2)^2}{3} \left[\frac{l^2}{2m(1-m)} T^{2m-4} + k_1 T^{-2m-4} \right]} \quad (31)$$

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\left[\frac{(m+1)(m+2)l^2}{4m(1-m)} - k_2 \right] T^{2m-4} + [k_1(2m+1) - k_3] T^{-2m-4}}{\frac{(m+2)^2}{3} \left[\frac{l^2}{2m(1-m)} T^{2m-4} + k_1 T^{-2m-4} \right]} \quad (32)$$

Hence, the total energy density parameter is obtained as:

$$\Omega = \Omega_{NHDE} + \Omega_m = \frac{\frac{(m+1)(m+2)l^2}{4m(1-m)} T^{4m} + k_1(2m+1)}{\frac{(m+2)^2}{3} \left[\frac{l^2}{2m(1-m)} T^{4m} + k_1 \right]} \quad (33)$$

4. Some Physical and Geometrical Properties of the Model:

From Fig. 1, it is clear that initially q is positive and decreases as time evolves, and approaches -1 asymptotically. The negative value of q indicates that the cosmic expansion is accelerating at late times.

From Fig. 2, we see that the new holographic dark energy density is a decreasing function of time.

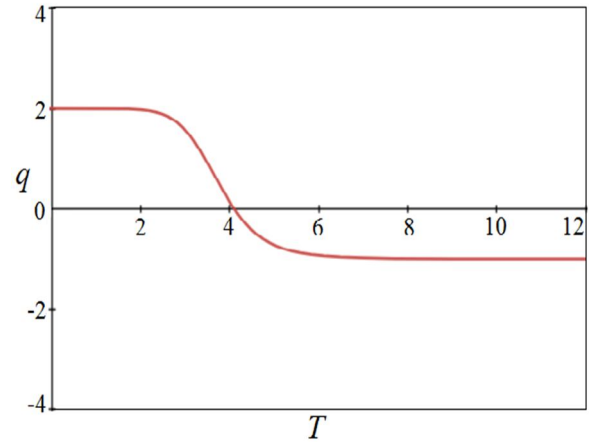


FIG 1. Deceleration parameter q vs T graph with $m = 2, k_1 = -1, l = 0.01$ (in natural units).

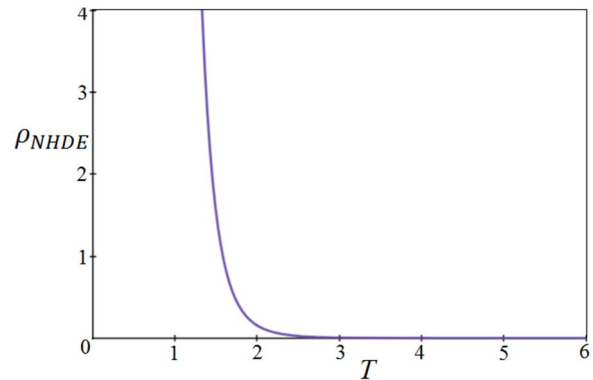


FIG 2. New holographic dark energy density ρ_{NHDE} vs T graph with $m = 2, k_1 = -1, l = 0.01, \delta = 1.5, \beta = 3$ (in natural units).

From Fig. 3, we see that the EoS parameter of NHDE is increasing with time, entering eventually the quintessence region $-1 < \omega_{NHDE} < -\frac{1}{3}$.

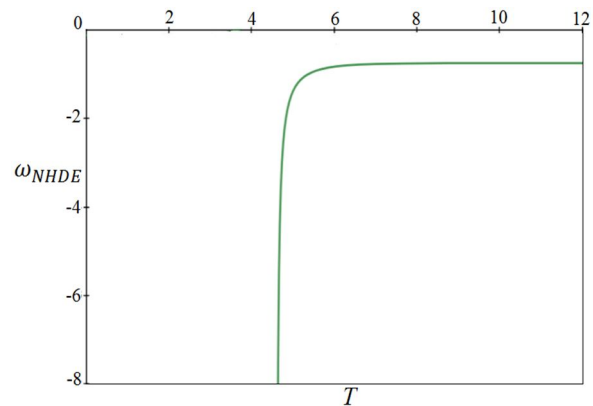


FIG 3. The EoS parameter of NHDE ω_{NHDE} vs T graph with $m = 2, k_1 = -1, l = 0.01, \delta = 1.5, \beta = 3$ (in natural units).

Fig. 4 describes the behavior of the total energy density. From the graph, we see that it is an increasing function of time and has values near 1 throughout the evolution of the universe. At a certain epoch, the total energy density crosses the isotropic background, that is, $\Omega = 1$. This shows that the model never reaches the isotropic background during late times, although it remains near the isotropic background.

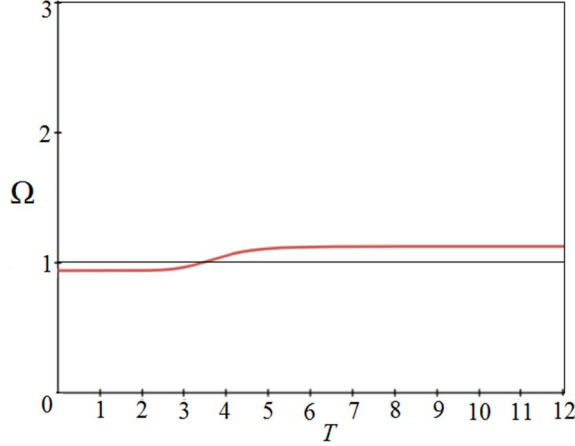


FIG 4. Total energy density parameter Ω vs T graph with $m = 2, k_1 = -1, l = 0.01$ (in natural units).

Also, since $m \neq 1$, therefore, $A_m \neq 0$ for all m . Hence, our model is anisotropic throughout the evolution.

It is also clear that at $T = 0, V = 0$ and the other parameters H, θ and σ^2 all diverge. Hence, the universe characterised by our model evolves with initial Big Bang type singularity and goes on expanding as $T \rightarrow \infty$.

5. Correspondence between New Holographic Dark Energy and Quintessence Scalar Field Model

In this section, we compare the EoS parameters and the energy densities for the new holographic dark energy and quintessence scalar field ϕ whose action is given by [45]:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi) \right] \quad (34)$$

where $V(\phi)$ is the quintessence potential of the scalar field ϕ . The Lagrangian density for the quintessence model is:

$$\mathcal{L}_\phi = -\frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi)$$

and the energy-momentum tensor of the quintessence field is

$$T_{ij}^{(\phi)} = \partial_i \phi \partial_j \phi - g_{ij} \left[\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right]$$

The energy density and pressure for the quintessence scalar field ϕ are given by:

$$\rho_\phi = -T_0^{0(\phi)} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (35)$$

$$p_\phi = T_i^{i(\phi)} = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad (36)$$

Thus, the equation of state for the quintessence scalar field ϕ is:

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \quad (37)$$

Now from (30) and (37), we get:

$$\frac{(2-3m)(m+1)l^2 T^{4m} + (2m+1)k_1}{k_2 T^{4m} + k_3} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \quad (38)$$

Again comparing (27) and (35), we get:

$$k_2 T^{2m-4} + k_3 T^{-2m-4} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (39)$$

From Eq. (38), we get:

$$\begin{aligned} \frac{1}{2} \dot{\phi}^2 = & \\ & \frac{(2-3m)(m+1)l^2 T^{4m} + (2m+1)k_1 + k_2 T^{4m} + k_3}{k_2 T^{4m} + k_3 - \frac{(2-3m)(m+1)l^2 T^{4m} - (2m+1)k_1}{4m(1-m)}} V(\phi) \end{aligned} \quad (40)$$

From (39) and (40), the potential term $V(\phi)$ is derived as:

$$V(\phi) = \frac{1}{2} \left[k_2 T^{2m-4} + k_3 T^{-2m-4} - \frac{(2-3m)(m+1)l^2 T^{2m-4} - (2m+1)k_1 T^{-2m-4}}{4m(1-m)} \right] \quad (41)$$

This type of potential function may be responsible for the accelerated expansion of the universe.

6. Conclusion

In this paper, we study the LRS Bianchi Type-II universe filled with pressureless cold dark matter and non-interacting new holographic dark energy (NHDE). To obtain an exact solution of Einstein's field equations, we assume the scalar of expansion θ to be proportional to the eigenvalue of the shear tensor σ_i^j which gives the relation between the directional scale factors as $A = lB^m$, where l and m are positive constants. Physical and geometrical properties of some cosmological parameters of our model are discussed. We find that:

- The universe evolves with a Big Bang singularity and goes on expanding with time.

- The early universe is decelerating ($q > 0$) while the present as well as the future universe is accelerating ($q < 0$).
- Although the universe is close to isotropic background, it remains anisotropic throughout its evolution.
- The new holographic dark energy density decreases with time.
- The EoS parameter of the model approaches the quintessence region $-1 < \omega_{HDE} < -\frac{1}{3}$ at late times.

We also find the correspondence between the new holographic dark energy and the quintessence scalar field and reconstruct the quintessence scalar potential.

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