Volume 17, Number 5, 2024. pp. 565-575

Jordan Journal of Physics

ARTICLE

Calculating Resistance in an Infinite Triangular Lattice Network

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Doi: **https://doi.org/10.47011/17.5.5**

Received on: 10/04/2023; *Accepted on: 27/08/2023*

Abstract: The lattice Green's function method is employed to analyze the resistance of a triangular network consisting of identical resistors in both perfect and perturbed scenarios, where a single interstitial resistance is added. In the perfect lattice scenario, all resistors are assumed to have equal resistance, and the network is considered ideal with no imperfections. To calculate the lattice Green's function and resistance of the perturbed lattice, Dyson's equation is utilized. This equation relates the perturbed and unperturbed Green's function and the interstitial resistance value. To validate the dependability of the lattice Green's function method in predicting the behavior of intricate networks, the accuracy of the computed values is evaluated by comparing them with the measured values for an infinite triangular network.

Keywords: Resistors, Lattice Green’s function, Perturbation.

1. Introduction

The lattice Green's function (LGF) is a significant aspect of the study of condensed matter physics and solid-state physics, particularly when dealing with impure solids [1]. It is utilized in various fields such as quantum mechanics, quantum field theories, and classical field theories. In quantum theory, the problem involves solving linear operator equations while taking into account specific boundary conditions. Green first introduced the fundamental concepts of Green’s function (GF) in potential theory, where he focused on solving Laplace's and Poisson's equations with different boundary conditions. Additionally, LGF is a critical tool in investigating the statistical mechanics of the spherical model [2]. Furthermore, the utilization of (LGFs) in physics presents a wide range of compelling benefits that permeate various aspects of condensed matter physics. These functions exhibit their remarkable versatility by finding applications in a diverse array of problems, spanning from lattice vibrations and luminescence to diffusion in solids and the dynamic behavior of spin waves [3]. Furthermore, LGFs play a crucial role in the theory of random walks on lattices [4] and in complex calculations of effective resistance in resistor networks [5]. These diverse applications highlight the indispensable nature of LGFs in exploring and understanding condensed matter phenomena.

Despite their utility, LGFs have certain limitations. These include their reliance on simplifying assumptions, the computational challenges associated with large systems, and their primary applicability to systems with translational symmetry [6]. Nevertheless, LGFs remain a powerful tool for analyzing quantum particles on lattices, offering significant insights into condensed matter physics and related fields.

For decades, electric circuit theorists have tackled the well-known problem of computing the resistance between two nodes in a network of resistors. This problem has been extensively studied since the time of Kirchhoff in 1847 and continues to be of interest today [7]. A variety of

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techniques have been developed over time to calculate the effective resistance between any two arbitrary sites in a resistor network [8-10]. Kirchhoff's work on electric networks, which dates back over 150 years, laid the groundwork for this research. Atkinson and Steewijk developed the approach originally used to compute the resistance between any two arbitrary nodes in an infinite lattice of identical resistors with square, triangular, hexagonal, and hypercubic geometries, as outlined in reference [11]. Jeng [12] introduced a novel approach that maps a random walk problem onto a resistor network, enabling the calculation of effective resistance between any two nodes in an infinite two-dimensional square lattice of unit resistors.

Cserti [5] proposed an alternative method leveraging the lattice Green's function (LGF) instead of using the superposition of current distributions. Expanding on this work, Cserti *et al.* [13] investigated perturbed networks by removing a bond (resistor) from a perfect lattice. They expressed the resistance between arbitrary nodes in terms of the resistance in the unperturbed lattice.

In an innovative contribution, Wu [14] presented a method for determining the effective resistance between any two nodes in a resistor network, applicable to both finite and infinite networks. Wu's approach uses the eigenvalues and eigenfunctions of the Laplacian matrix associated with the network. Furthermore, Wu derived equations tailored to one-, two-, and three-dimensional networks with various boundary conditions to compute the resistance.

Asad *et al.* [15] examined multiple infinite networks of capacitors using the LGF, while Hijjawi *et al*. [16] determined the capacitance of infinite networks by removing a single bond from a perfect lattice. Asad *et al*. [17] used the superposition principle and charge distribution to explore infinite networks of identical capacitors, while Cserti *et al.* [18] developed a general method for obtaining the electric resistance between any two nodes in an infinite lattice structure of resistors that forms a periodic tiling of space. Our research aims to investigate the impact of adding a single interstitial resistance to an infinite network. In this research, we utilized the powerful GF method to investigate the resistance characteristics of a perfect and infinite triangular network consisting of identical resistors. The results obtained through this approach highlight the intrinsic symmetry present in the lattice structures. By deriving recurrence relations, we precisely calculated the resistances between any two arbitrary sites on the infinite triangular lattice. For analyzing the resistance in a perturbed lattice network, we employed the remarkable efficiency and elegance of the Dirac notation, which represents the pinnacle of problem formulation.

The article is organized into five sections. Section 2 introduces the theoretical framework, detailing the formulae that relate the resistance in perfect, infinite triangular networks of identical resistors to the LGF of the Tight Binding Hamiltonian. Section 3 focuses on the LGF and resistance in a perturbed infinite triangular network, with particular attention to the case of a single interstitial resistance. Section 4 applies the theoretical concepts to computational models, presenting results derived using both analytical methods and Circuit Maker software. Finally, Section 5 concludes the study by summarizing the key findings and insights gained from the research.

2. Perfect Triangular Lattice

This section outlines the methodology used to calculate the LGF and effective resistance in a flawless, infinite triangular network composed of resistors with uniform resistance R. The notation $I(\vec{r}_l)$ is employed to indicate the current flowing into a specific site $\vec{r}_l$, while $V(\vec{r}_l)$ represents the potential at that site [5].

$$\sum_{i=1}^{3}[\,V(\vec{r}_l+\vec{a}_i)-2V(\vec{r}_l)+V(\vec{r}_l-\vec{a}_i)]=-I(\vec{r}_l)R \tag{1}$$

Here, $a_i(i=1,2)$ are the independent primitive lattice vectors, each of which has the same magnitude a, referred to as the lattice constant.

If we consider a three-dimensional lattice and assign the basis vector $|l\rangle$ to represent the lattice point $\vec{r}_l$, then:

$$V(\vec{r}_l)=\langle l|V\rangle \; and \; I(\vec{r}_l)=\langle l|I\rangle \tag{2}$$

It is assumed that the set of basis vectors $|l\rangle$ is complete and orthonormal, meaning that the Kronecker delta function is used to define the inner product between any two basis vectors, and the sum of the outer product of all basis vectors gives the identity matrix. By expressing the vectors $|V\rangle$ and $|l\rangle$ in terms of the lattice basis, it is possible to represent these vectors as linear combinations of the basis vectors.

$$|V\rangle = \sum_l |l\rangle \langle l|V\rangle = \sum_l |l\rangle V(\vec{r}_l) \quad (3)$$

$$|I\rangle = \sum_l |l\rangle \langle l|I\rangle = \sum_l |l\rangle I(\vec{r}_l) \quad (4)$$

Equation (1) takes the form:

$$\sum_n \sum_{i=1}^{3} [\langle l+i|n\rangle \langle n|V\rangle - 2\langle l|n\rangle \langle n|V\rangle + \langle l-i|n\rangle \langle n|V\rangle = -\langle l|I\rangle R] \quad (5)$$

Equation (5) may be expressed as:

$$\sum_n \sum_{i=1}^{3} [\delta_{l+i,n} - 2\,\delta_{l,n} + \delta_{l-i,n}] \langle n|V\rangle = -\langle l|I\rangle R \quad (6)$$

in which we have utilized:

$$V(\vec{r}_l) = \langle l|V\rangle = \sum_n \langle l|n\rangle \langle n|V\rangle = \sum_n \delta_{l,n} V(\vec{r}_n) \quad (7)$$

Equation (1) is represented using the Dirac vector space notation as:

$$L_o\, |V\rangle = -R\, |I\rangle \quad (8)$$

The operator L_o is referred to as the Laplacian operator of the unperturbed lattice.

The LGF for a perfect lattice is defined as follows [6]:

$$L_o\, G_o = -1 \quad (9)$$

The resistance value between two specific sites, $\vec{r}_i$ and $\vec{r}_j$, can be determined by the following formula [5]:

$$R_o(i,j) = 2R\,[G_o(i,i) - G_o(i,j)] \quad (10)$$

where the diagonal and off-diagonal elements of the LGF are represented by $G_o(i,i)$ and $G_o(i,j)$, respectively.

The definition of the LGF for the triangular lattice is as follows [5]:

$$G_o(l,m) = \int_{-\pi}^{\pi} \frac{dx_1}{2\pi} \int_{-\pi}^{\pi} \frac{dx_2}{2\pi} \frac{e^{i(lx_1 + mx_2)}}{2\sum_{i=1}^{3}(1-\cos x_i)} \quad (11)$$

By applying Eq. (11), one can express the resistance between the origin and the lattice point $\vec{r}_l = (l,m)$ in an infinite triangular lattice as:

$$R_o(l,m) = R \int_{-\pi}^{\pi} \frac{dx_1}{2\pi} \int_{-\pi}^{\pi} \frac{dx_2}{2\pi} \frac{1-e^{i(lx_1 + mx_2)}}{\sum_{i=1}^{3}(1-\cos x_i)} \quad (12)$$

3. Single Interstitial Resistance in a Perfect Lattice

In this section, the GF technique is employed to calculate the resistance of a modified lattice known as the perturbed lattice. This lattice is created by introducing an additional resistance, referred to as an interstitial resistance, between the ends of a diagonal bond ($\vec{r}_{io}$,$\vec{r}_{jo}$). To better understand this concept, we examine a triangular configuration of identical resistors, denoted by R, in which one of the triangular unit cells is diagonally shunted by an extra resistance of $R' = \alpha R$, as shown in Fig. 1. In this context, α represents a constant with specific values. Specifically, we consider two values for α: 0.5 and 1.

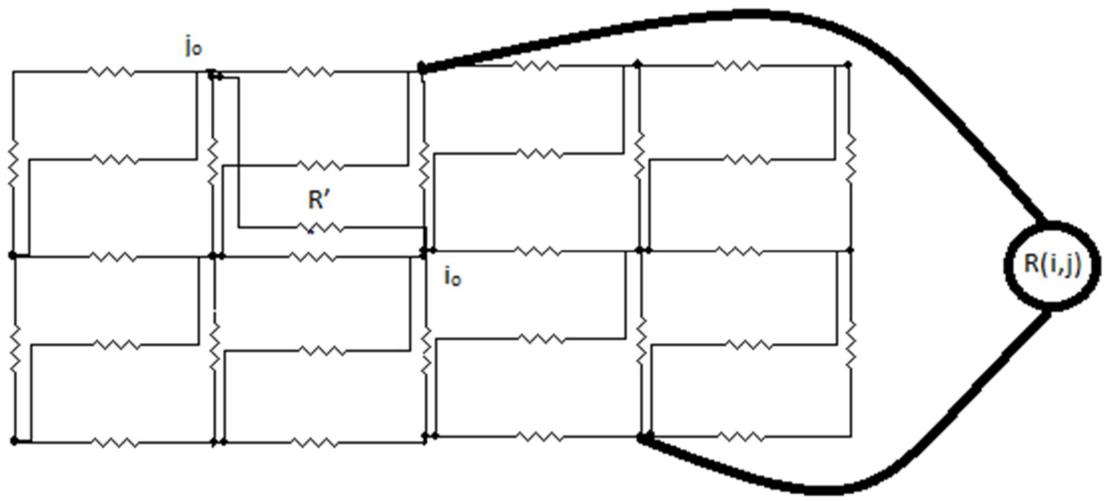

FIG. 1. Perturbation of a triangular lattice of resistors R by inserting an interstitial resistance R' between the endpoints ($\vec{r}_{io}$,$\vec{r}_{jo}$) of the diagonal bond.

In order to find the total resistance between two endpoints of the interstitial resistance, we need to calculate the equivalent resistance. This can be done by combining the interstitial resistance, represented by R', with the resistance across the diagonal bond ($\vec{r}_{io}$,$\vec{r}_{jo}$) in an ideal lattice, in parallel. The current contribution at site $\vec{r}_i$ that results from the diagonal bond

$(\vec{r}_{io},\vec{r}_{jo})$ in the perfect lattice is referred to as $\delta I(\vec{r}_i)$:

$$\delta I(\vec{r}_i)R = \delta(\vec{r}_i,\vec{r}_{io})\left[V(\vec{r}_{io}) - V(\vec{r}_{jo})\right] + \delta(\vec{r}_i,\vec{r}_{jo})[V(\vec{r}_{jo}) - V(\vec{r}_{io})] \quad (13)$$

The introduction of the interstitial resistance $R^{'}$ between the ends of the diagonal bond $(\vec{r}_{io},\vec{r}_{jo})$ results in a current contribution $\delta I^{'}(\vec{r}_i)$ at site $\vec{r}_i$, which can be expressed as follows:

$$\delta I^{'}(\vec{r}_i) = \frac{R\,\delta I(\vec{r}_i)}{R^{'}} \quad (14)$$

Multiply both sides of Eq. (14) by R to get:

$$\delta I^{'}(\vec{r}_i)R = \frac{R\,\delta I(\vec{r}_i)R}{R^{'}}$$

$$\delta I^{'}(\vec{r}_i)R = \frac{R}{R^{'}}[\delta(\vec{r}_i,\vec{r}_{io})\left[V(\vec{r}_{io}) - V(\vec{r}_{jo})\right] + \delta(\vec{r}_i,\vec{r}_{jo})[V(\vec{r}_{jo}) - V(\vec{r}_{io})]] \quad (15)$$

Using $\delta(\vec{r}_i,\vec{r}_j) = \langle i|j\rangle$ and $(\vec{r}_i) = \langle i|V\rangle$, we have:

$$\delta I^{'}(\vec{r}_i)R = \frac{R}{R^{'}}[\langle i|[|i_o\rangle\,(\langle i_o| - \langle j_o|)] + [|j_o\rangle\,(\langle j_o| - \langle i_o|)]\,|V\rangle] \quad (16)$$

Equation (16) may be rewritten as:

$$\delta I^{'}(\vec{r}_i)R = \langle i|L^{'}|V\rangle \quad (17)$$

The interstitial resistance gives rise to a disturbance in the system, which is represented by the mathematical operator $L^{'}$.

$$L^{'} = \frac{R}{R^{'}}\,(|i_o\rangle - |j_o\rangle)\,(\langle i_o| - \langle j_o|) = \frac{R}{R^{'}}\,|x\rangle\,\langle x| \quad (18)$$

where $|x\rangle = |i_o\rangle - |j_o\rangle$

The current at $\vec{r}_i$ in the perturbed lattice can be calculated by Ohm's law, and the formula for this current is:

$$I(\vec{r}_i)R = (-L_o V)(\vec{r}_i) + \delta I^{'}(\vec{r}_i)R \quad (19)$$

When we substitute Eq. (17) in Eq. (19), we obtain:

$$\langle i|I\rangle\,R = -\langle i|L_o|V\rangle + \langle i|L^{'}|V\rangle \quad (20)$$

We may write:

$$L\,|V\rangle = -R\,|I\rangle \quad (21)$$

The perturbed lattice has a Laplacian operator $L^{'}$ which is different from the Laplacian operator L_o of the perfect lattice. Therefore, the Laplacian operator of the perturbed lattice can be expressed as:

$$L = L_o - L^{'} \quad (22)$$

The GF for a lattice that has been perturbed is expressed as follows:

$$L\,G = -1 \quad (23)$$

By merging Eq. (22) and Eq. (23), we obtain:

$$G = -L^{-1} = (1 + G_o L^{'})^{-1}\,G_o \quad (24)$$

To determine G, Eq. (24) is expanded into an infinite geometric series [18]:

$$G = G_o - G_o L^{'} G_o + G_o L^{'} G_o L^{'} G_o - G_o L^{'} G_o L^{'} G_o L^{'} G_o + \ldots \quad (25)$$

Due to the uncomplicated nature of the perturbation $L^{'}$, the summation in Eq. (25) can be computed accurately, resulting in a particular form:

$$G = G_o - \left(\frac{R}{R^{'}}\right) G_o|x\rangle\left[1 - \left(\frac{R}{R^{'}}\right)\langle x|G_o|x\rangle + \left(\frac{R}{R^{'}}\right)^2 (\langle x|G_o|x\rangle)^2 - \left(\frac{R}{R^{'}}\right)^3 (\langle x|G_o|x\rangle)^3 + \ldots\right]\langle x|\,G_o$$

$$= G_o - \left(\frac{R}{R^{'}}\right) G_o\,|x\rangle\left[\left(1 + \frac{R}{R^{'}}\langle x|G_o|x\rangle\right)^{-1}\right]\langle x|\,G_o$$

$$G = G_o - \frac{R G_o(|i_o\rangle - |j_o\rangle)\,(\langle i_o| - \langle j_o|)\,G_o}{R^{'} + R\,(\langle i_o| - \langle j_o|)\,G_o\,(|i_o\rangle - |j_o\rangle)} \quad (26)$$

The perturbed LGF can be expressed in terms of G_o as:

$$G(i,j) = G_o(i,j) - \frac{R[G_o(i,i_o) - G_o(i,j_o)][G_o(i_o,j) - G_o(j_o,j)]}{R^{'} + 2R\,[G_o(i_o,i_o) - G_o(i_o,j_o)]} \quad (27)$$

To determine the resistance between two positions, $\vec{r}_i$ and $\vec{r}_j$, we use the relation:

$$R(i,j) = R\,[G(i,i) - G(i,j) - G(j,i) + G(j,j)] \quad (28)$$

The presence of a perturbation has caused a disturbance to the regular pattern of a lattice structure, resulting in a loss of its translational symmetry, which can be observed by the fact that the $G(i,i)$ and $G(j,j)$ are not equal. However, $G(i,j)$ remains symmetric, indicating that $G(i,j) = G(j,i)$. Consequently, Eq. (28) can be simplified accordingly.

$$R(i,j) = R\,[G(i,i) + G(j,j) - 2G(i,j)] \quad (29)$$

Based on Eqs. (27) and (29), it is possible to determine the resistance between $\vec{r}_{io}$ and $\vec{r}_{jo}$ using G_o.

$$R(i,j) = 2R\,[G_o(i,i) - G_o(i,j)] - \frac{(R)^2\,[G_o(i,i_o) - G_o(i,j_o) - G_o(j,i_o) + G_o(j,j_o)]^2}{R^{'} + 2\,R\,[G_o(i_o,i_o) - G_o(i_o,j_o)]} \quad (30)$$

Ultimately, by utilizing Eq. (27), it becomes feasible to express the earlier equation in terms of the resistance of an ideal or perfect lattice, denoted as R_o:

$$R(i,j,;R^{'}) = R_o(i,j) - \frac{[R_o(i,j_o)+R_o(j,i_o)-R_o(i,i_o)-R_o(j,j_o)]^2}{4[R^{'}+R_o(i_o,j_o)]} \quad (31)$$

The given equation represents the ultimate outcome for the resistance that exists between any two lattice positions, $\vec{r}_i$ and $\vec{r}_j$, within a network, whether it be finite or infinite, as long as the interstitial resistance is linked between the ends of the diagonal bond (i_o, j_o).

Using Eq. (31), it is possible to determine the resistance that exists across the interstitial resistance.

$$R(i_o,j_o,;R^{'}) = R_o(i_o,j_o) - \frac{[R_o(i_o,j_o)+R_o(j_o,i_o)-0-0]^2}{4[R^{'}+R_o(i_o,j_o)]}$$

$$R(i_o,j_o,;R^{'}) = \frac{R^{'}R_o(i_o,j_o)}{R^{'}+R_o(i_o,j_o)} \quad (32)$$

As previously stated, $R_o(i_o, j_o)$ represents the resistance that exists between the ends of the diagonal bond (i_o, j_o) within a perfect lattice. Using Eq. (31), it is possible to demonstrate that when $R^{'}$ approaches infinity, the problem simplifies to that of a perfect lattice. The formula provided in Eq. (31) holds true for any type of lattice configuration in which every individual cell contains only one lattice site.

4. Numerical Results

This section includes numerical findings for both perfect and perturbed triangular lattices.

4.1 The Perfect Triangular Lattice Network

Table 1 displays the numerical resistance values between the origin and specific nodes, along with the calculated resistance values for both the infinite and 10 x 10 unperturbed triangular lattice. The computational values for resistance were obtained through the utilization of CIRCUIT MAKER software.

TABLE 1. The values of the resistance in units of R for infinite and computational values of the resistance (10 x 10) perfect triangular lattice.

The site (i,j)	The values of $R_o(i,j)/R$	
	infinite	10 x 10
(1, 0)	0.3333	0.3371
(2, 0)	0.4614	0.4771
(3,0)	0.5362	0.5749
(4,0)	0.5892	0.6713
(5,0)	0.6302	0.8386
(6,0)	0.6637	0.8682
(7,0)	0.6920	0.8735
(8,0)	0.7166	0.9141
(9,0)	0.7382	0.9501
(10,0)	0.7576	0.9827
(-1,0)	0.3333	0.3371
(-2,0)	0.4614	0.4771
(-3,0)	0.5362	0.5749
(-4,0)	0.5892	0.6713
(-5,0)	0.6302	0.8386
(-6,0)	0.6637	0.8682
(-7,0)	0.6920	0.8735
(-8,0)	0.7166	0.9141
(-9,0)	0.7382	0.9501
(-10,0)	0.7576	0.9827
(1,1)	0.3333	0.3366
(2,2)	0.4614	0.4764
(3,3)	0.5362	0.5786
(4,4)	0.5892	0.6953
(5,5)	0.6302	0.9562
(6,6)	0.6637	0.9993

The site (i,j)	The values of $R_o(i,j)/R$	
	infinite	10 x 10
(7,7)	0.6920	1.0397
(8,8)	0.7166	1.1269
(9,9)	0.7382	1.2095
(10,10)	0.7576	1.2888
(-1,-1)	0.3333	0.3366
(-2,-2)	0.4614	0.4764
(-3,-3)	0.5362	0.5786
(-4,-4)	0.5892	0.6953
(-5,-5)	0.6302	0.9562
(-6,-6)	0.6637	0.9993
(-7,-7)	0.6920	1.0397
(-8,-8)	0.7166	1.1269
(-9,-9)	0.7382	1.2095
(-10,-10)	0.7576	1.2888

4.2 Single Interstitial Resistance

Within this section, we utilize Eq. (31) to compute the resistance that exists between the origin and a specific point $j = (j_x, j_y)$ within a triangular array of resistors. This array includes additional interstitial resistance that connects to one of the unit-cell triangles in a diagonal manner.

$$R(j_x - i_x, j_y - i_y; R') = R_o(j_x - i_x, j_y - i_y) - \frac{[R_o(j_{ox} - i_x, j_{oy} - i_y) + R_o(j_x - i_{ox}, j_y - i_{oy}) - R_o(i_x - i_{ox}, i_y - i_{oy}) - R_o(j_x - j_{ox}, j_y - j_{oy})]^2}{4\,[R' + R_o(j_{ox} - i_{ox}, j_{oy} - i_{oy})]} \quad (33)$$

Suppose we add a shunt resistance ($R' = \frac{1}{2} R$) between the points $i_o = (0,0)$ and $j_o = (1,-1)$. in a triangular lattice. Using the formula and resistance values from Table 1, the total resistance between nearest neighbors in the modified infinite triangular lattice was calculated. The results are presented in Table 2. Additionally, Tables 2 and 3 summarize the computed resistance values for both the infinite lattice and the 10 × 10 perturbed networks.

TABLE 2. Equivalent resistances in terms of R between (0,0) and sites $j = (j_x, j_y)$ for an infinite perturbed triangular lattice. $R' = 0.5\,R$ is introduced between the sites $i_o = (0,0)$ and $j_o = (1,-1)$.

The site (i,j)	The values of $\frac{R(i,j)}{R}$	The site (i,j)	The values of $\frac{R(i,j)}{R}$
	infinite		infinite
(0,0)	0	(0,0)	0
(1,0)	0.29997	(-1,0)	0.31735
(2,0)	0.39753	(-2,0)	0.43764
(3,0)	0.47982	(-3,0)	0.50931
(4,0)	0.53896	(-4,0)	0.56070
(5,0)	0.58377	(-5,0)	0.60070
(6,0)	0.61976	(-6,0)	0.63357
(7,0)	0.64972	(-7,0)	0.66137
(8,0)	0.67557	(-8,0)	0.68569
(9,0)	0.69808	(-9,0)	0.70692
(10,0)	0.71829	(-10,0)	0.72624

TABLE 3. As in Table 2. but for 10 x 10 perturbed triangular lattice.

The site (i,j)	The values of $\frac{R(i,j)}{R}$ 10 x 10 lattice	The site (i,j)	The values of $\frac{R(i,j)}{R}$ 10 x 10 lattice
(0,0)	0	(0,0)	0
(1,0)	0.2842	(1,1)	0.3117
(2,0)	0.3858	(2,2)	0.4380
(3,0)	0.4879	(3,3)	0.5354
(4,0)	0.5871	(4,4)	0.6505
(5,0)	0.7540	(5,5)	0.9111
(6,0)	0.8008	(6,6)	0.9737
(7,0)	0.8639	(7,7)	1.0584
(8,0)	0.9326	(8,8)	1.1388
(9,0)	0.9967	(9,9)	1.2162
(10,0)	1.1082	(10,10)	1.3254
(-1,0)	0.3214	(-1,-1)	0.3117
(-2,0)	0.4527	(-2,-2)	0.4380
(-3,0)	0.5471	(-3,-3)	0.5354
(-4,0)	0.6424	(-4,-4)	0.6505
(-5,0)	0.8097	(-5,-5)	0.9111
(-6,0)	0.8394	(-6,-6)	0.9737
(-7,0)	0.9152	(-7,-7)	1.0584
(-8,0)	0.9871	(-8,-8)	1.1388
(-9,0)	1.0545	(-9,-9)	1.2162
(-10,0)	1.1198	(-10,-10)	1.3254

We will now examine the impact of the interstitial resistance, denoted as $R' = R$, which is connected between the points $i_o = (0,0)$ and $j_o = (-1,1)$. The outcome of this analysis is presented in Tables 4 and 5.

TABLE 4: As in Table 2, but at $R' = R$.

The site (i,j)	The values of $\frac{R(i,j)}{R}$ infinite	The site (i,j)	The values of $\frac{R(i,j)}{R}$ infinite
(0,0)	0	(0,0)	0
(1,0)	0.32333	(-1,1)	0.31247
(2,0)	0.44655	(-2,0)	0.42148
(3,0)	0.51939	(-3,0)	0.50096
(4,0)	0.57139	(-4,0)	0.55780
(5,0)	0.61176	(-5,0)	0.60118
(6,0)	0.64487	(-6,0)	0.63624
(7,0)	0.67286	(-7,0)	0.66558
(8,0)	0.69728	(-8,0)	0.69096
(9,0)	0.71865	(-9,0)	0.71312
(10,0)	0.73800	(-10,0)	0.73303

TABLE 5. As in Table 2, but for 10 x 10 perturbed triangular lattice at $R' = R$.

The site (i,j)	The values of $\frac{R(i,j)}{R}$ 10 x 10 lattice	The site (i,j)	The values of $\frac{R(i,j)}{R}$ 10 x 10 lattice
(0,0)	0	(0,0)	0
(1,0)	0.3268	(1,1)	0.3203
(2,0)	0.4612	(2,2)	0.4512
(3,0)	0.5567	(3,3)	0.5503
(4,0)	0.6524	(4,4)	0.6660
(5,0)	0.8197	(5,5)	0.9267

The site (i,j)	The values of $\frac{R(i,j)}{R}$ 10 x 10 lattice	The site (i,j)	The values of $\frac{R(i,j)}{R}$ 10 x 10 lattice
(6,0)	0.8402	(6,6)	0.9646
(7,0)	0.8897	(7,7)	1.0040
(8,0)	0.9244	(8,8)	1.0902
(9,0)	0.9649	(9,9)	1.1717
(10,0)	0.9935	(10,10)	1.2499
(-1,0)	0.3025	(-1,-1)	0.3203
(-2,0)	0.4173	(-2,-2)	0.4512
(-3,0)	0.5179	(-3,-3)	0.5503
(-4,0)	0.6161	(-4,-4)	0.6660
(-5,0)	0.7832	(-5,-5)	0.9267
(-6,0)	0.8114	(-6,-6)	0.9646
(-7,0)	0.8291	(-7,-7)	1.0040
(-8,0)	0.8717	(-8,-8)	1.0902
(-9,0)	0.9126	(-9,-9)	1.1717
(-10,0)	0.9607	(-10,-10)	1.2499

Figure 2 displays the resistances of both an infinite and a 10 x 10 perfect triangular lattice as a function of j_x. The figure illustrates the resistance within the infinite triangular lattice, which exhibits symmetry under the transformation $(n, m) = (m, n)$ due to the lattice's inversion symmetry.

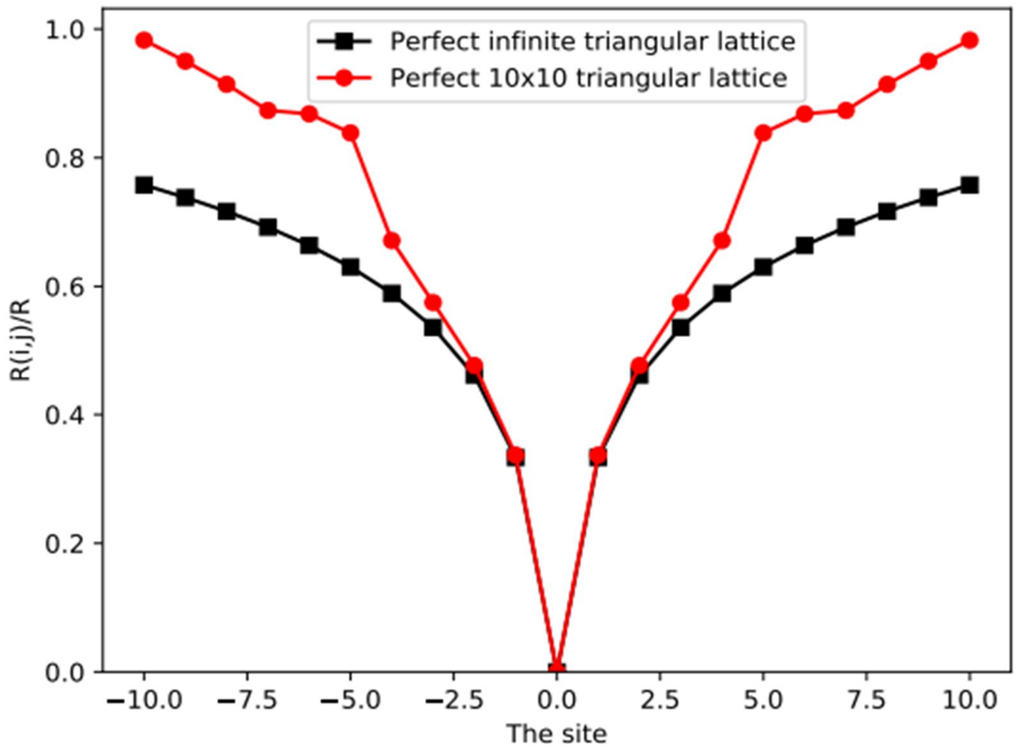

FIG. 2. The total resistance in terms of R of the infinite (squares) and 10 x 10 (circles) perfect triangular lattices between (0,0) and $(j_x, 0)$ as functions of j_x.

Figures 3 and 4 present the calculated theoretical resistances for perfect and perturbed infinite triangular lattices, respectively. The resistance calculations take into account the interstitial resistances and are plotted as a function of j_x from the origin to the site $(j_x, 0)$. In Figs. 5 and 6, the total resistance is plotted against j_x for 10 x 10 perturbed lattices. The perturbed lattice has a consistently lower resistance compared to the perfect (ideal) lattice, as shown by the negative value of the second term in Eq. (31).

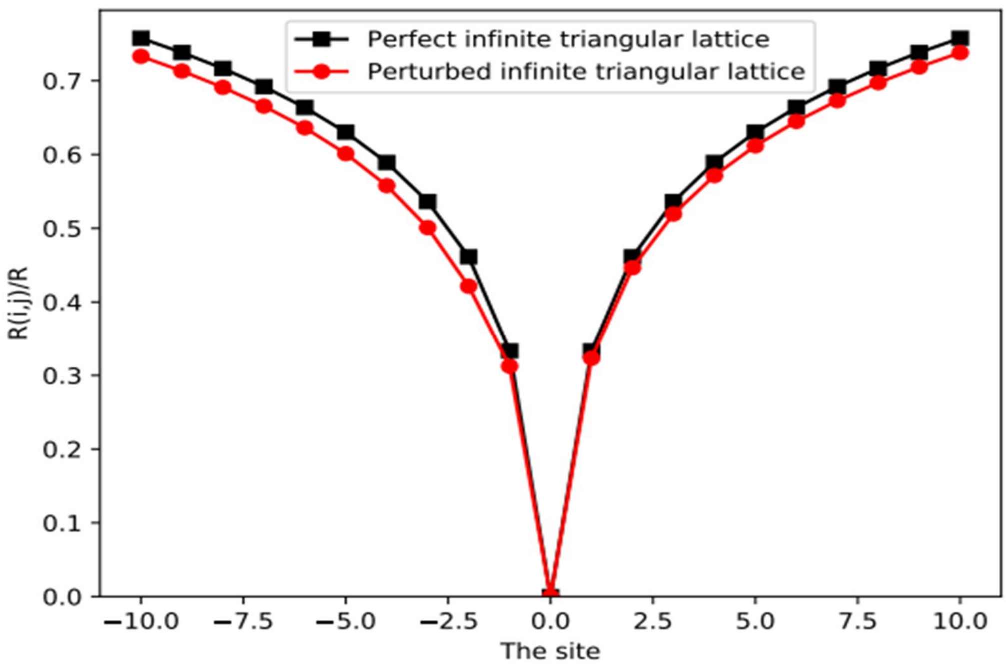

FIG. 3. The resistance of the infinite triangular lattice, both perturbed and perfect, is determined in units of R. The calculation is done between two sites: $\vec{r}_i = (0,0)$ and $\vec{r}_j = (j_x, 0)$ as a function of j_x. The interstitial resistance $R^{\prime} = R$ is inserted between $\vec{r}_{io} = (0, 0)$ and $\vec{r}_{jo} = (-1, +1)$.

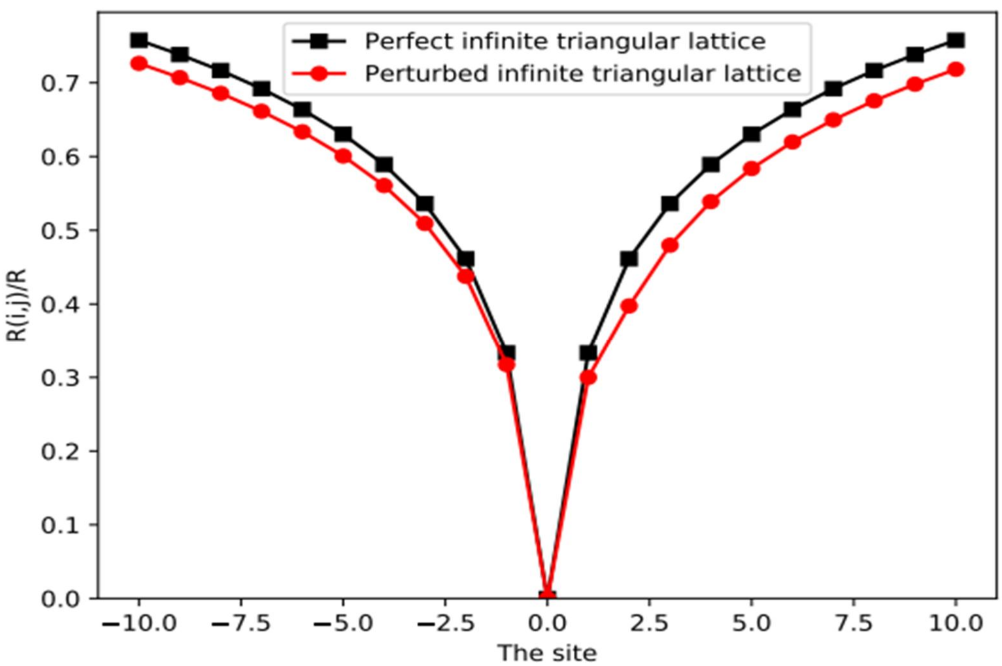

FIG. 4. As in Fig. 3, but the interstitial resistance $R^{\prime} = \frac{R}{2}$ is inserted between $\vec{r}_{io} = (0,0)$ and $\vec{r}_{jo} = (+1, -1)$.

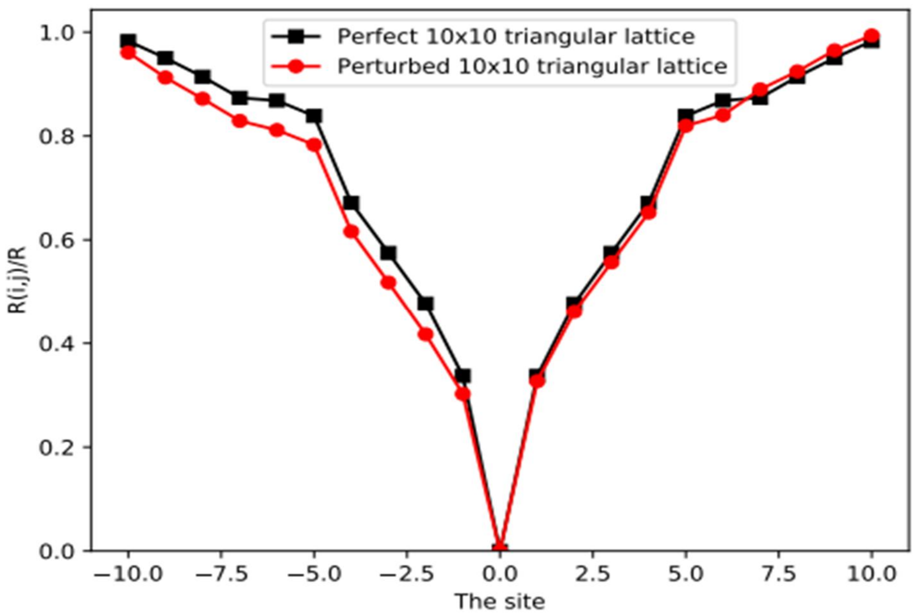

FIG. 5 As in Fig. 3, but for 10 x 10 triangular lattice and the interstitial resistance $R^{'} = R$ is added between $\vec{r}_{io} = (0,0)$ and $\vec{r}_{jo} = (-1,+1)$.

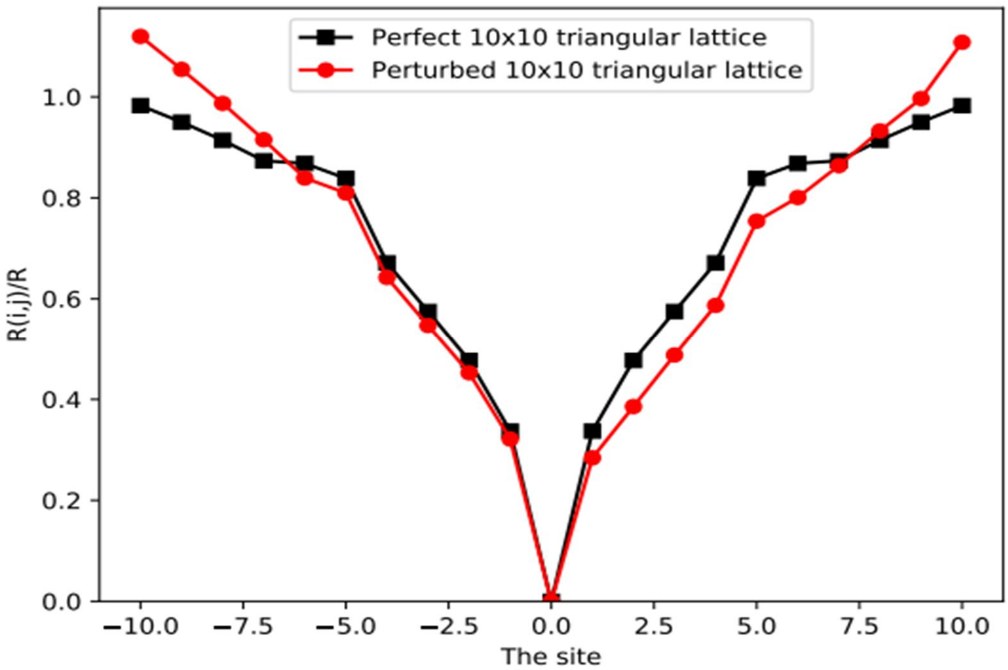

FIG. 6. As in Fig. 3, but for 10 x 10 triangular lattice and the interstitial resistance $R^{'} = R/2$ is added between $\vec{r}_{io} = (0,0)$ and $\vec{r}_{jo} = (+1,-1)$.

The implemented grid provides the recorded resistance values depicted in Figs. 2 and 5. This indicates that employing a mesh composed of 10 × 10 elements allows for effective examination of the equivalent resistance. However, when approaching the boundary, the measured resistance surpasses the calculated value, which can be attributed to the influence of the edge effect. Furthermore, it can be observed from Figs. 3–6 that the resistance approaches that of the perfect lattice when the distance between sites $\vec{r}_i = (0,0)$ and $\vec{r}_j = (j_x, 0)$ is increased. However, it is important to note that the resistance symmetry is lost as $j_x \to -j_x$, due to the disruption of translational symmetry in the perturbed lattice.

In brief, the values obtained for the perfect triangular lattice exhibit strong agreement with bulk values derived from different calculation methods. Specifically, the results are in remarkable alignment with those obtained using Cserti's method [5], Atkinson and Steenwijk's method [11], and Igor's method [19]. This consistency enhances the credibility of our measurements, confirming their accuracy and reliability.

5. Conclusions

The resistance of a triangular network between two random sites in an infinite resistor network has been determined through both theoretical and computational methods, for both the perfect and perturbed scenarios.

Our observations indicate that in both the perfect and perturbed finite networks, the effective resistance between any two nodes is higher than that in the corresponding infinite network. If the interstitial resistance is greater (smaller) than R, then the effective resistance between any two nodes in the perturbed lattice is greater (smaller) than that in the perfect lattice. Moreover, the theoretical resistance values for both the perfect and perturbed lattices agree with those obtained from computational methods using Circuit Maker.

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