Volume 17, Number 4, 2024. pp. 411-422

Jordan Journal of Physics

ARTICLE

Calculation the Low Orbits and More Stable for a Satellite around Mars

Duaa Deyaa Abood and **Abdulrahman H. Saleh**

Department of Astronomy and Space, College of Science, University of Baghdad, Iraq.

Doi: **https://doi.org/10.47011/17.4.4**

Received on: 15/12/2022; *Accepted on: 25/06/2023*

Abstract: This research examines various types of orbits around Mars. The effects of Mars's non-spherical shape, atmospheric drag, and solar attraction on these orbits were included in the calculations. The objective was to determine the optimal orbital elements to obtain a stable orbit of a satellite around Mars. The values of angles w and Ω were taken as 40° and 20°, respectively, while the orbit inclination was examined at three experimental values: i = 88°, 89°, and 90°. The perigee height above the Martian surface was assessed at three altitudes (hp = 50, 100, and 150 km) considering all perturbations except atmospheric drag. The orbital eccentricity was tested at values of e = 0.01, 0.02, 0.05, 0.08, and 0.1. The findings indicate that the most stable orbit was achieved with a low-altitude perigee (hp = 50 km), low eccentricity (e = 0.05), and an inclination of i = 90°.

Keywords: Mars, Orbiting spacecraft, Orbital elements, Inclination, Eccentricity.

1. Introduction

When a spacecraft moves around Mars, the perturbation forces must be accounted for. The most significant perturbation is due to the non-spherical shape of Mars, which introduces periodic variations in the orbit. By applying relationships between Keplerian orbital elements and Cartesian coordinates, orbital perturbations in the position and velocity vectors can be measured [1].

Spacecraft exploring the solar system often orbit various planets before escaping and re-entering an orbit around Earth. These spacecraft are transferred to their targets and follow orbits around the Sun as they travel between planets. When a spacecraft is close to its target, the planet's gravitational field deflects it into a modified orbit, causing it to either gain or lose energy.

To enter an orbit around a planet, the spacecraft or satellite's relative velocity must be reduced using a rocket when the satellite is near to its new orbit. This step allows the spacecraft to be captured into an elliptical orbit. Finally, to return to Earth, the spacecraft must gain enough momentum to complete the process in the opposite direction successfully [2].

2. The Orbital Elements of the Elliptical Orbit

An orbit is called an ellipse when its plane is inclined, and it is characterized by six orbital elements: semi-major axis (a), eccentricity (e), inclination (i), argument of perigee (w), right ascension of ascending node (Ω), and mean anomaly (M). To decrease the size of the transfer orbit and achieve a stable final orbit for a satellite around Mars, it is necessary to increase the height of the perigee (hp) while decreasing both the semi-major axis (a) and an eccentricity (e) [3].

Corresponding Author: Duaa Deyaa Abood **Email**: Duaa.abood1107@sc.uobaghdad.edu.iq

The inclination (i) is the angle between the orbital plane and the plane of the equator, calculated as:

$$i = tan^{-1}\frac{\sqrt{h_x^2+h_y^2}}{h_z} \quad (1)$$

The argument of perigee (w) is the angle between the ascending node and the perigee, calculated as:

$$w = tan^{-1}\frac{zh}{yh_x-xh_y} \quad (2)$$

The right ascension of the ascending node (Ω) is the angle between the vernal equinox and the ascending node on the equatorial plain, calculated as:

$$\Omega = tan^{-1}(-\frac{h_x}{h_y}) \quad (3)$$

Equations (1), (2), and (3), known as the Euler angles, describe the orbit's direction in space. Here, hx, hy, and hz are the angular momentum components of x, y, and z directions, and h is given by:

$$h = \sqrt{h_x^2+h_y^2+h_z^2}$$

The mean anomaly (M) is calculated from perigee to the position of the satellite. It is defined as the fraction of an orbit period and is measured by the equation:

$$M = E - e\ sinE \quad (4)$$

or

$$M = n(t-t_p) \quad (5)$$

where E is the eccentric anomaly, n is the mean motion, t is epoch time, and tp is the time at which the satellite passes the perigee point [3, 4].

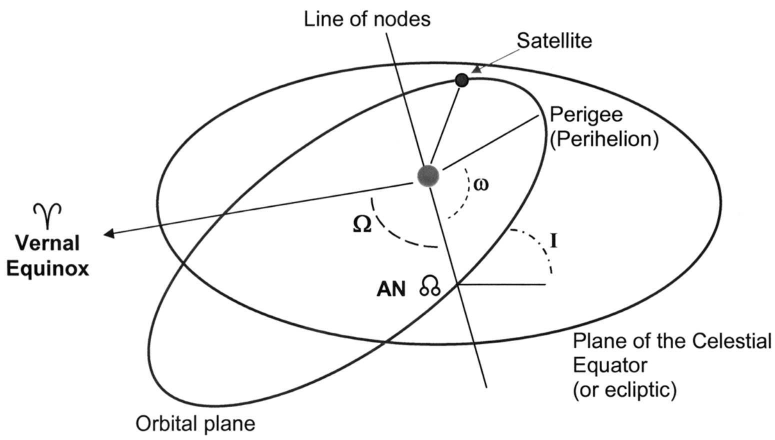

FIG. 1. The orbital elements of a satellite during it is spin around a planet [5].

TABLE 1. The main six orbital elements and their coefficients for Mars orbit around the Sun for the year 1900 [6].

Orbital element	a_0	a_1	a_2	a_3
L	293.737334	+19141.69551	+0.0003107	---------
a	1.5236883	-----------	---------	---------
e	0.09331290	+0.000092064	- 0.000000077	---------
i	1.850333	-0.0006750	+0.0000126	---------
w	285.431761	+1.0697667	+0.0001313	+0.00000414
Ω	48.786442	+0.7709917	-0.0000014	-0.00000533

The orbital elements = $a_0+a_1T+a_2T^2+a_3T^3$ (6)

$$\text{where } T = \frac{JD-2415020.0}{36525} \quad (7)$$

JD = INT(365.25 y) + INT(30.6001(m+1)) + DD.dd + 1720994.5 + B (8)

where Y, m, and DD.dd are the year, month, and day with time, respectively. B is the Gregorian correction on the date, where B = 0 before 15/10/1582 AD.

The longitude of the perihelion point (α) is calculated as:

$$\alpha = w + \Omega \quad (9)$$

The mean anomaly of the planets is calculated as:

$M = L - w - \Omega$ (10)

For the major axis:

$2a = r_p + r_a$

where r_p is the perigee distance and, r_a is the apogee distance, calculated as:

$r_p = a(1 - e)$ (11)

$r_a = a(1 + e)$ (12)

TABLE 2. Mars Euler angles values for the year 2000 [6].

Euler angle	a_0	a_1	a_2	a_3
I	1.845113	-0.0081839	-0.00002305	-0.000000045
W	285.597172	+0.7385934	+0.00046647	+0.000006962
Ω	49.319212	-0.2940497	-0.00064435	-0.000008182

3. Perturbation Forces

3.1. Solar Radiation Pressure

Solar radiation pressure is a force exerted on a satellite due to the momentum flux of sunlight reaching Mars. For most satellites, this force acts radially away from the Sun. The magnitude of the resulting acceleration on the satellite is given by:

$a_{srp} = k * p * \left(\frac{A}{m}\right) * \left(\frac{r_{mars} - r_{sat}}{R_{sun-mars}}\right)$ (13)

where:

- asrp is solar radiation pressure acceleration,
- k is a constant = 1.3 and (p) is = 4.56 x 10^{-6} ,
- A/m is a cross-sectional area of the satellite,
- r_{mars} is the Mars position,
- r_{sat} is the satellite position,
- $R_{sun-mars}$ is the distance between the Sun and Mars.

In general, it is assumed that the sun-line a perpendicular to the cross-sectional area, because it is taken as the maximum cross-sectional area to calculate the worst possible case [5].

3.2. Atmospheric Drag

Assume that the cross-sectional area of the satellite (A/m) is perpendicular to its velocity vector, which maximizes the atmospheric drag force. By changing the satellite's orientation, we can adjust the atmospheric drag from zero to its maximum value. If we further assume that the only other force acting on the satellite is Mars's gravitational attraction, then as soon as the atmospheric drag on the satellite is set to zero, that satellite would rotate smoothly in an ideal Kepler orbit from its current position and velocity [7, 8].

The lower atmosphere of Mars extends from the planet's surface to about 7 km. Within this range, temperature (T) decreases linearly, and pressure (P) decreases exponentially. The relationships between T and P are as follows:

If h < 7 km, then, T = -31 – 0.000998 h, p = $0.699 * e^{-0.00009\,h}$

If h > 7 km, then, T = - 23.4 – 0.00222 h, p = $0.699 * e^{-0.00009\,h}$

$\rho = p / (0.1921 * (T + 273.1))$ (17)

where ρ is density [4, 7].

The perturbing acceleration of the satellite can be represented as:

$a_{Drag} = -\frac{1}{2} C_D \frac{A}{m} \rho\, v_{r\,(mag)}{}^2$ (18)

where $v_{r(mag)}$ is the relative speed between the satellite and the atmosphere, C_D is the drag coefficient, and ρ is the air density at the satellite's altitude [9].

A precise prediction of the satellite motion under the influence of drag involves a good density model of the upper atmosphere. An empirical atmospheric density model is used for this purpose. The velocity of the satellite relative to the atmosphere is defined as [10]:

$v_r = v_{in} + r \times w_{Earth}$

3.3. Planetary Oblateness

Planetary oblateness is a measure of how much a planet is flattened by its rotation. It is a unitless magnitude.

Several Martian satellites, such as MRO and MGS, are positioned in Sun-synchronous orbits (SSO).

Recent studies suggest that to achieve a stable, Martian frozen orbit, the initial values of eccentricity (e) and the angle of ascending node (ω) should be carefully chosen based on given values for the semi-major axis (a) and inclination (i). For Mars, the J2 = 1.9555×10^{-3}, and J3= 3.14498×10^{-5} [5-7].

The J2 perturbation, which affects satellite orbits in three Cartesian components, is calculated using the following equation:

$$J2 = \left(2\frac{\epsilon_M}{3}\right) - \left(\frac{R_M^3 \omega_M^2}{3GM_M}\right) \quad (19)$$

where ε_M is Mars's oblateness, R_M is Mars's mean equatorial radius, ω_M is the Mars rotation rate, G is the universal gravitational constant, and M_M is Mars's mass.

3.4. Solar Gravity

The main external perturbations come from the Sun and the planet's moons. In this study, we focused only on the Sun's gravitational effect, excluding the perturbation effects from Mars's moons. The satellite receives a stronger gravitational pull when it is closer to these external influences. The gravitational attraction exerted by the Sun on a satellite around Mars is denoted by the symbol (Mus) and is calculated using Newton's universal law of gravitation [8]:

$$F(Mus) = G * \frac{M*m}{R^2} \quad (20)$$

Mus = $1.6100839093*10^{21}$ N.

4. Mars Coordinates from the Sun

To determine the coordinates of Mars relative to the Sun, the following results can be obtained using a MATLAB program:

$$l = 293.737334+19141.69551*T+0.0003107*T2 \quad (21)$$

$$a = 1.5236883 \quad (22)$$

$$e = 0.0933129+0.000092064*T-0.000000077*T^2 \quad (23)$$

$$i = 1.857806-0.0081565 * T_2 -0.00002304 * T_2^2 + 0.000000044 * T_2^3 \quad (24)$$

$$w = 285.762379 + 0.7387251 * T_2 + 0.00046551 * T_2^2 + 0.000006939 * T_2^3 \quad (25)$$

$$\Omega = 49.852347 - 0.2941821 * T_2 -0.00064344 * T_2^2 - 0.000008159 * T_2^3 \quad (26)$$

where $T = \frac{j3-2415020}{36525}$, $T_2 = \frac{j3-2451545.5}{36525}$, $j3 = \frac{j2+tp}{24*3600}$, and j2 is Julian date.

5. Practical Section and Discussion of Results

5-1. For a proposed satellite orbit around Mars, we analyzed orbits over 2000 periods with initial orbital elements set to Ω = 20°, ω = 40°, and i = 88°. Using eccentricity values e = 0.01, 0.02, 0.05, 0.08, 0.1 and altitudes above Mars's surface hp = 50, 100, and 150 km, we generated graphical data through MATLAB. Our analysis of these graphs identified that the most stable orbital configuration—characterized by minimal variations in orbital elements—occurred at a low altitude of 50 km from the Mars surface and with an eccentricity magnitude of 0.05, as shown in Fig. 2.

Note that not all the obtained graphs were included because it is not possible to include all of them in this research.

5-2. Further analysis was conducted using inclinations i = 89° and i = 90° while maintaining Ω = 20° and ω = 40° for 2000 periods. Eccentricities remained set to e = 0.01, 0.02, 0.05, 0.08, 0.1, and satellite altitudes above Mars were hp = 50, 100, and 150 km. By observing and comparing graphical outputs for each configuration, it was evident that the optimal stability was achieved with I = 90°, an altitude of 50 km, and an eccentricity of 0.05, as illustrated in Figs. 3-10.

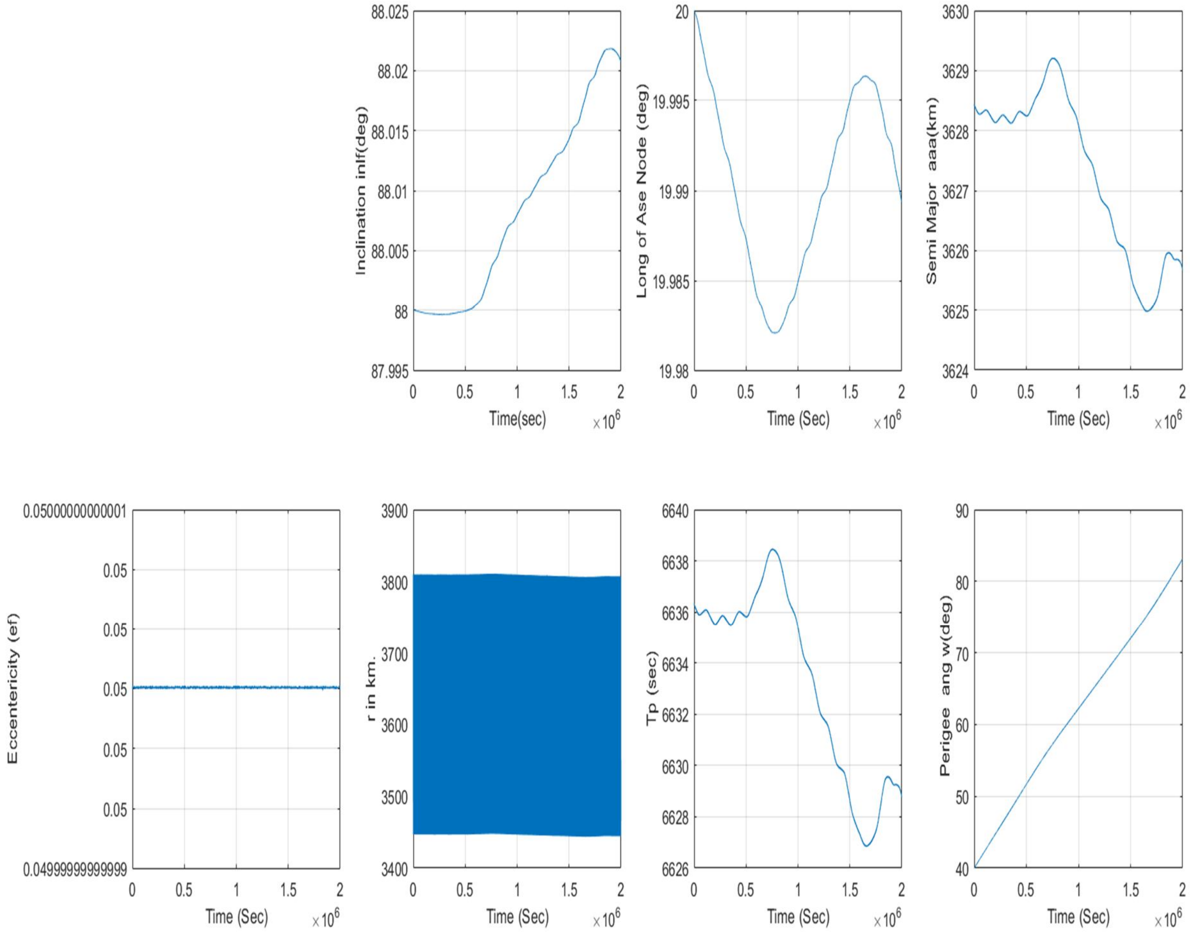

FIG. 2. The orbital elements of the satellite orbit around Mars at (e=0.05, hp=50km, i=88) for 2000 periods i=88 deg.

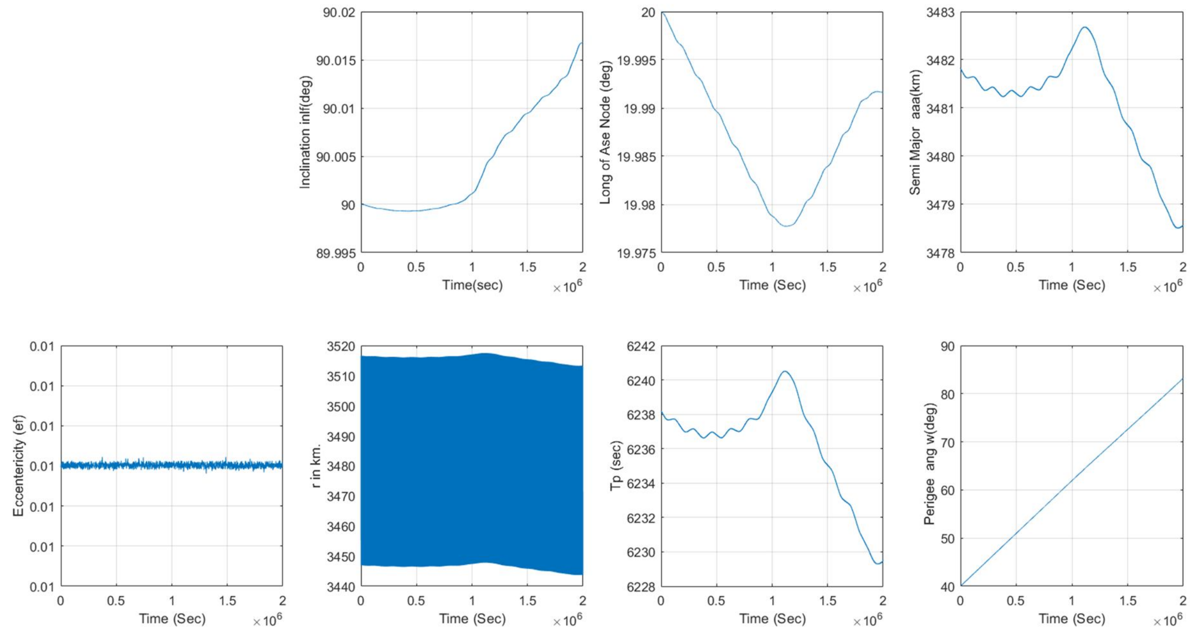

FIG. 3. The orbital elements of the satellite orbit around Mars at (e = 0.01, hp = 50 km, i = 90) for 2000 periods.

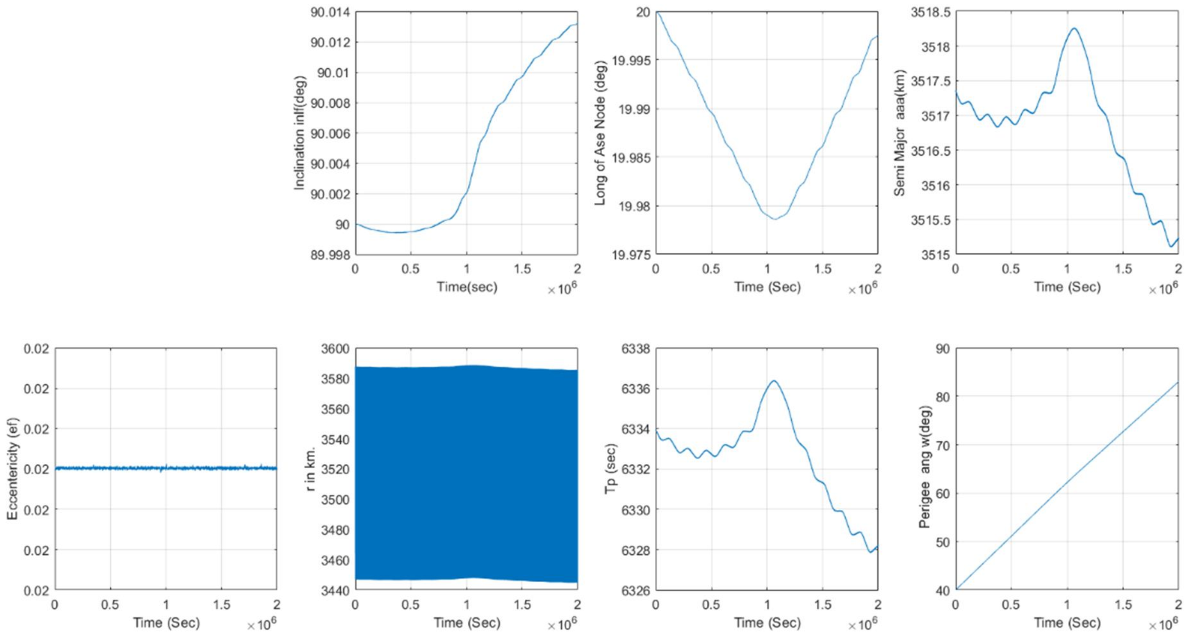

FIG. 4. The orbital elements of the satellite orbit around Mars at (e = 0.02, hp = 50 km, i = 90) for 2000 periods.

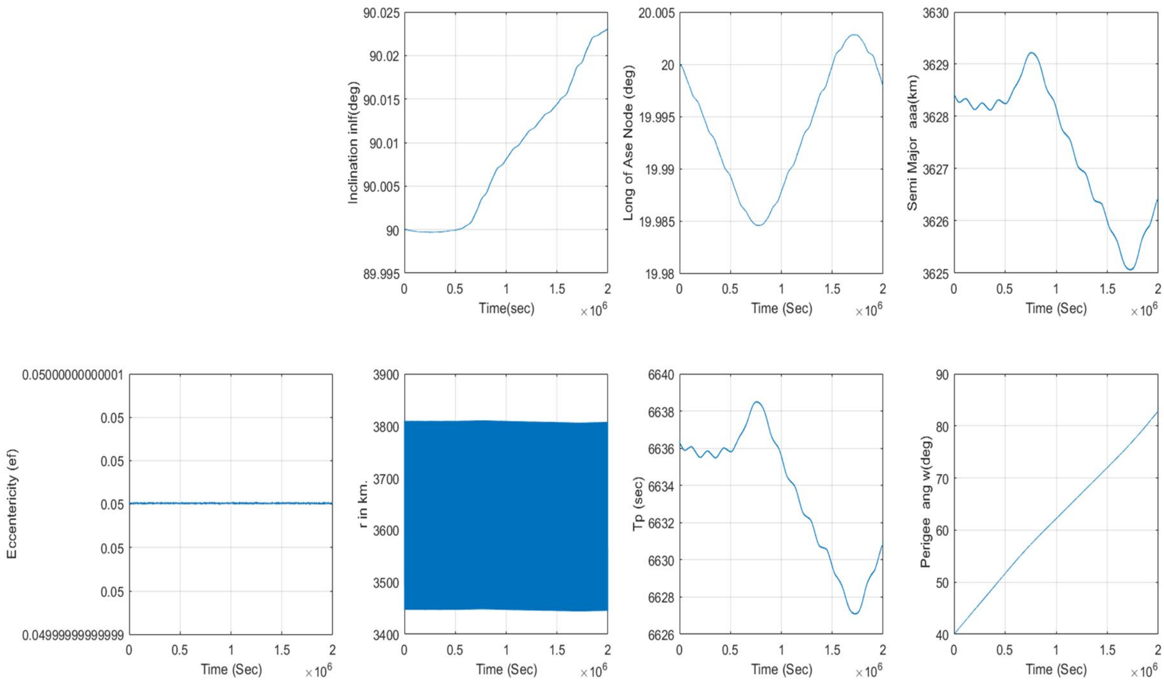

FIG. 5. The orbital elements of the satellite orbit around Mars at (e = 0.05, hp = 50 km, i = 90) for 2000 periods.

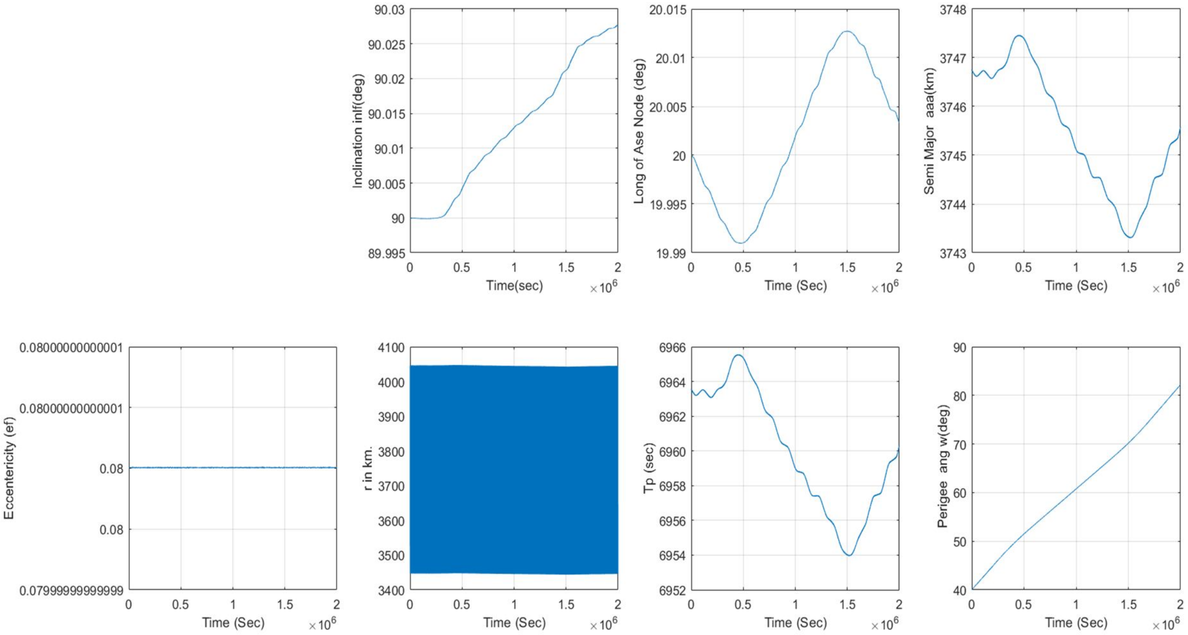

FIG. 6. The orbital elements of the satellite orbit around Mars at (e = 0.08, hp = 50 km, i = 90) for 2000 periods.

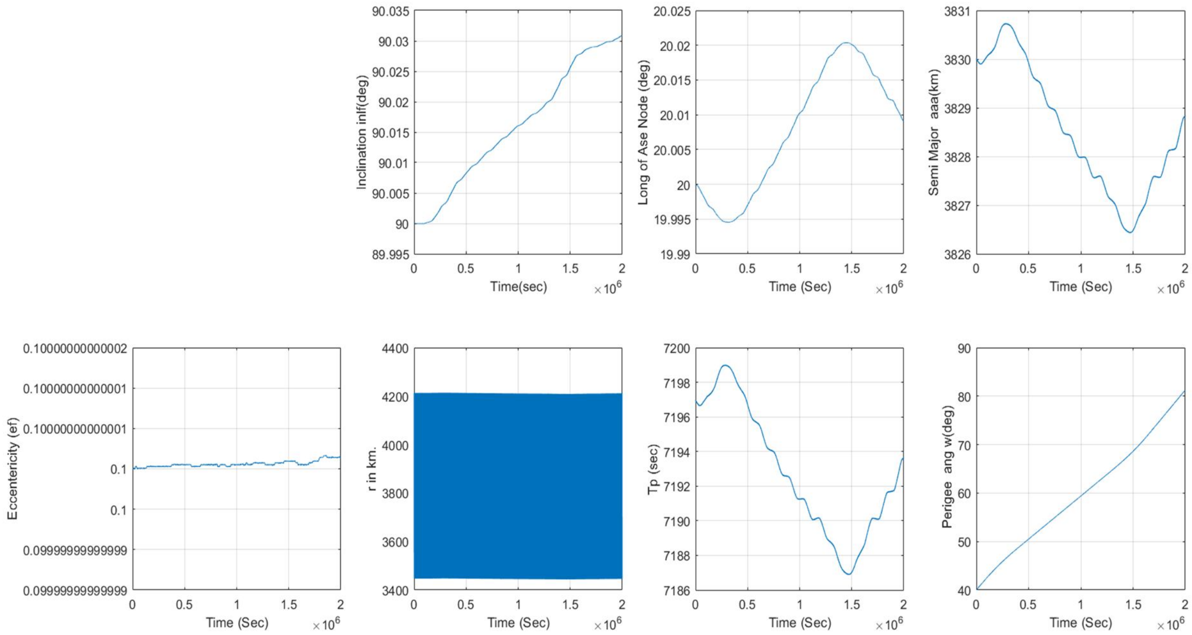

FIG. 7. The orbital elements of the satellite orbit around Mars at (e = 0.1, hp = 50 km, i = 90) for 2000 periods.

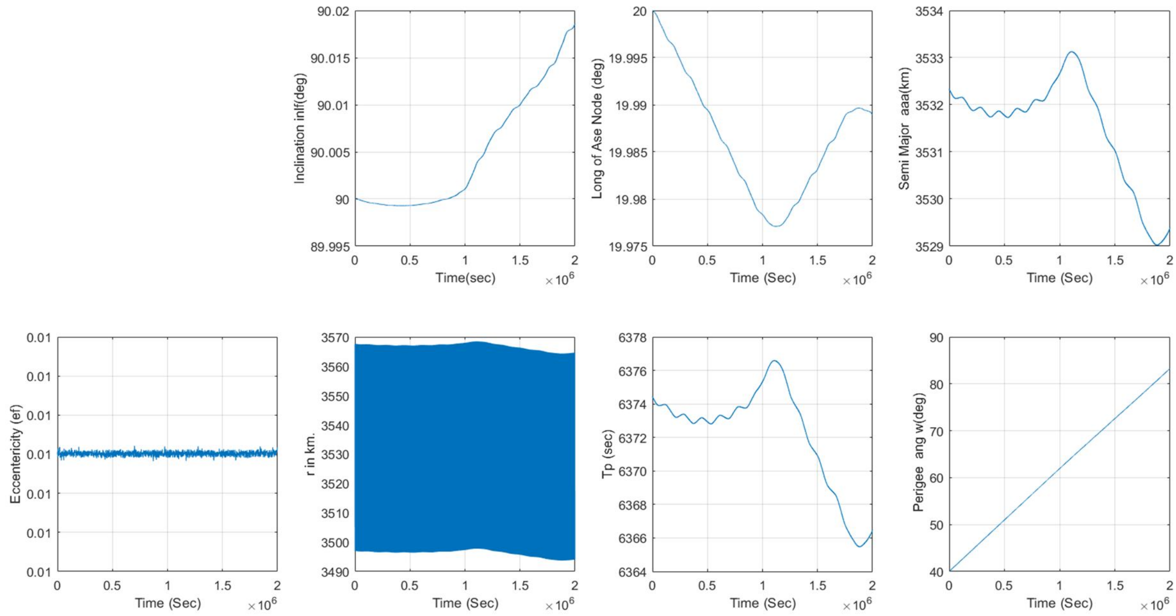

FIG. 8. The orbital elements of the satellite orbit around Mars at (e = 0.01, hp = 100 km, i = 90) for 2000 periods.

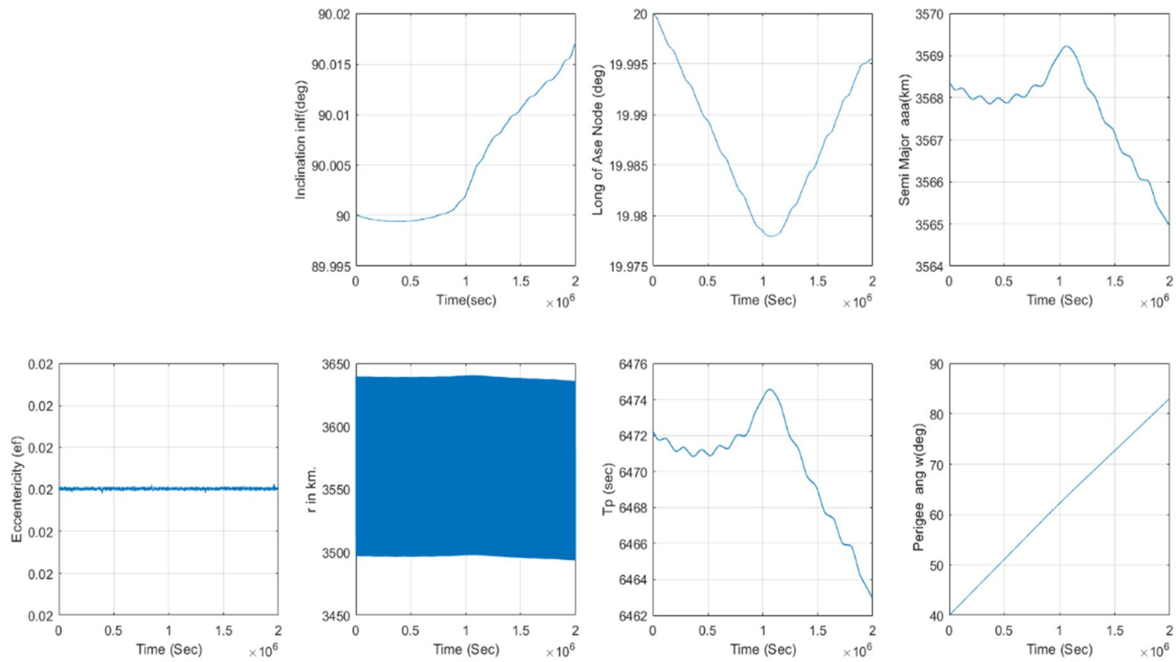

FIG. 9. The orbital elements of the satellite orbit around Mars at (e = 0.02, hp = 100 km, i = 90) for 2000 periods.

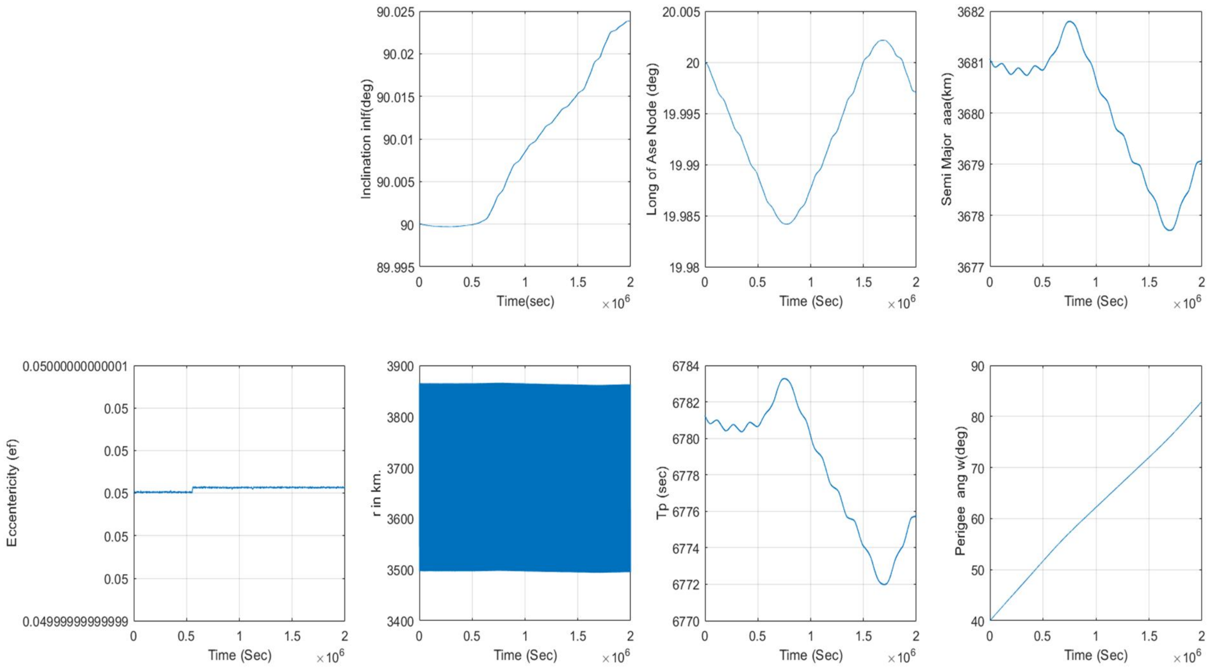

FIG. 10. The orbital elements of the satellite orbit around Mars at (e = 0.05, hp = 100 km, i = 90) for 2000 periods.

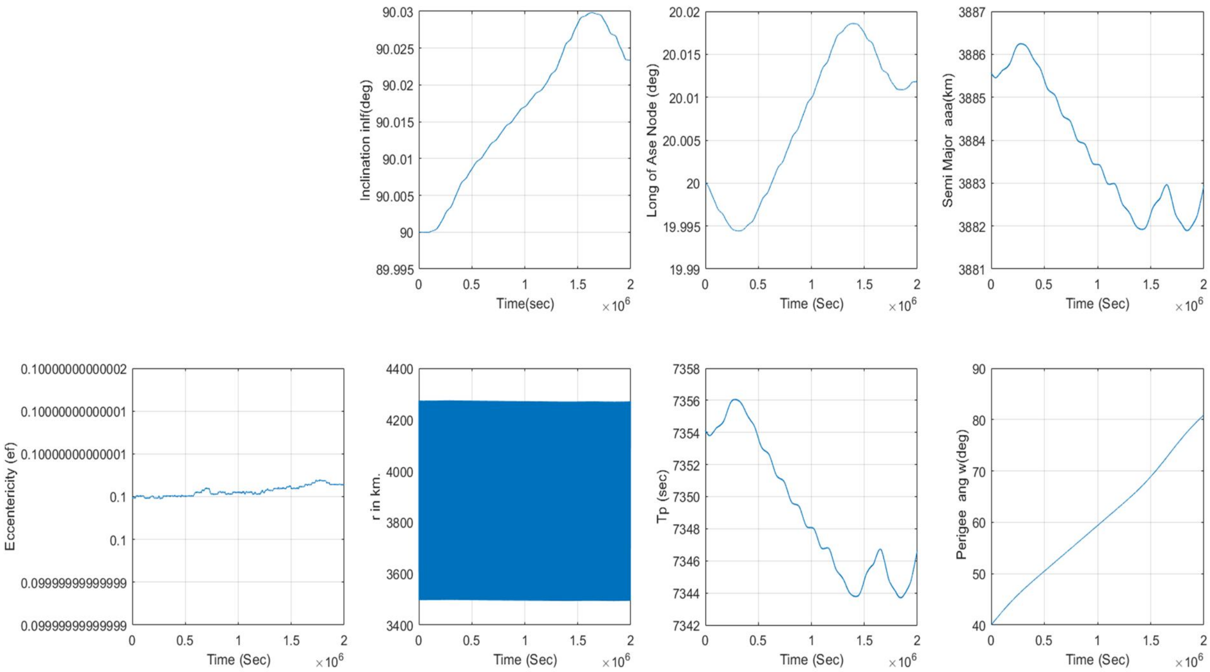

FIG. 11 The orbital elements of the satellite orbit around Mars at (e = 0.1, hp = 100 km, i = 90) for 2000 periods.

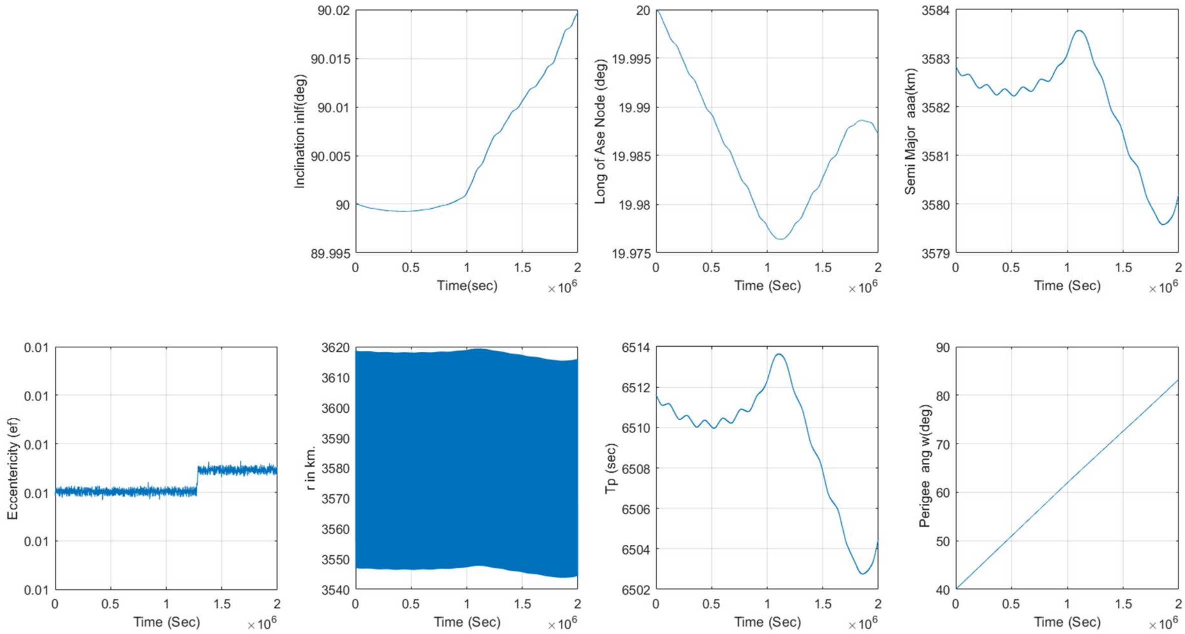

FIG. 12. The orbital elements of the satellite orbit around Mars at (e = 0.01, hp = 150 km) for 2000 periods.

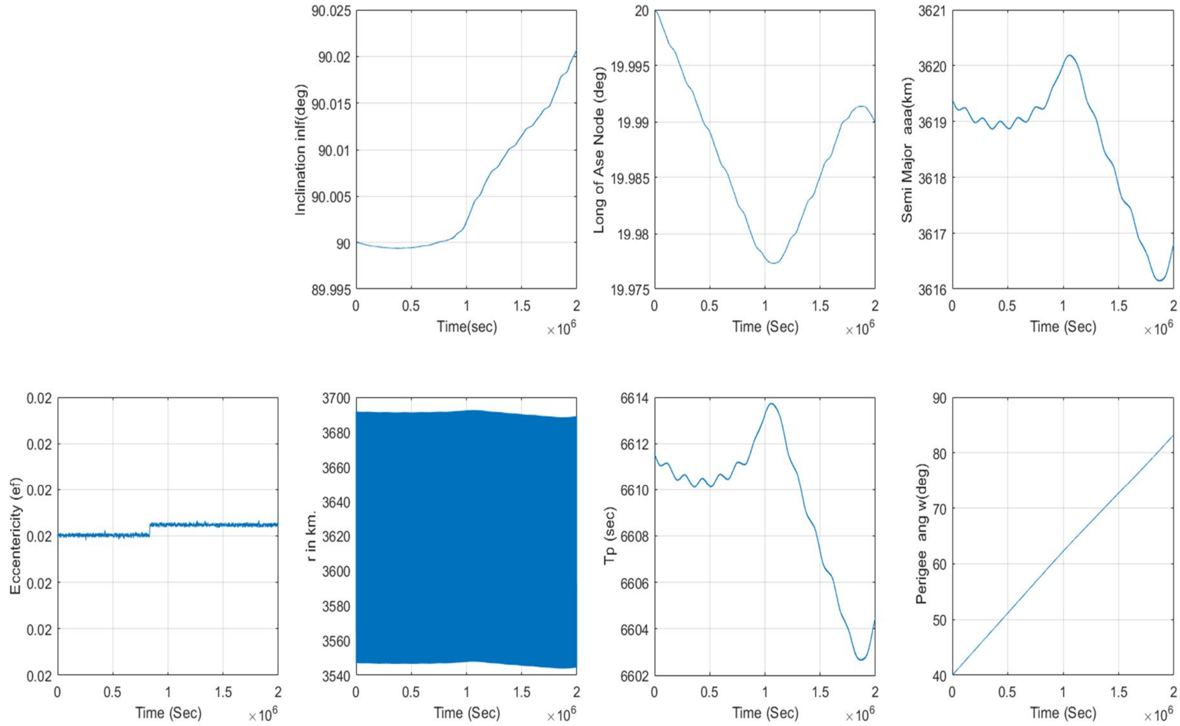

FIG. 13. The orbital elements of the satellite orbit around Mars at (e = 0.02, hp = 150 km, i = 90) for 2000 periods.

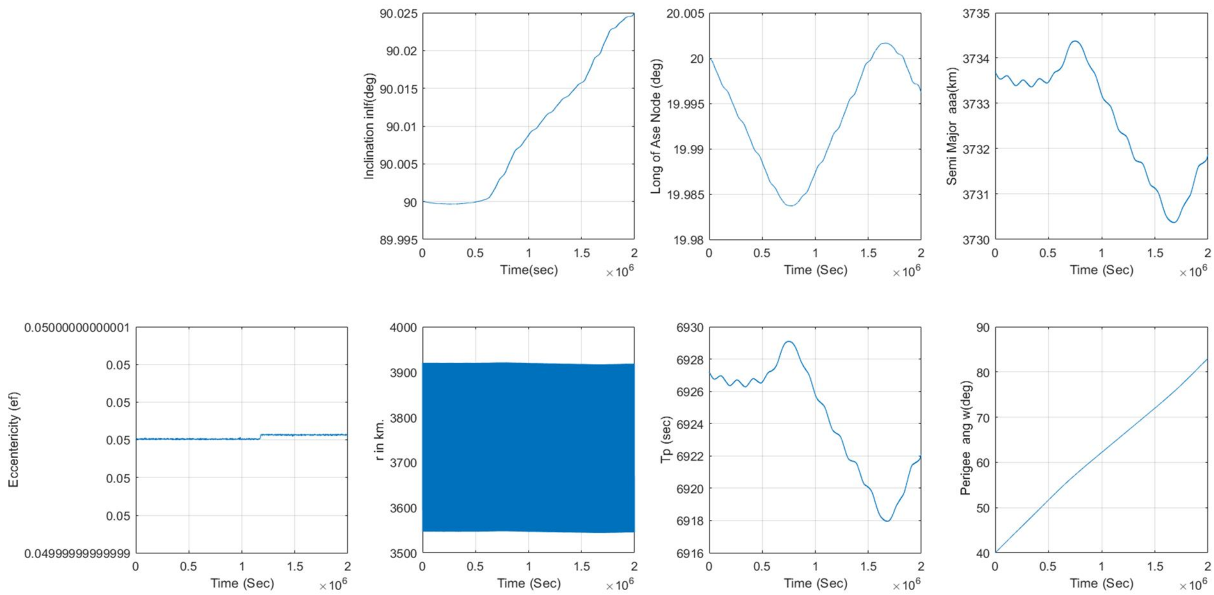

FIG. 14. The orbital elements of the satellite orbit around Mars at (e = 0.05, hp = 150 km, i = 90) for 2000 periods.

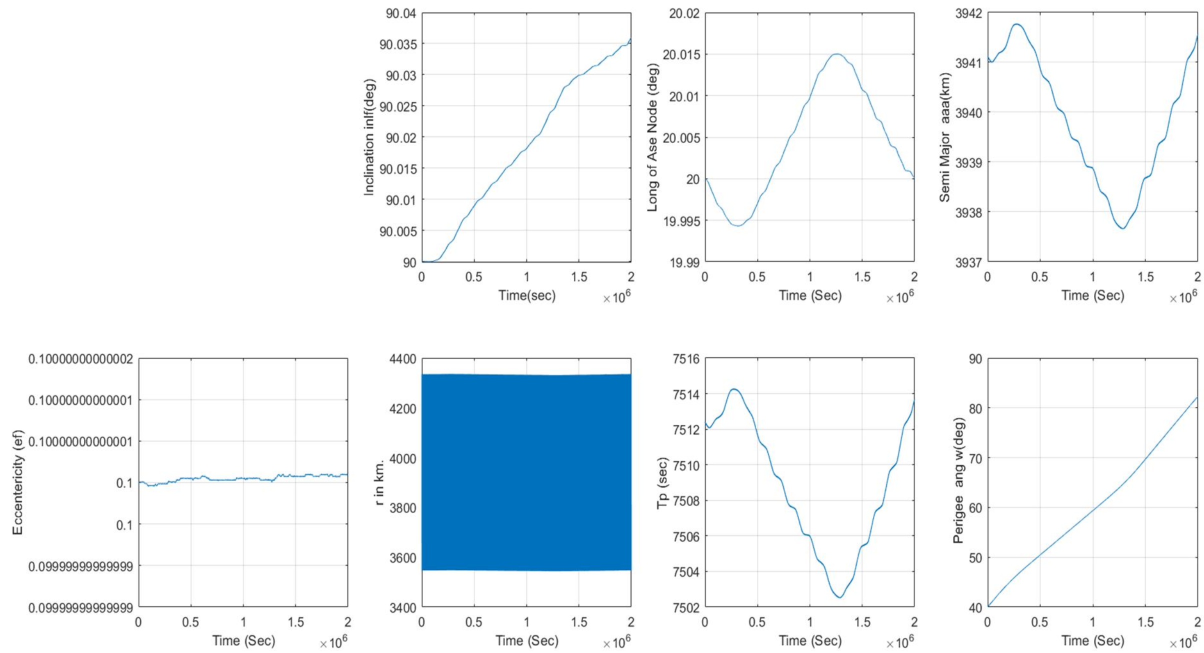

FIG. 15. The orbital elements of the satellite orbit around Mars at (e = 0.1, hp = 150 km, i = 90) for 2000 periods.

References:

[1] He, Z.-Z. and Huang, C.-L., Res. Astron. Astrophys. J., 15 (1) (2015) 107.

[2] https://courses.lumenlearning.com/astronomy/chapter/motions-of-satellites-and-spacecraft/.

[3] Fadhil, O. A. and Saleh, A. H., J. Phys.: Conf. Ser., 1664 (2020), 012005.

[4] Glenn Research Center, NASA, https://www.grc.nasa.gov/www/k-12/airplane/ atmosmrm.html.

[5] Oliveira, A. C. et al., J. Phys.: Conf. Ser., 1365 (2019) 012028.

[6] Meeus, J., “Astronomical Formula for Calculators”, 4th Ed., (Enlarged & Revised, 1988).

[7] Sinclair, A. T., Royal Astron. Soc., NASA Astrophys. Data Syst., I55 (1972) 249.

[8] Noullez, A. and Tsiganis, K., Adv. Space Res., 67 (2021) 477.

[9] Seeber, G., "Satellite Geodesy", 2nd Ed. (Walter de Gruyter, Berlin, NY, 2003).

[10] Montenbruck, O. and Gill, E., "Satellite Orbits Models Methods and Applications", 2nd Ed., (Springer-Verlag Berlin Heidelberg, 2001).