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ARTICLE

The Many-Worlds Interpretation versus the Copenhagen Interpretation: A Case Discussion with the Hydrogen Atom

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Abstract: The main objective of this work is to compare the interpretations of the hydrogen atom spectrum according to two famous schools in quantum mechanics: Copenhagen and many-worlds. The Schrodinger equation is solved using the many-worlds interpretation, and the results are then compared to those obtained using the Copenhagen interpretation. While the energy spectra are similar in both cases, the interpretations of these results differ. In the many-worlds interpretation, the eigenvectors are entangled across multiple worlds, whereas, in the Copenhagen interpretation, they are superimposed. The hydrogen atom, being a system of only one electron and without electron-electron interaction, serves as a clear and accessible example for comparing these interpretations. In this case, the wave function depends on independent coordinates and is written as a tensor product of independent functions, even before solving the Schrödinger equation. In more complex systems where there are electron-electron, electron-nucleus, nucleus-nucleus, and other interactions, the wave function should be written as a tensor product of entangled states after solving the Schrodinger equation. The aim of this study is to demonstrate to physics and chemistry teachers and students that there are different ways to view the quantum world. The many-worlds interpretation is simply another way of interpreting the solutions of the Schrodinger equation, rather than a new mathematical approach. The present work emphasizes the importance of understanding different interpretations of quantum mechanics and their implications for understanding the physical world.

Keywords: Hydrogen atom spectrum, Schrodinger equation, Copenhagen school, Many-worlds interpretation.

1. Introduction

The meaning of quantum mechanics, both in concept and description, has been the subject of various interpretations, leading to debates between different schools of thought. Although the exact number of interpretations is not precisely defined, there are about sixteen commonly discussed schools of interpretation. Hugh Everett proposed the many-worlds interpretation in 1957, suggesting the existence of multiple parallel universes. This interpretation is universally considered the second most significant interpretation and has received

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support from notable physicists and philosophers, including Nobel Prize laureates Stephen Hawking, Murray Gell-Mann, and Richard Feynman.

According to Everett's concept, the universe branches into multiple copies, each corresponding to a different possible outcome of that measurement. These branches, or "worlds", exist independently and are not directly observable or accessible to one another [1]. In the many-worlds interpretation, the universe consists of multiple worlds. Each world follows deterministic and reversible laws, with no non-deterministic or irreversible wavefunction. Consequently, there is no collapse of wave functions associated with scaling [2]. Everett considered the wavefunction to be a real entity, suggesting that all possible outcomes of a quantum measurement exist as separate branches or parallel universes. This interpretation can be seen as a departure from the traditional collapse of the wavefunction and a return to a view where all mathematical entities in a physical theory are considered real [3]. Similar to how Maxwell's electromagnetic fields and Dalton's atoms were treated in classical physics, Everett treated the wavefunction as a real object. He assumed that the wavefunction obeyed the same equation during observation as it did at all other times [4].

In the Everett interpretation, quantum systems, such as particles, become entangled when they interact. According to this interpretation, any system can be considered an observer, and when it interacts with another system, it effectively performs a measurement or observation. As a result of this interaction, the observer system and the observed system split into multiple copies or branches. Each branch corresponds to one of the possible outcomes of the measurement, and each copy of the observer perceives only one specific outcome while remaining unaware of the other possible outcomes [1].

Interactions between systems and their environments lead to communication between observers. This communication transmits correlations and induces the splitting, or decoherence, of the universal wavefunction into multiple branches. These branches represent equally real but mutually unobservable worlds. Unlike in other interpretations, the wavefunction does not collapse at the moment of observation. Instead, it continues to evolve deterministically, encompassing all possibilities within it [5]. The outcomes exist simultaneously but do not interfere further with each other; every single prior world has split into mutually unobservable but equally real worlds.

Everett demonstrated that once there is a possibility that an object is in any state, the universe of that object transforms into a series of parallel universes equal to the number of possible states in which this object can exist, each universe containing a single possible state of this object. It should be noted that Everett was not entirely comfortable with talking about "many worlds"; it seemed less important to him how this language described pure wave mechanics. Rather, his emphasis was on the empirical understanding and cognitive implications of his theory [6].

Several recent works on Everett's interpretation, viewed as a realist interpretation of quantum formalism, highlight how it can benefit from advances in the metaphysics of dispositions [7], removing action at a distance and randomness from quantum theory [8]. There is also a work on the discussion of the possibility of multiverses beyond the universe we live in and the alternatives to the multi-world interpretation, as proposed by Christianto *et al.* [9].

The Copenhagen interpretation, developed by Bohr [10] and Heisenberg [11] is the most famous interpretation of quantum mechanics, which is still used and followed in the teaching of quantum mechanics worldwide. However, it is not a deterministic interpretation. Despite the preponderance of the Copenhagen reading of quantum phenomena, numerous questions concerning the interpretation of quantum mechanics continue to be the subject of energetic debates, affirming that its fundamental basis is far from being definitively settled [12].

The Copenhagen interpretation proposed by Bohr *et al.* is based on a dualistic core idea: the description of the microscopic world by quantum mechanics must be supplemented by an external classical world, which causes the wave packet collapse [13]. However, Albert Einstein and Erwin Schrodinger did not agree with the Copenhagen school [14].

Currently, many physicists find quantum theory to be full of contradictions and paradoxes that are difficult to resolve consistently. The disagreement focuses primarily on the problem of describing observations [9]. This leads to the question of what happens when quantum theory is applied unrestrictedly to the entire universe. Some argue that if we allow unlimited application of Schrödinger's equation, assuming the quantum state is something physically real, contradictions may arise.

In this context, our original work is based on the discussion of the hydrogen atom's spectrum treated according to two famous schools: the many-worlds interpretation and the Copenhagen interpretation. Before proceeding with this research, it is important to note that the key difference between these schools lies in how they interpret measurement results. Therefore, we would like to recall some important definitions from algebra, which are necessary to clarify this work.

2. Measurements

Measurements in general are a "function" of application, between observers and observables, in order to determine a certain physical quantity of the observable, such as its position, its momentum, its energy, etc.

2.1 Functions

A function is defined as a relation "application", that assigns each element of group "A" to only one element of group "B". Both sets "A" and "B" must be non-empty. Thus, "f: A → B" is a function of this type; for each element a_i ∈ A, there is a unique corresponding " image" element b ∈ B [15].

2.2 Types of Functions:

a) Subjective Function

It is a function that assigns two or more elements of "A" to the same element of set "B". The elements of "A" have the same figure in the other set. So for every b_i in the set "B", there is at least one "a_i" in the set "A" [15].

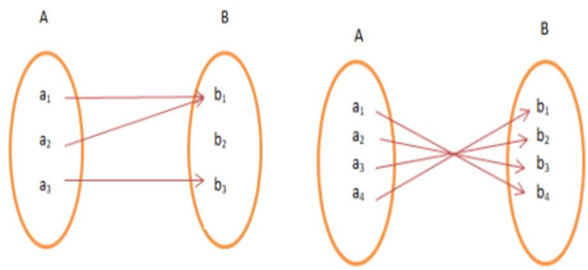

FIG. 1. Sets A and B.

b) Bijective Function

A function f is bijective if it links each element of "A" with a distinct element of "B" and each element of "B" has a pre-image in "A". In other words, for every "b_i" in set "B" there is exactly one "a_i" in set "A" [15]. In measurement, there is no meaning to many observers to observe or measure just one observable.

Now. we present the corresponding physical meanings of the mathematical definitions above. The measurement process is a relational process, or "application", that connects the group of observers "B" to the set of observables, set "A". In the quantum case, set "B" represents the different set of observes, $\{\widehat{H_i}\}$, where $\{\widehat{H_i}\}$ represents the Hamiltonian or energy operators in set "B"; its role is to measure its eigenvalues a_i in set "A".

In the Copenhagen interpretation, Bohr and his team considered that there is just one observer, $\widehat{H}$, in set "B" that observes various observables, $\{a_i\}$, in set "A", using a set of eigenfunctions that are "not necessary parallel". Conversely, in the many-worlds interpretation, it is assumed that multiple observes, ", depending on the existing potential, $\widehat{H}_i$ in set "B", "each observer observes just one observable", $\{a_i\}$, in set "A" using a different set of eigenfunctions, "which should be parallel".

In conclusion, in the Copenhagen school, the relationship between observers and observables is subjective, while in the many-worlds interpretation, this relationship is bijective.

FIG. 2. Everett's and Copenhagen's view.

3. Solving and Interpreting Schrodinger Equation for the Hydrogen Atom, According to the Many-Worlds Interpretation.

Before starting, we would like to present a summary of our modest modeling of entangled states according to the many-worlds interpretation [16]. Once there is a possibility of an object being in any state, the universe of that object turns or splits into a series of parallel "branched" universes equal to the number of possible states where the object can exist. Each universe contains one possible unique state for that object [1]. We symbolize the universe by $|\Psi\rangle$, and because of the parallel states, we write:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \,.........\otimes |\psi_n\rangle$$
$$|\Psi\rangle = \prod_{j=1}^{n\geq 2} |\psi_j\rangle \otimes |\psi_{j+1}\rangle, \quad j(\leq n) \tag{1}$$

Parallel universes imply no interference (superposition) between them. Because of this, no information is exchanged between the states. Therefore, the universal wave function that describes the universal world is a tensor product of states or worlds.

When a measurement is taken in universe $|\Psi\rangle = \prod_{j=1}^{n\geq 2} |\psi_j\rangle \otimes |\psi_{j+1}\rangle$ using an observer, represented by an operator $\hat{A}_i$, this operator acts on $|\Psi\rangle$, measuring only its corresponding eigenvalue λ_i, and so on, for other observers $\hat{A}_k$, $k = 1,2,.. \neq i$. Each observer measures its specific eigenvalue $\lambda_k \neq \lambda_i$. The measurement operation for a single observer, $\hat{A}_j$, acting on $|\Psi\rangle$, is as follows:

$$\hat{A}_j \prod_{j=1}^{n} |\psi_j\rangle \otimes |\psi_{j+1}\rangle$$
$$= \lambda_i \prod_{j=1}^{n} |\psi_j\rangle \otimes |\psi_{j+1}\rangle$$

The expectation value $\langle \hat{A}_j \rangle$ is given by $\Rightarrow$ (2)

$$\left\{\prod_{j=1}^{n} \langle {}_{j+1}\psi| \otimes \langle {}_{j}\psi|\right\} \hat{A}_j \left\{\prod_{j=1}^{n} |\psi_j\rangle \otimes |\psi_{j+1}\rangle\right\}$$

$= \lambda_j$. The normalisation of $|\Psi\rangle$ gives:

$$\langle \Psi|\Psi\rangle = 1 \Rightarrow \langle \psi_i|\psi_j\rangle = \delta_{i,j}$$

When measurements are taken by all observers ($\sum_{j=1}^{n} \hat{A}_j$), each observer $\hat{A}_j$ measures only its correspondent eigenvalue λ_j in its own world $|\psi_j\rangle$. Due to the parallel nature of these worlds, "there is no superposition of states". The general form of the universe $|\Psi\rangle$ as a function of all single universes $|\psi_j\rangle$ can be expressed as:

$$|\Psi\rangle = \prod_{j\neq i}^{n\geq 2} \sum_{i=1}^{n\geq 2} (-1)^{i-1} \alpha_i \alpha_j |\psi_i\rangle \otimes |\psi_j\rangle . \tag{3}$$

The factor $(-1)^{i-1}$ is due to the mirroring (reflection) property.

In $\alpha_i \alpha_j |\psi_i\rangle \otimes |\psi_j\rangle \neq \alpha_i |\psi_i\rangle \otimes \alpha_j |\psi_j\rangle$, the first part, $\alpha_i \alpha_j |\psi_i\rangle \otimes |\psi_j\rangle$, indicates entangled states, while the second one, $\alpha_i |\psi_i\rangle \otimes \alpha_j |\psi_j\rangle$, refers to separated states. The coefficient $\alpha_{j,j+1}$ represents the entanglement factor. Since the universe, represented by the wave function $|\Psi\rangle$, is normalized to unity, we write:

$$\langle\Psi|\Psi\rangle = \int_{-\infty}^{+\infty} \Psi^*\Psi\, dv = 1$$
$$= \prod_{j\neq i}^{n\geq 2} \sum_{i=1}^{n\geq 2} (-)^{i-1}|\alpha_i|^2|\alpha_j|^2 = 1 \quad . \tag{4}$$
$$i = 0,1,.....n$$

where the entanglement factor $\alpha_i\, \alpha_{j,} = \sqrt{\frac{1}{n}}$.

All the wave functions are real and reversible, with no wave collapse and no eigenvalue degeneration. Due to state entanglement, once any observer measures its eigenvalue in its state, all other eigenvalues of the other states are instantly known, even though there is no exchange of information between them. As a result, the Hilbert space is constructed from the tensor product of vector states.

Application to the Hydrogen Atom Case

Our aim is to apply the many-worlds interpretation to solve the stationary Schrodinger equation for the hydrogen atom.

$$\widehat{H}|\Psi\rangle = E|\Psi\rangle$$

The above equation in the hydrogen atom case, "independent coordinates", will take the following form under the many-worlds interpretation:

$$\hat{H}_i|\Psi\rangle = E_i|\Psi\rangle \tag{5}$$

The observer $\hat{H}_i$ in the entangled states can observe only its eigenvalue E_i on the world or branch $|\psi_i\rangle$. As previously discussed, Everett believed that the electron of the hydrogen atom, if it has the potential to exist in multiple states, will have its initial state divided into many other states, ($|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$,....). Each state $|\psi_i\rangle$ is composed of many other states, which we call "orbits", *(S_i, P_i, d_i...)*, with each orbit containing a single electron copy.

Therefore, in the hydrogen atom case, we can write the state $|\psi\rangle$ as a tensor product of its different states or branches, even before solving the Schrodinger equation, as follows:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes .... \otimes |\psi_i\rangle \otimes ...... \otimes |\psi_n\rangle \tag{6}$$

where $\widehat{H}_i = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$, and $V(r) = \frac{-q^2}{4\pi\varepsilon_0\, r}$ is the potential due to the single hydrogen proton in the hydrogen nucleus at a distance r (considering the nucleus to be static). The observer (operator) $\hat{H}_i$ within $|\psi_i\rangle$ measures only the observable E_i. Consequently, Eqs. (1) and (5) are rewritten in the following form:

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + \frac{-q^2}{4\pi\varepsilon_0\, r}\right\}(|\psi_i\rangle \otimes |\psi_{i+1}\rangle) \otimes \left[\prod_{j=1}^{n}|\psi_j\rangle \ \otimes\ |\psi_{j+1}\rangle\ \right] = E_i(|\psi_i\rangle \otimes |\psi_{i+1}\rangle) \otimes \left[\prod_{j=1}^{n}|\psi_j\rangle \ \otimes\ |\psi_{j+1}\rangle\ \right] \quad j \neq i \quad . \tag{7}$$

Since there is no superposition between states (referred to as "parallel states"), the above equation, after subtraction the term

$$|\psi_{i+1}\rangle \otimes \left[\prod_{j=1}^{n}|\psi_j\rangle \ \otimes |\psi_{j+1}\rangle\ \right]$$

takes the following form on both sides:

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + \frac{-q^2}{4\pi\varepsilon_0\, r}\right\}|\psi_i\rangle = E_i|\psi_i\rangle \tag{8}$$

That means that observer $\widehat{H}_i$ measures only the observable E_i on its state $|\psi_i\rangle$, which is entangled with other possible states:

$$\left[\prod_{j=1}^{n}|\psi_j\rangle \ \otimes |\psi_{j+1}\rangle\ \right] \quad i \neq j\, . \tag{9}$$

Note that Eq. (8) is the same as in the Copenhagen interpretation.

Because the hydrogen electron moves around the nucleus in different positions $\vec{r}_i$, we rewrite Eq. (8) using spherical coordinates as follows:

$$-\frac{\hbar^2}{2m_e}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_i}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_i}{\partial\theta}\right) + \frac{1}{r^2(\sin\theta)^2}\frac{\partial^2\psi_i}{\partial^2\phi}\right] - \frac{e^2}{4\pi\varepsilon_0 r}\psi_i = E\psi_i \tag{10}$$

The solutions to this equation in the Copenhagen interpretation are:

$$\psi_{ilm}(\vec{r},\theta,\varphi) = R_i(r)Y_{lm}(\theta,\varphi)$$

$$E_i = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{i^2}\,(i = 1; 2; ..i ...; n) \quad (11)$$

According to the many-worlds interpretation, each world « i » is described by the following wave function:

$$\psi_{ilm}(\vec{r},\theta,\varphi) = R_i(r)Y_{lm}(\theta,\varphi) \otimes \left[\prod_{j=1}^{n} \mathrm{R_j}\ (\mathrm{r})\mathrm{Y_{lm}}(\theta,\phi) \otimes \mathrm{R_{j+1}}\ (\mathrm{r})\mathrm{Y_{lm}}(\theta,\phi)\right] \quad j \neq i \quad (12)$$

The combined wave function for all "worlds," representing all observers, is given by:

$$|\Psi\rangle = \prod_{j\neq i}^{n\geq 2} \sum_{i=1}^{n\geq 2} (\pm)^{i-1}\alpha_i\alpha_j\, |\psi_{ilm}(r,\theta,\phi)\rangle_i \otimes |\psi_{ilm}(r,\theta,\phi)\rangle_j \quad (13)$$

4. Results and Interpretations According to the Many-Worlds School

The energy and its correspondent world are defined as:

$$E_i = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{i^2}\,(i = 1; 2; ..i ...; n) \quad (14)$$

$$\psi_{ilm}(\vec{r},\theta,\varphi) = R_i(r)Y_{lm}(\theta,\varphi) \otimes \left[\prod_{j=1}^{n} \mathrm{R_j}\ (\mathrm{r})\mathrm{Y_{lm}}(\theta,\phi) \otimes \mathrm{R_{j+1}}\ (\mathrm{r})\mathrm{Y_{lm}}(\theta,\phi)\right] \quad j \neq i \quad (15)$$

This can be interpreted as follows: the observer in the world $\psi_{ilm}(\vec{r},\theta,\varphi)$, which is entangled with all the other possible worlds (states), notices or measures the energy:

$$E_i = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{i^2}. \quad (14)$$

To clarify how many worlds are interpreted, we assume that the hydrogen electron has the possibility to be in sp^3 worlds or orbits. In this case, the hydrogen electron simultaneously exists in $s, p_x, p_y, and\ p_z$ states without probability, contrasting with the Copenhagen interpretation. So, the observer in the s-world, which is entangled with the $p_x, p_y, and\ p_z$ worlds measures the energy:

$$E_{s1} = -\frac{mZ^2e^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{1^2} = -13.6\ eV\ ,$$

while its world expression is:

$$\psi_{1,0,0} = R_{1,0}(r)Y_{0,0}(\theta,\varphi) \otimes R_{2,0}(r)Y_{0,0}(\theta,\varphi) \otimes R_{2,1}(r)Y_{1,1}(\theta,\varphi) \otimes R_{2,1}(r)Y_{1,-1}(\theta,\varphi) \otimes R_{2,1}(r)Y_{1,0}(\theta,\varphi)$$

$$=2\left(\frac{1}{a_0}\right)^{3/2}\exp\left(-\frac{r}{a_0}\right)Y_{0,0} \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\left(2-\frac{r}{a_0}\right).\exp\left(-\frac{r}{2a_0}\right)Y_{0,0} \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,-1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,0}(\theta,\varphi). \quad (16)$$

Observers in the worlds (s_2, p_x, p_y, p_z) measure the same energy:

$$E_{s2} = E_{p_x} = E_{p_y} = E_{p_z} = -\frac{mZ^2e^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{2^2} = -3.4\ eV$$

Their mathematical representations are as follows:

$$\psi_{2,0,0} = \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\left(2-\frac{r}{a_0}\right).\exp\left(-\frac{r}{2a_0}\right)Y_{0,0} \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,-1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,0}(\theta,\varphi) \otimes 2\left(\frac{1}{a_0}\right)^{3/2}\exp\left(-\frac{r}{a_0}\right)Y_{0,0}$$

$$\psi_{2,1,1} = \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,-1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,0}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\left(2-\frac{r}{a_0}\right).\exp\left(-\frac{r}{2a_0}\right)Y_{0,0} \otimes 2\left(\frac{1}{a_0}\right)^{3/2}\exp\left(-\frac{r}{a_0}\right)Y_{0,0}$$

$$\psi_{2,1,-1} = \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,-1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,1}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\frac{r}{\sqrt{3}a_0}\exp\left(-\frac{r}{2a_0}\right)Y_{1,0}(\theta,\varphi) \otimes \left(\frac{1}{2a_0}\right)^{\frac{3}{2}}\left(2-\frac{r}{a_0}\right).\exp\left(-\frac{r}{2a_0}\right)Y_{0,0} \otimes 2\left(\frac{1}{a_0}\right)^{3/2}\exp\left(-\frac{r}{a_0}\right)Y_{0,0}$$

$$\psi_{2,,1,0} = \left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \frac{r}{\sqrt{3}a_0} \exp\left(-\frac{r}{2a_0}\right) Y_{1,0}(\theta,\varphi) \otimes$$
$$\left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \frac{r}{\sqrt{3}a_0} \exp\left(-\frac{r}{2a_0}\right) Y_{1,1}(\theta,\varphi) \otimes$$
$$\left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \frac{r}{\sqrt{3}a_0} \exp\left(-\frac{r}{2a_0}\right) Y_{1,-1}(\theta,\varphi) \otimes$$
$$\left(\frac{1}{2a_0}\right)^{\frac{3}{2}} \left(2-\frac{r}{a_0}\right).\exp\left(-\frac{r}{2a_0}\right) Y_{0,0} \otimes$$
$$2\left(\frac{1}{a_0}\right)^{3/2} \exp\left(-\frac{r}{a_0}\right) Y_{0,0} \tag{17}$$

The wave function describing the entirety of these worlds is:

$$|\Psi\rangle = \sqrt{\frac{1}{5}}\ \psi_{1,0,0} - \sqrt{\frac{1}{5}}\ \psi_{2,0,0} + \sqrt{\frac{1}{5}}\ \psi_{2,1,1} - \sqrt{\frac{1}{5}}\ \psi_{2,1,-1} + \sqrt{\frac{1}{5}}\ \psi_{2,,1,0}$$

The results indicate that when the hydrogen electron of $\left|\Psi_{1,0,0}\right\rangle$ world, or $|s\rangle$, has the potential to be in other states (for example, in the $\left|sp^3\right\rangle$ worlds), the initial world $|s\rangle$ splits into $\left|sp^3\right\rangle$ worlds. Each $\left|sp^3\right\rangle$ world contains a copy of the electron, and each observer in each state measures only the energy corresponding to their respective state (see the figure below).

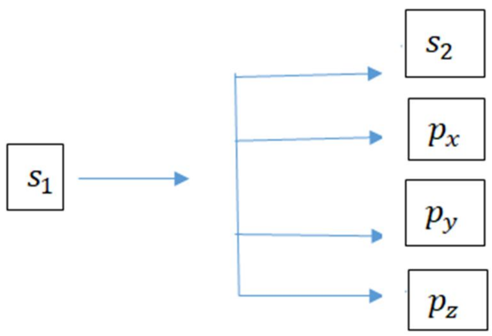

FIG. 3. Electron states.

According to the Copenhagen school, the general wave function is represented by the following relation:

$$\Psi(r,\theta,\varphi) = \sum_{n,l,m} c_{n,l,m}\psi_{n,lm}(r,\theta,\varphi);\ where,$$

$$c_{n,l,m} = \left\langle \Psi(r,\theta,\varphi) \middle| \psi_{n,l,m}(r,\theta,\varphi) \right\rangle,$$

$$and\ \left|c_{nl,m}\right|^2\ present\ the\ probability$$

$$E_n = -\frac{m_e Z^2 e^4}{8\varepsilon_0^2 h^2 n^2},\quad n = 1,2,...i....$$

The results mean that before any measurement, the hydrogen electron, as observed by a single observer, exists in a superposition of all possible states, represented by a combination of wave functions. This concept is discussed in the works of Held [17] and Saxon [18]

$$\Psi(r,\theta,\varphi) = \sum_{n,l,m} c_{n,l,m}\psi_{n,lm}(r,\theta,\varphi),\quad n = 1,2,...i...,.$$

Upon measurement by the observer, the general wave function, $\Psi(r,\theta,\varphi) = \sum_{n,l,m} c_{n,l,m}\psi_{n,lm}(r,\theta,\varphi)$, collapses into a single state:

$$\psi_{i,l,m}(r,\theta,\varphi). \tag{18}$$

5. Conclusion

In classical mechanics, the determinism principle implies that the observer's presence does not affect the observables; measurements in classical physics are deterministic.

However, many interpretation schools have emerged in quantum mechanics, some fundamentally differing from classical mechanics' determinism.

The Copenhagen interpretation, one of the most influential schools, is based on the indeterminism principle. It does not require an observer-induced wave packet collapse, nor does it prioritize the observer's perspective, rejecting ideas of subjectivism and positivism in measurement.

Conversely, deterministic schools such as Hugh Everett's "many-worlds interpretation" hold that quantum mechanics follows a deterministic framework similar to classical mechanics. According to the many-worlds interpretation, there is no action at a distance. Instead, this theory gives a set of local descriptions that wholly describe the entire physical universe. It provides metaphysical neutrality between observers' perspectives on different branches of the universal wave function, as opposed to one-world theories that give a privileged perspective on reality to an observer.

Many students have no idea about these different interpretations at the microscopic scale and cannot explain why some interpretations are deterministic while others are not.

This paper aims to explore these two interpretations in the context of the hydrogen atom's spectrum. By solving the Schrödinger equation under the many-worlds interpretation, our results are compared to those derived from the Copenhagen interpretation. Our findings underscore the mathematical viability of the deterministic school in explaining quantum mechanics because it resolves most paradoxes like the collapse of the quantum wave.

Finally, the many-worlds interpretation presents itself as a reasonable interpretation of quantum mechanics. However, it is necessary to work on the problem of probability and consider modifications to the standard theory.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

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