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ARTICLE

Behaviour of Bianchi Type-V Dark Energy Model in $f(\mathrm{R},\mathrm{T})$ Gravity with a Specific Form of Hubble Parameter

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Abstract: In this paper, we have constructed a Bianchi type V cosmological model, in the presence of bulk viscous fluid and within the framework of $f(\mathrm{R},\mathrm{T})$ theory of gravity with an appropriate choice of the functional $f(\mathrm{R},\mathrm{T})$in the form $f(\mathrm{R},\mathrm{T}) = R + 2f(T)$, where R and T are respectively Ricci scalar and trace of energy momentum tensor. In order to obtain a deterministic solution, we have considered two general forms of hyperbolic scale factors. The different forms of scale factors considered here produce time-varying deceleration parameters in all the cases that simulate the cosmic transition. The state finder diagnostic pair is found to be in the acceptable range. The physical parameters are constrained from different representative values to build up a realistic cosmological model aligned with the observational behaviour.

Keywords: Bianchi type V, Dark energy, $f(\mathrm{R},\mathrm{T})$ gravity, Variable deceleration parameter.

1. Introduction

In recent years, several modified gravity theories, like $f(R)$ gravity, $f(G)$ gravity, $f(T)$ gravity and so on, were investigated by many researchers. A large class of cosmological models has explained the acceleration of the universe in terms of a component with negative pressure, the so-called dark energy (DE). The limitations of general relativity in providing a satisfactory explanation of this phase of evolution have led cosmologists to adopt hypotheses and study their implications in this context. The hypotheses include those assigning (I) the time–dependence of the gravitational constant and cosmological term (II) some other geometries or physical fields associated with the universe and (III) modified or alternative theories of gravity. Modified gravity theories certainly provide a way of understanding the problem of DE and the possibility to reconstruct the gravitational field theories that would be capable to reproduce the late–time acceleration of the universe. In an effort to address the cosmic speed-up issue, Harko et al. [1] introduced a modified gravity theory known as $f(\mathrm{R},\mathrm{T})$ gravity. Several studies were made in this theory addressing different contexts, such as energy conditions (Alvarenga et al. [2]; Kiani & Nozari [3]), wormhole solution (Azizi [4]; Moraes et al. [5]), anisotropy cosmology (Sharif & Zubair [6]; Mishra et al. [7-8]), higher dimensions (Troisi [9]) and non-interacting Chaplygin gas (Shabani [10]; Shabani & Farhoudi [11]). Sharma and Singh [12] have studied the string cosmological model with magnetic field in Bianchi Type II space-time. With a rescaled functional of $f(\mathrm{R},\mathrm{T})$gravity, extensive investigations were carried out in Bianchi type VIh space-time to understand the dynamical behaviour of the aniotropic universe (Mishra et al. [13-14]). Zubair et al. [15] have investigated the anisotropy source with the dynamical analysis of cylindrically symmetric space-time, whereas Mishra and Vadrevu [16] have constructed a cylindrically symmetric

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model with the exact solution. Aktas and Aygun [17] have shown that magnetized field vanishes in FRW universe for $f(\mathrm{R,T})$gravity. Many more Bianchi type cosmological models have been developed in recent past (Shamir [18]; Chaubey & Shukla [19]; Pawar [20-23]; Samanta [24] ; Reddy et al. [25-27]; Shri Ram [28]; Nasr Ahmed [29]). The extraordinary phenomena of $f(\mathrm{R,T})$gravity may provide some significant signatures and effects which could distinguish and discriminate between various gravitational models. Therefore, this theory has attracted many researchers to explore different aspects of cosmology and astrophysics in isotropic and in anisotropic space-times (See for example Khade and Wasnik [30]; Chakraborty [31]; Houndjo et al. [32]; Pasqua et al.[33]; Singh and Singh [34]; Baffou et al. [35]; Santos and Ferst [36]; Noureen et al. [37]; Shamir [38]; Singh and Singh [39]; Alhamzawi and Alhamzawi[40]; Yousaf et al. [41]; Alves et al. [42]; Zubair et al. [43]; Sofuoglu [44]; Momeni et al. [45]; Das et al. [46]; Salehi and Aftabi [47]; Singh and Beesham [48]; Srivastava and Singh [49]; Sharif and Anwar [50]; Tiwari and Beesham [51]; Shabani et al. [52]; Rajabi and Nozari [53]; Baffou et al. [54]; Lobato et al. [55]; Tretyakov [56]; Elizalde and Khurshudyan [57]; Ordines and Carlson [58]; Maurya and Tello-Ortizb [59]; Esmaeili [60] and references therein).

Bulk viscosity is the only dissipative phenomenon occurring in FRW models and is significant in causing the accelerated expansion of the universe known as inflationary phase as discussed by Setaren et al. [61]. Several cosmologists have discussed the role of bulk viscosity in the early evolution of the universe in different physical contexts. The cosmological and astrophysical implications of $f(\mathrm{R,T})$ gravity theory in the presence of perfect fluids and bulk viscous fluids have been studied by several cosmologists. Shri Ram et al. [62] investigated Bianchi type–I and –V bulk viscous fluid cosmological models. Sahu et al. [63] discussed cosmic transits and anisotropic models of Bianchi type–III. Further, Sahoo et al. [64-69] studied cosmological models in $f(\mathrm{R,T})$ theory with variable deceleration parameters. This motivates the theorists to construct various models of different Bianchi space–times in different contexts.

Spatially homogeneous and anisotropic Bianchi models have been widely studied in the framework of general relativity to describe the early stage of evolution of the universe. The theoretical studies and observational data of cosmic microwave background (CMB) and the large structure have stimulated the study of anisotropic models. The study of anisotropic models has also been extended to modified gravitational theories. Pradhan et al. [70], Aktas et al. [71], Yilmaz et al.[72] and Sharif and Zubair [73, 74] are some of the authors who have investigated several aspects of anisotropic Bianchi models in $f(\mathrm{R,T})$gravity.

Recently, the dark energy models, which are inspiring many astrophysicists, are the holographic dark-energy models. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite.

In this paper, we have investigated the physical behaviour of the cosmological model obtained with Bianchi type–V space-time in the presence of hyperbolic scale factor in two different cases. The present paper is organized as follows. The field equations of $f(\mathrm{R,T})$ gravity have been derived in Section 2. The model and basic framework have been presented in Section 3. In Sections 4, 6, the derivation and analysis of parameters have been derived for cases I, II. In Sections 5, 7, the physical properties of the model have been discussed for cases I, II, respectively. Finally, the conclusion is given in Section 8.

2. Field Equations

We assume that the cosmic matter may be represented by the energy-momentum tensor of an imperfect bulk viscous fluid as:

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p} g_{ij} \tag{1}$$

where $\bar{p}$ is the bulk viscous pressure given by:

$$\bar{p} = p - \varsigma u^i; i \tag{2}$$

satisfying a linear equation of state:

$$p = \epsilon\rho, 0 \leq \epsilon \leq 1. \tag{3}$$

Here, p is the equilibrium pressure, ρ is the energy density of matter, ς is the coefficient of bulk viscosity and u^i is the flow vector of the fluid satisfying $u_i u^i = 1$. The semicolon stands for covariant differentiation. On thermodynamic grounds, bulk viscosity coefficient ς is positive, assuring that the viscosity pushes the dissipative pressure $\bar{p}$ towards negative values. However, the correction applied to the thermodynamical pressure p due to bulk viscous pressure is very small. Therefore, the dynamics of cosmic evolution is not fundamentally influenced by the inclusion of the viscous term in the energy-momentum tensor.

For the field equations in $f(R,T)$ modified gravity model, we assume that the function $f(R,T)$ is given by:

$$f(R,T) = R + 2f(T) \quad (4)$$

where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor of matter. The gravitational field equation is immediately given by:

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f^{'}(T)T_{ij} + [2\bar{p}f^{'}(T) + f(T)]g_{ij} \quad (5)$$

where the prime denotes a derivative with respect to the argument.

The simplest cosmological model can be obtained by choosing the function $f(T)$, so that $f(T) = \lambda T$, where λ is a constant.

3. The Model and Basic Framework

The diagonal form of the metric of Bianchi type-V cosmological model is given by:

$$ds^2 = dt^2 - A^2dx^2 - e^{2\beta x}[B^2dy^2 + C^2dz^2]. \quad (6)$$

Here, $A = A(t)$, $B = B(t)$ and $C = C(t)$ are cosmic scale factors and β is an arbitrary constant.

The spatial volume V and the average Hubble's parameter H are defined as:

$$V = a^3 = ABC, \quad (7)$$

$$3H = \frac{\dot{V}}{V} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (8)$$

where a dot denotes differentiation with respect to cosmic time t.

The shear scalar σ and anisotropy parameter Am are defined as follows:

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2 \quad (9)$$

$$Am = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2 \quad (10)$$

where $\triangle H_i = H_i - H, (i = 1,2,3)$ and $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters.

For the metric (6), Eqs. (1), (4) and (5) in comoving coordinates lead to the following set of equations:

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\beta^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda\bar{p} \quad (11)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\beta^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (12)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\beta^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (13)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\beta^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (14)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \quad (15)$$

After integrating Eq. (15) and absorbing integration constant into B or C, we get:

$$A^2 = BC. \quad (16)$$

We have five highly non-linear differential equations with six unknowns; namely, $A, B, C, \rho, \bar{p}, \varsigma$. Therefore, to find a consistent solution to these equations, subtracting Eq. (13) from Eq. (12), Eq. (14) from Eq. (13), Eq. (14) from Eq. (12) and integrating the resulting equations, we obtain the following three relations (Saha and Rikhvitsky [75,76]), respectively:

$$\frac{A}{B} = m_1 exp\left[k_1\int\frac{dt}{a^3}\right] \quad (17)$$

$$\frac{A}{C} = m_2 exp\left[k_2\int\frac{dt}{a^3}\right] \quad (18)$$

$$\frac{B}{C} = m_3 exp\left[k_3\int\frac{dt}{a^3}\right] \quad (19)$$

where $m_1, m_2, m_3, k_1, k_2, k_3$ are constants of integration.

Using Eq. (7), we write the metric functions from (17)-(19) in explicit form as:

$$A = ad_1 exp\left[\alpha_1\int\frac{dt}{a^3}\right] \quad (20)$$

$$B = ad_2 exp\left[\alpha_2\int\frac{dt}{a^3}\right] \quad (21)$$

$$C = ad_3 exp\left[\alpha_3\int\frac{dt}{a^3}\right] \quad (22)$$

where:

$$d_1 = \sqrt[3]{m_1m_2}, d_2 = \sqrt[3]{m_1^{-1}m_3}, d_3 = \sqrt[3]{(m_2m_3)^{-1}}, \quad (23)$$

and

$$\alpha_1 = \frac{k_1+k_2}{3}, \alpha_2 = \frac{k_3-k_1}{3}, \alpha_3 = \frac{-(k_2+k_3)}{3}. \quad (24)$$

The constants d_1, d_2, d_3 and $\alpha_1, \alpha_2, \alpha_3$ satisfy the following two relations:

$$\alpha_1 + \alpha_2 + \alpha_3 = 0;\ d_1\, d_2 d_3 = 1. \quad (25)$$

Substituting Eq. (16) in Eqs. (20)-(22), we obtain:

$$A = a \quad (26)$$

$$B = adexp\left[\alpha \int \frac{dt}{a^3}\right], \quad (27)$$

$$C = ad^{-1}exp\left[-\alpha \int \frac{dt}{a^3}\right] \quad (28)$$

where:

$$d_1 = 1, d_2 = d_3^{-1} = d, \alpha_1 = 0, \alpha_2 = -\alpha_3 = \alpha\ .$$

4. Derivation and Analysis of Parameters

Case (i): For $H = \eta tanh(\eta t)$

For the explicit determination of the cosmic parameters, we need one more condition. Recently, Pacif and Mishra [77] as well as Esmaeili and Mishra [78] have obtained cosmological model in Bianchi types geometry with a specific variation of the Hubble parameter in general relativity, which is a good approximation concerning the current late-time acceleration of the universe. Following the same here, we consider that:

$$H = \eta tanh(\eta t). \quad (29)$$

With the form of H given by Eq. (29), we obtain the average scale factor as:

$$a = \delta cosh(\eta t). \quad (30)$$

Using Eqs. (26)-(28) with the help of (30), we obtain the metric functions as:

$$A = \delta cosh(\eta t). \quad (31)$$

$$B = d\delta cosh(\eta t) exp\left[\frac{\alpha}{2\eta\delta^3}(sech(\eta t)tanh(\eta t) + 2arctane^{\eta t})\right]. \quad (32)$$

$$C = d^{-1}\delta cosh(\eta t) exp\left[\frac{-\alpha}{2\eta\delta^3}(sech(\eta t)tanh(\eta t) + 2arctane^{\eta t})\right]. \quad (33)$$

$$\rho = \frac{1}{(8\pi+2\lambda)(8\pi+4\lambda)}\left\{6\eta^2(4\pi+\lambda)tanh^2(\eta t) - 2\lambda\eta^2 sech^2(\eta t) - \frac{\alpha^2}{\delta^6}(2\pi+\lambda)\left[2sech^3(\eta t) - sech(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right]^2 - \frac{8(3\pi+\lambda)\beta^2}{\delta^2 cosh^2(\eta t)}\right\}. \quad (34)$$

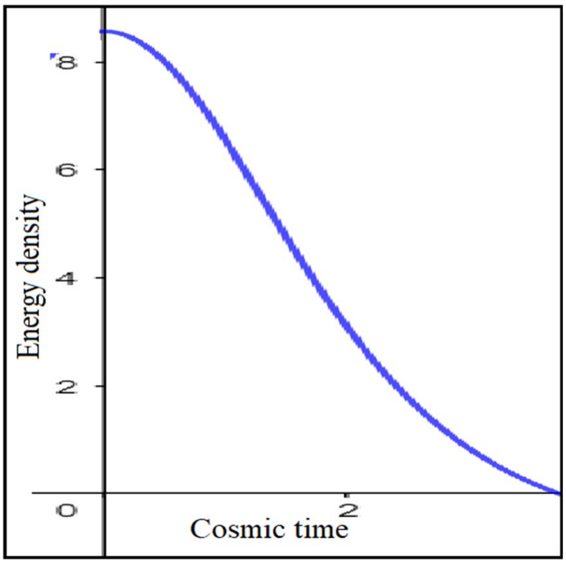

FIG. 1. Energy density *vs*. time for $\eta = 0.5, \alpha = 2, \lambda = -6.5, \delta = 1, \beta = 2$.

The energy density in Fig. 1 lies in the positive domain. It has been observed that the energy density is high in the early time of the universe and then gradually decreases to null. It may be noted here that since ρ needs to be positive, the first term of (34) should dominate the second. Therefore, the behaviours of the parameters are constrained accordingly within the admissible limits.

$$\bar{p} = \frac{1}{(8\pi+2\lambda)(8\pi+4\lambda)}\left\{-6\eta^2(4\pi+\lambda)tanh^2(\eta t) - 2(8\pi+3\lambda)\eta^2 sech^2(\eta t) - \frac{\alpha^2}{\delta^6}(2\pi+\lambda)\left[2sech^3(\eta t) - sech(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right]^2 + \frac{8\pi\beta^2}{\delta^2 cosh^2(\eta t)}\right\}. \quad (35)$$

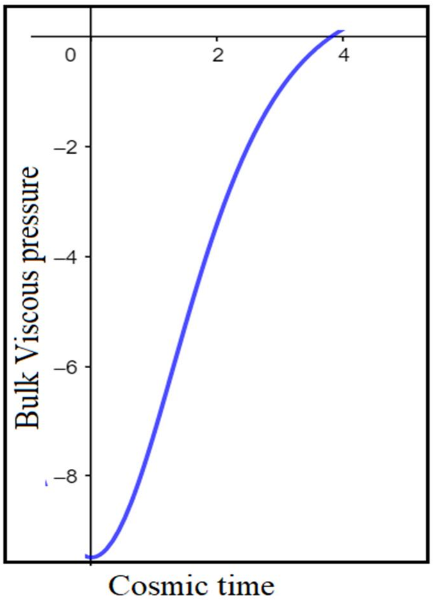

FIG. 2. Bulk viscous pressure $\bar{p}$ *vs*. time for $\eta = 0.5, \alpha = 2, \lambda = -6.5, \delta = 1, \beta = 2$.

From Fig. 2, we observe that bulk viscous pressure $\bar{p}$ lies in the negative range to suffice the acceleration of the universe. The bulk viscous pressure of the universe is an increasing function of cosmic time t, which begins from a negative value and tends to zero at a present epoch. The accelerated expansion of the universe, as per the recent cosmological observations, is due to dark energy which is nothing but negative pressure. Thus, the derived model is in good agreement with the observation.

$$\varsigma = \frac{1}{3(8\pi+2\lambda)(8\pi+4\lambda)\eta\tanh(\eta t)}\Big\{6\eta^2(4\pi+\lambda)(\varepsilon+1)tanh^2(\eta t) + 2(8\pi+3\lambda-\varepsilon\lambda)\eta^2 sech^2(\eta t) - \frac{\alpha^2}{\delta^6}(1-\varepsilon)(2\pi+\lambda)\left[2sech^3(\eta t) - \mathrm{sech}(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right]^2 - \frac{8[(3\pi+\lambda)\varepsilon+\pi]\beta^2}{\delta^2 cosh^2(\eta t)}\Big\}. \quad (36)$$

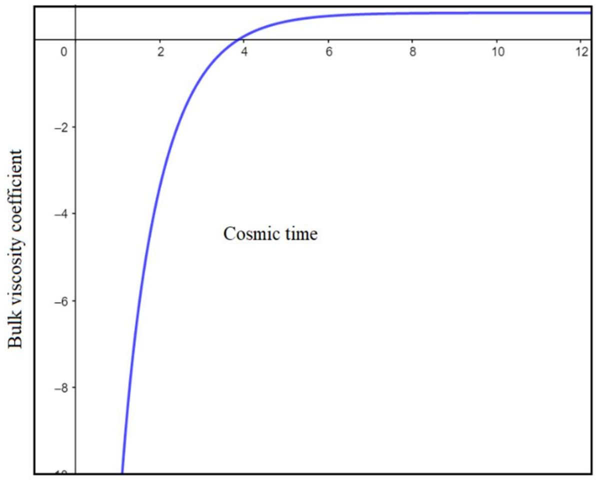

FIG. 3. Bulk viscosity coefficient *vs*. time for $\eta = 0.5, \alpha = 2, \lambda = -6.5, \delta = 1, \beta = 2, \varepsilon = 0.1$.

The barotropic equation of state parameter is used to obtain the coefficient of bulk viscosity. It can be observed that the bulk viscosity is negative at higher redshift (early time) and positive at lower redshift (late time) in the present model with bulk viscosity shown in Fig. 3. This means that the rate of entropy production is negative in the early epoch and positive in the later epoch. Thus, the model does not violate the law of entropy. It also shows the transition from negative to positive in due course of evolution, which indicates the earlier decelerating phase of the universe with positive pressure (suitable for structure formation) and present accelerating phase of the evolution with negative pressure.

5. Physical Properties of the Model

The spatial volume (V), the directional Hubble parameter (H_i), the expansion scalar (θ), the shear scalar (σ^2), the deceleration parameter (q) and the anisotropy parameter (Am) are, respectively, given by:

$$V = a^3 = \delta^3 cosh^3(\eta t). \tag{37}$$

$$H_1 = \frac{\dot{A}}{A} = \eta tanh(\eta t). \tag{38}$$

$$H_2 = \frac{\dot{B}}{B} = \frac{\alpha}{2\delta^3}\left[2sech^3(\eta t) - sech(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right] + \eta \tanh(\eta t). \tag{39}$$

$$H_3 = \frac{\dot{C}}{C} = \frac{-\alpha}{2\delta^3}\left[2sech^3(\eta t) - sech(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right] + \eta \tanh(\eta t). \tag{40}$$

$$\theta = 3\eta tanh(\eta t). \tag{41}$$

$$\sigma^2 = \frac{\alpha^2}{4\delta^6}\left[2sech^3(\eta t) - sech(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right]^2. \tag{42}$$

$$q = -coth^2(\eta t). \tag{43}$$

$$Am = \frac{1}{3}\left\{4 + \frac{\alpha^2}{2\eta^2\delta^6 tanh^2(\eta t)}\left[2sech^3(\eta t) - sech(\eta t) + \frac{2e^{\eta t}}{1+e^{2\eta t}}\right]^2\right\}. \tag{44}$$

The above results are useful to discuss the behavior of the model.

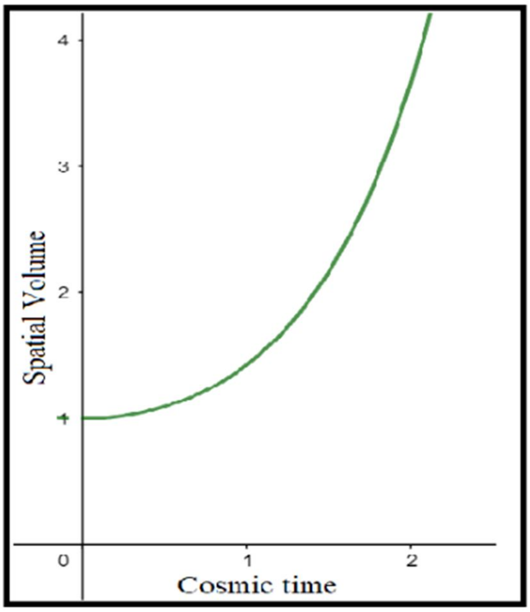

FIG. 4. Spatial volume *vs*. time for $\delta = 1, \eta = 0.5$.

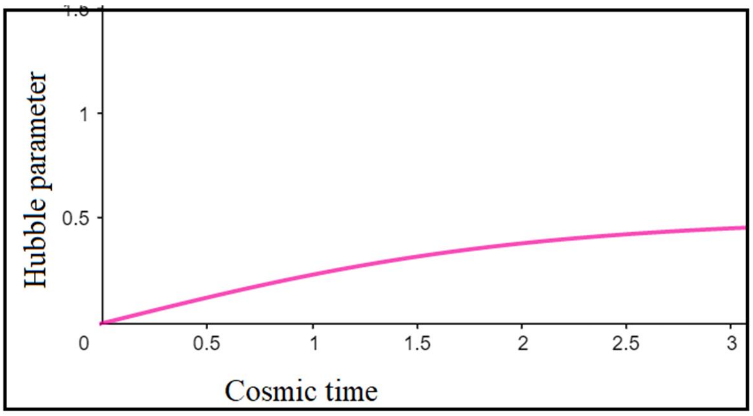

FIG. 5. Hubble parameter *vs*. time for $\delta = 1, \eta = 0.5$.

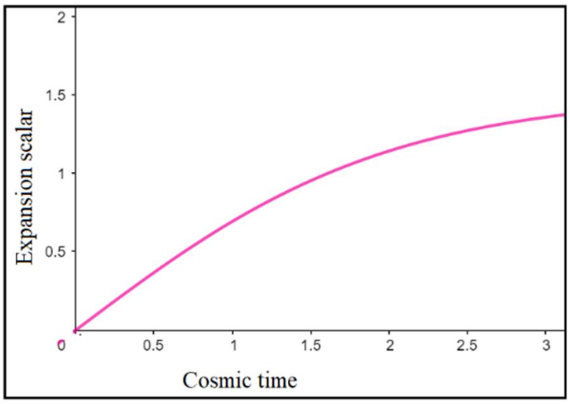

FIG. 6. Expansion scalar *vs*. time for $\delta = 1, \eta = 0.5$.

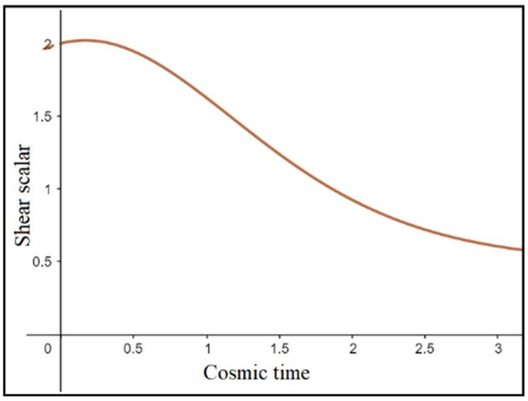

FIG. 7. Shear scalar *vs*. time for $\delta = 1, \eta = 0.5, \alpha = 2$.

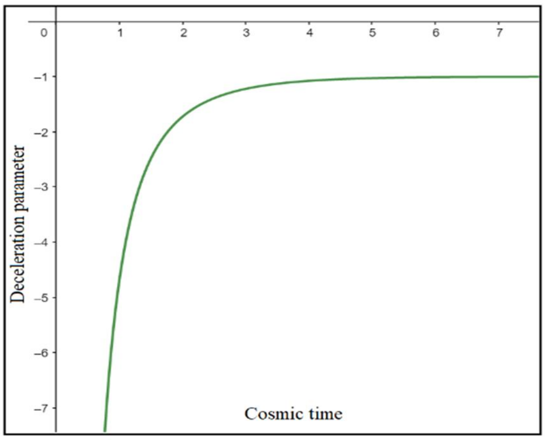

FIG. 8. Deceleration parameter *vs*. time for $\delta = 1$, $\eta = 0.5$.

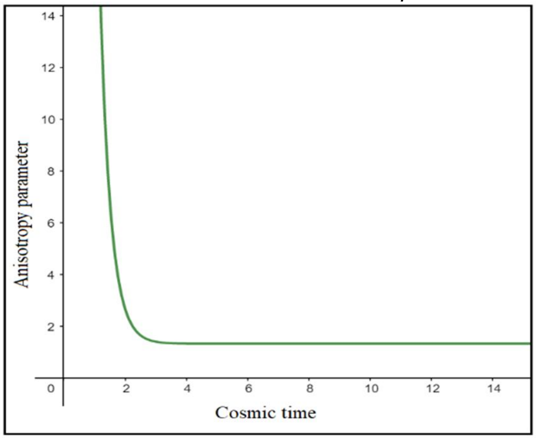

FIG. 9. Anisotropic parameter *vs*. time for $\delta = 1, \eta = 0.5$, $\alpha = 2$.

We have the following observations:

From Eq. (37), it can be observed that the volume scale factor is finite at the initial epoch and positive throughout the evolution. It increases gradually with the increase in time as shown in (Fig 4). The graphical representation of the Hubble parameter is shown in Fig. 5. The parameter is governed by the constant η and the cosmic time. Since we have already assumed a positive constant η to obtain a model that fits observationally, the parameter is now totally controlled by the cosmic time. Hubble parameter increases with the increase in time. Shear scalar decreases with the increase in time (Fig 7). Fig.6 depicts that the expansion scalar increases with the increase in time.

The deceleration parameter is found to be $q = -coth^2(\eta t)$. It indicates that the parameter always remains negative throughout the cosmic evolution for $\eta = 0.5$. Since the scale factor is hyperbolic and can never be negative, this confirms that the deceleration parameter will always remain in the negative domain. It is also observed from Fig. 8 that the accelerated expansion occurs in a reasonable time period, which can be termed as the transition period. The mean anisotropic parameter decreases exponentially and approaches null with an increase in time (Fig. 9).

6. Derivation and Analysis of Parameters

Case (ii): For $H = \eta coth(\eta t)$

$$H = \eta coth(\eta t). \quad (45)$$

With the form of H given by Eq. (45), we obtain the average scale factor as:

$$a = \delta sinh(\eta t). \quad (46)$$

Using Eqs. (26)-(28) with the help of (30), we obtain the metric functions as:

$$A = \delta sinh(\eta t). \quad (47)$$

$$B = d\delta sinh(\eta t) exp\left[\frac{-\alpha}{2\eta\delta^3}\left(cosech(\eta t)coth(\eta t) + logtanh(\frac{\eta t}{2})\right)\right]. \quad (48)$$

$$C = d^{-1}\delta sinh(\eta t)exp\left[\frac{\alpha}{2\eta\delta^3}\left(cosech(\eta t)coth(\eta t) + logtanh(\frac{\eta t}{2})\right)\right]. \quad (49)$$

$$\rho = \frac{1}{(8\pi+2\lambda)(8\pi+4\lambda)}\left\{6\eta^2(4\pi+\lambda)coth^2(\eta t) + 2\lambda\eta^2 cosech^2(\eta t) - \frac{\alpha^2}{\delta^6}(2\pi+\lambda)\left[cosech(\eta t)\left(cosech^2(\eta t) + coth^2(\eta t) - \frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right)\right)\right]^2 - \frac{8(3\pi+\lambda)\beta^2}{\delta^2\sinh^2(\eta t)}\right\}. \quad (50)$$

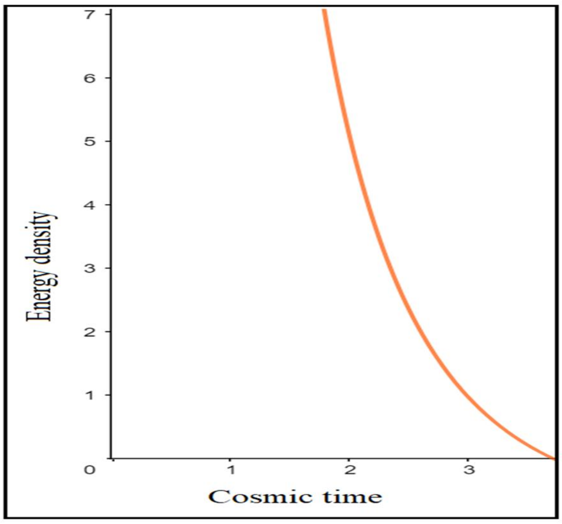

FIG. 10. Energy density *vs*. time for $\eta = 0.5, \alpha = 2, \lambda = -6.5, \delta = 1, \beta = 2$.

The energy density in Fig. 10 lies in the positive domain. It has been observed that the energy density decreases with the increase in time, but it is infinite at the initial epoch.

$$\bar{p} = \frac{1}{(8\pi+2\lambda)(8\pi+4\lambda)}\left\{-6\eta^2(4\pi+\lambda)coth^2(\eta t) + 2(8\pi+3\lambda)\eta^2 cosech^2(\eta t) - \frac{\alpha^2}{\delta^6}(2\pi+\lambda)\left[cosech(\eta t)\left(cosech^2(\eta t) + coth^2(\eta t) - \frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right)\right)\right]^2 + \frac{8\pi\beta^2}{\delta^2 sinh^2(\eta t)}\right\}. \quad (51)$$

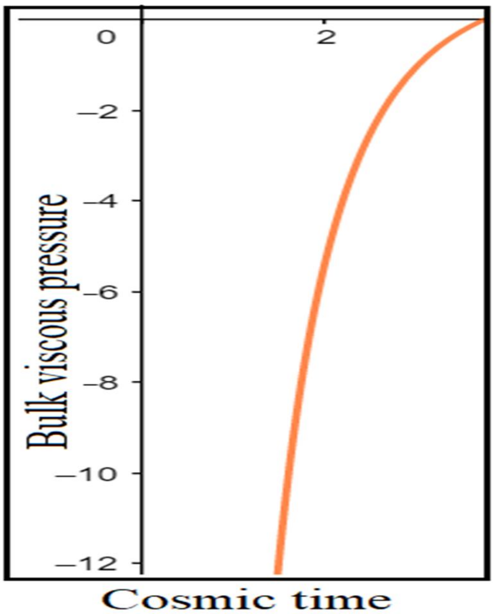

FIG. 11. Bulk viscous pressure $\bar{p}$ *vs*. time for $\eta = 0.5, \alpha = 2, \lambda = -6.5, \delta = 1, \beta = 2$.

From Fig. 11, we observe that bulk viscous pressure $\bar{p}$ is increasing as a function of time. It begins from a large negative value and tends to zero at the present epoch. The present study demonstrates the expanding behaviour of the universe and on the other hand, negative pressure indicates the cosmic accelerated expansion of the universe.

$$\varsigma = \frac{1}{3(8\pi+2\lambda)(8\pi+4\lambda)\eta\coth(\eta t)} \times \left\{ \begin{array}{c} 6\eta^2(4\pi+\lambda)(\varepsilon+1)coth^2(\eta t) \\ +2(8\pi+3\lambda-\varepsilon\lambda)\eta^2 cosech^2(\eta t) - \frac{\alpha^2}{\delta^6}(1-\varepsilon)(2\pi+\lambda) \\ \left[cosech(\eta t)\begin{pmatrix} cosech^2(\eta t) + coth^2(\eta t) \\ -\frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right) \end{pmatrix} \right]^2 \\ -\frac{8[(3\pi+\lambda)\varepsilon+\pi]\beta^2}{\delta^2 sinh^2(\eta t)} \end{array} \right\} \quad (52)$$

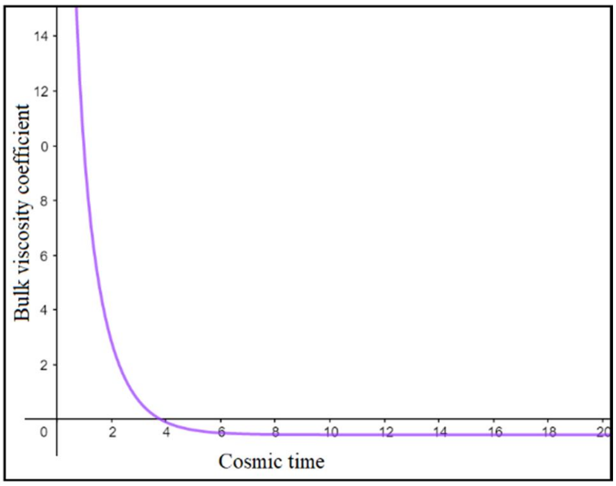

FIG. 12. Bulk viscosity coefficient *vs*. time for $\eta = 0.5, \alpha = 2, \lambda = -6.5, \delta = 1, \beta = 2, \varepsilon = 0.1$.

Using the equation of state parameter, the bulk viscosity coefficient is shown in Fig. 12. Bulk viscosity coefficient is infinite at the initial epoch and has a transition from positive to negative. It also shows the transition from positive to negative in due course of evolution, which indicates the earlier accelerating phase of the universe with negative pressure (suitable for structure formation) and the present decelerating phase of the evolution with positive pressure.

7. Physical Properties of the Model

The spatial volume (V), the directional Hubble parameter (H_i), the expansion scalar (θ), the shear scalar (σ^2), the deceleration parameter (q) and the anisotropy parameter (Am) are, respectively, given by:

$$V = a^3 = \delta^3 sinh^3(\eta t). \quad (53)$$

$$H_1 = \frac{\dot{A}}{A} = \eta coth(\eta t). \quad (54)$$

$$H_2 = \frac{\dot{B}}{B} = \frac{\alpha}{2\delta^3}\left[cosech(\eta t)\left(cosech^2(\eta t) + coth^2(\eta t) - \frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right)\right)\right] + \eta\coth(\eta t). \quad (55)$$

$$H_3 = \frac{\dot{C}}{C} = \frac{-\alpha}{2\delta^3}\left[cosech(\eta t)\left(cosech^2(\eta t) + coth^2(\eta t) - \frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right)\right)\right] + \eta\coth(\eta t). \quad (56)$$

$$\theta = 3\eta coth(\eta t). \quad (57)$$

$$\sigma^2 = \frac{\alpha^2}{4\delta^6}\left[cosech(\eta t)\left(cosech^2(\eta t) + coth^2(\eta t) - \frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right)\right)\right]^2. \quad (58)$$

$$q = -1 + sech^2(\eta t). \quad (59)$$

$$Am = \frac{1}{3}\left\{4 + \frac{\alpha^2}{2\eta^2\delta^6 coth^2(\eta t)}\left[cosech(\eta t)\left(cosech^2(\eta t) + coth^2(\eta t) - \frac{1}{2}cosech\left(\frac{\eta t}{2}\right)\coth\left(\frac{\eta t}{2}\right)\right)\right]^2\right\}. \quad (60)$$

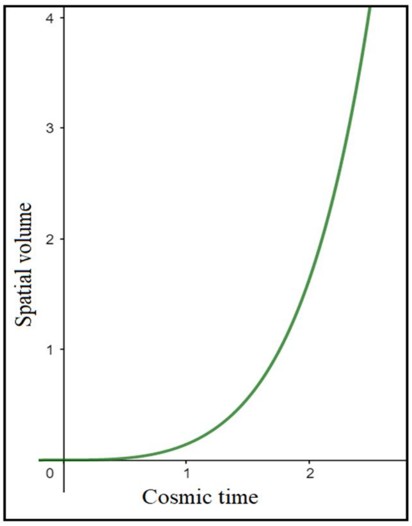

FIG. 13. Spatial volume *vs*. time for $\delta = 1, \eta = 0.5$.

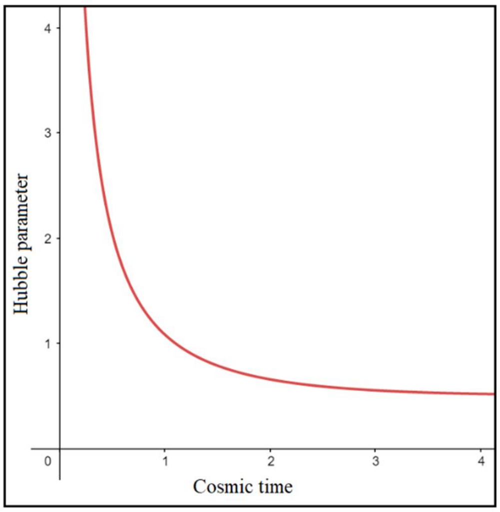

FIG.14. Hubble parameter *vs*. time for $\delta = 1, \eta = 0.5$.

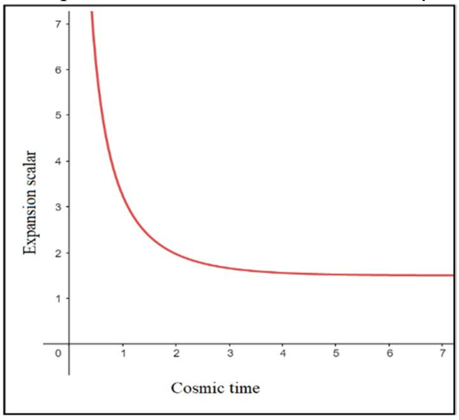

FIG. 15. Expansion scalar *vs*. time for $\delta = 1, \eta = 0.5$.

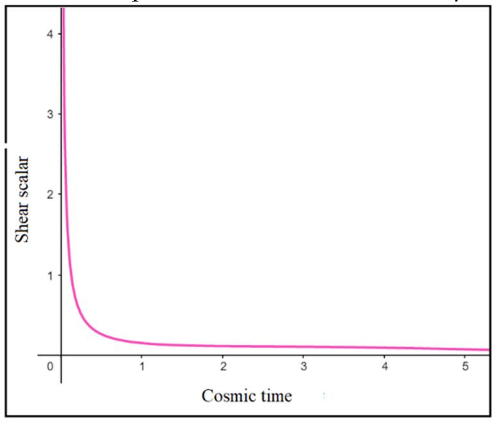

FIG. 16. Shear scalar *vs*. time for $\delta = 1, \eta = 0.5, \ \alpha = 2$.

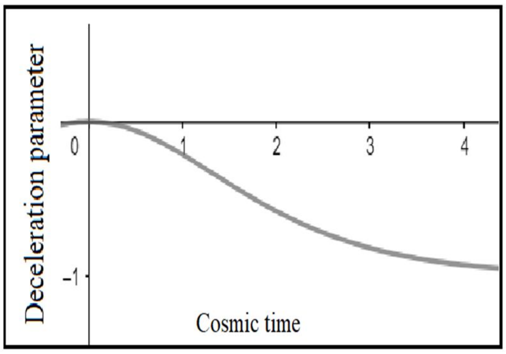

FIG. 17. Deceleration parameter *vs*. time for $\delta = 1$, $\eta = 0.5$.

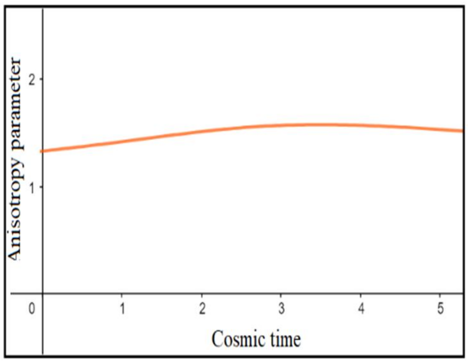

FIG. 18. Anisotropic parameter *vs*. time for $\delta = 1$, $\eta = 0.5, \ \alpha = 2$.

We have the following observations:

Spatial volume is zero when t = 0 and it increases as time increases. This means that the expansion of the universe starts with finite volume and it is expanding as t increases (Fig. 13). The average Hubble parameter (H), expansion scalar (θ) and shear scalar (σ) are functions of time t, have a singularity at t = 0 and tend to zero for large t. The expansion scalar and shear scalar diverge at an early stage of the universe and tend to zero for infinitely large values. It is observed that the universe initially evolves with an infinite expansion rate and shows a constant expansion at a later epoch. Also, H decreases as t increases (Fig. 14). The Hubble parameter H approaches zero for infinitely large time. Expansion scalar (θ) also decreases as t increases (Fig. 15), but the positive values of Hubble parameter and expansion scalar throughout the evolution show that the universe is expanding gradually. Here, the anisotropy parameter is finite at initial time and is uniform throughout the evolution of the universe.

8. Conclusion

We have considered two different hyperbolic forms of Hubble parameter to construct some DE cosmological models of the universe in the framework of $f(R,T)$ gravity. The space-time considered is the spatially homogeneous anisotropic Bianchi V metric. With the help of the forms of Hubble parameter and Hubble expansion rate along different directions, the anisotropic behaviour of the dark energy-driven cosmological model has been simulated. For case I, energy density is finite, whereas in case II, energy density is infinite for $t = 0$. In both cases, energy density is a decreasing function of time and lies in the positive domain. For the first case, the bulk viscous pressure of the universe is an increasing function of cosmic time t, which begins from a negative value and tends to zero at the present epoch, whereas in the second case, bulk viscous pressure $\bar{p}$ is an increasing function of time. It begins from a large negative value and tends to zero at the present epoch. In both cases, the model provides an accelerating behaviour of the universe at late time of the evolution. More or less, the physical behaviour of both models appears to be the same at least at late times. In both cases, the model represents an expanding, shearing and accelerating universe. In both cases, the model has no initial singularity. Since the metric potential of the universe A(t) and B(t) are constant at $t = 0$, we observed that in both cases, the positive value of the Hubble parameter and the deceleration parameter q $\rightarrow -1$ for infinite time throughout the evolution show that the universe is expanding and accelerating exponentially. In the first case, there is no initial singularity, whereas in the second case, we have infinite energy density, infinite internal pressure for initial time. This means that our universe has an initial singularity for case II. In the present work, we observed that different DE anisotropic models depend on Hubble parameter. It is concluded that the bulk viscous pressure anisotropy in DE fluid plays a very important and interesting role, so that bulk viscous pressure anisotropy needs to be investigated for further better understanding of the accelerated expansion of the universe.

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