

Treatment of Perturbation in Energy Levels using Hypothetical Perturbers: A Case Study of Mg X

Salman Raza^a, Naveed Ali^a, Muhammad Raza^b, Adeel Tahir^c
and Zaheeruddin^a

^a Department of Physics, University of Karachi, Karachi, Pakistan.

^b Department of Physics Karakoram International University Gilgit, Gilgit-Baltistan Pakistan.

^c Department of Physics, Federal Urdu University of Science Art and Technology, Karachi, Pakistan.

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Abstract: Rydberg atoms exhibit hydrogen-like behavior in which the motion of a single electron is governed by the effective potential of the ionic core, consisting of the nucleus and the remaining electrons. The energies of hydrogenic atoms are commonly calculated using the Rydberg-Ritz formula, which can also be adapted for lithium-like systems by introducing a non-integer principal quantum number. The deviation from the integer quantum number is referred to as the quantum defect. This approach successfully describes the energy levels of most Rydberg atoms and ions. However, when perturbations occur between nearby energy levels, the standard Rydberg-Ritz formula becomes inadequate and must be replaced by the Extended Martin formula.

Accurate determination of perturbed energy levels generally requires detailed knowledge of the interacting perturber states, which are often difficult to identify experimentally. This study presents an alternate and effective method for treating perturbed Rydberg levels using hypothetical perturbers. The proposed approach was applied to four perturbed series of Mg X, and the energies of all four series were successfully determined using a new technique based on hypothetical levels to account for perturbations arising from foreign configuration interaction.

The calculated Rydberg energies were further validated through the computation of transition probabilities, oscillator strengths, and line strengths. In total, 68 Rydberg energy levels and 935 spectroscopic parameters were calculated. Among these, 29 Rydberg levels and 751 transition probabilities, oscillator strengths, and line strengths are not currently available in the National Institute of Standards and Technology (NIST) database.

Keywords: Rydberg levels, Transition probabilities, Oscillator strength, Perturbation theory, Mg X.

1. Introduction

In this study, precise energy values for lithium-like Mg X were obtained from the National Institute of Standards and Technology (NIST) [1]. The calculations were performed using atomic structure codes developed in Python based on the one-electron approximation within the framework of the weakest bound electron potential model theory (WBEPMT) [2],

combined with the numerical Coulomb approximation (WBEPMCA) [3] and quantum defect theory (QDT) [4]. The calculated results showed excellent agreement with the reference data reported by NIST, confirming the accuracy and reliability of the computational approach.

Magnesium (Mg), an alkaline earth metal, is characterized by its silvery-white metallic appearance. Owing to its high chemical reactivity, elemental magnesium is rarely found in its free form in nature despite its significant cosmic and terrestrial abundance [5]. Magnesium makes up 13% of the planet's mass and around 2% of its crust. It is the ninth most plentiful element in the universe and the eighth most prevalent in the earth's crust. It is the fourth most frequent element on Earth after silicon, iron, and oxygen. Additionally, it is added to zinc alloys to enhance their dimensional stability and mechanical qualities [6]. Magnesium alloys are highly favored for use in aerospace or transportation technologies due to their excellent strength-to-weight ratio. However, their corrosion behavior makes them less attractive than aluminum alloys. This study investigates the fundamental mechanisms underlying magnesium corrosion and provides a basis for the development of new magnesium alloys with improved corrosion resistance [7]. Since its discovery, Mg has played an important role in technological and industrial development. In the early stages of its application, military demands significantly accelerated its advancement and utilization [8]. For example, magnesium was used in the manufacture of flares, incendiary bombs, and ammunition that were later employed in World War II. Its high combustibility contributed to intense fires and extensive destruction [9]. Following World War II, researchers discovered that magnesium was readily available and had a special combination of qualities that made it appealing for various uses. Magnesium is now utilized in consumer electronics, automotive, and aerospace engineering. It also has its uses in medicines and organic chemistry [10] and is employed in constructing several general-purpose items, including office supplies, sporting goods, and domestic items [8]. Furthermore, Mg is becoming increasingly popular among researchers and industry professionals as a viable biomaterial because of its exceptional biological qualities, particularly its capacity to biodegrade in vivo [11]. Transition rate data for Mg X were previously investigated by Peach et al. in studies related to atomic data for opacity calculations [12]. Their work included a detailed comparison between theoretical and experimental results, demonstrating good computational accuracy for

opacity-related applications; however, the study was limited to low-lying energy levels.

In 1981, Fawcett analyzed the emission spectra of plasmas generated by a 4GW neodymium-glass laser incident on solid planar targets. The reported transitions involved energy levels corresponding to principal quantum numbers $n = 3$ and $n = 4$ [13].

In 1998, Yan et al. calculated highly accurate energy levels and oscillator strengths for Mg X using a Hylleraas-type variational method [14]. In related studies, energy levels, lifetimes, and transition probabilities for transitions among calculated states from Be-like to Ne-like ion sequences were also reported. The corresponding wavefunctions were obtained using the multiconfiguration Hartree–Fock (MCHF) method, with relativistic corrections incorporated through the Breit–Pauli Hamiltonian, excluding the orbit–orbit interaction term. The calculations included both allowed electric dipole (E1) transitions and forbidden magnetic pole (M1), electric quadrupole (E2), and magnetic quadrupole (M2) transitions. [15]. The results demonstrated that perturbation effects under relativistic corrections remain significant below the Mg X ionization limit.

In addition, fully relativistic calculations of collision strengths for Li-like ions were performed using distorted-wave methods, through which transition energies and electric dipole oscillator strengths were determined [16]. In 2003, Podobedova et al. compiled the Mg X far-ultraviolet spectrum and provided accuracy estimations for the reported spectral lines [17]. The available literature on Mg X spectroscopy remains relatively limited and primarily focuses on selected spectroscopic characteristics and low-lying transitions. A later compilation by Kelleher and Podobedova in 2008 incorporated transition-rate data sourced from multiple references [12, 17], while the accuracy of the compiled values was evaluated using the standard deviation of the mean [18]. Since then, little attention has been given to high-lying transition rates and their corresponding energy levels in Li-like Mg X systems.

In theoretical techniques for determining radiative properties of atomic/ionic systems, only low electron state properties are computed due to the demanding calculations involved. Recently, however, few successful studies have

been conducted for high electron states in atoms/ions of different elements using one-electron approximation [19, 20, 21, 22, 23, 24, 25], and the computed values are in good agreement with previously published literature and the values listed at NIST [1]. In this computation, WBEPMA is employed on Mg X. Unlike the solution of alkali metals, the correlation between the core and valence shell electrons is considered. In lithium-like Mg X, owing to the higher shielding effect, the condition is distinct from that of alkali metals such as lithium. Based on the potential model, the ionic system is subdivided into weakest bound electrons (WBE) and non-weakest bound electrons (NWBE's) based on their distinguished behavior in excitation and ionization. Therefore, the wave function of the model potential for many-electron systems can be modified into a one-electron system, which can be called a one-electron approximation and can be computed easily for the radiative properties of ionic systems [26].

2. Theory

In a multi-electron system, the outermost electron is supposed to be the most loosely bound compared to the rest of the electronic cloud; thus, it can most easily be excited. From the perspective of electronic excitation, the behavior of the weakest bound electron (WBE) differs significantly from that of the remaining electrons in the atomic system. Consequently, the WBE can be treated independently, allowing a complex multi-electron problem to be simplified into an effective single-electron problem [28]. This concept was later combined with quantum defect models by Zheng et al. in the development of the Weakest Bound Electron Potential Model (WBEPM) theory [26]. The foundation of WBEPM is based on the assumption that, in any atomic system, the orbiting electrons can be classified into the weakest bound electron (WBE) and the non-weakest bound electrons (NWBEs) [27-33]. The WBE is assumed to move within an effective field generated by the nucleus and the inner-core electrons. When the WBE is excited to a higher energy level, it occupies a larger orbit and exhibits a longer lifetime. Consequently, the coupling between the core and the WBE decreases [34]. The single-electron Hamiltonian and the corresponding nonrelativistic

Schrödinger equation for the WBE (i) are given as follows:

$$H_i = -\frac{1}{2} + \nabla_i^2 + V(r_i) \quad (1)$$

$$\left[-\frac{1}{2} + \nabla_i^2 + V(r_i)\right]\psi_i = \varepsilon_i\psi_i \quad (2)$$

Using the polar coordinates and separating the angular part, we get the radial part:

$$\frac{d^2R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + 2 \left(E - \frac{A}{r_i} - \frac{B}{r_i^2} - \frac{l(l+1)}{r_i^2} \right) = 0 \quad (3)$$

Here, $A/r_i - B/r_i^2 = V(r_i)$ is the potential observed by the WBE's, $A = -Z^*$ is the effective nuclear charge where r_i is the distance between WBE's (i) and the nucleus.

$B = \frac{d(d+1)+2dl}{2}$ herein, the factor d has been introduced due to quantum defects. It is the modification of integral values of n and l into non-integral values n^* and l^* . Hence Equ.3 becomes:

$$\frac{d^2R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + 2 \left(E + \frac{Z^*}{r_i} - \frac{l^*(l^*+1)}{r_i^2} \right) = 0 \quad (4)$$

Here, $2B + l(l+1) = l^*(l^*+1)$ and $l^* = l - \delta_n$, $n^* = n - \delta_n$, are effective orbital- and principal- quantum numbers, respectively, herein, δ_n is the quantum defect and is given as a function of n :

$$\delta_n = a_0 + \frac{a_1}{(n-\delta_o)^2} + \frac{a_2}{(n-\delta_o)^4} + \frac{a_3}{(n-\delta_o)^6} \quad (5)$$

herein, δ_o is the lowest value of quantum defect. The variables a_i 's can be calculated by fitting the first few energies of Rydberg levels. The energy E of the weakest bound electron is given as:

$$E = -\frac{Z^{*2}}{2n^{*2}} = -\frac{Z^{*2}}{2(n-\delta_n)^2} \quad (6)$$

herein, " n^* " denotes the effective principle quantum number, while " Z^* " denotes the effective nuclear charge. They can be found by;

$$\frac{Z^*}{n^*} = \frac{Z_{net}}{n-\delta_n} \quad (7)$$

" Z_{net} " in Eq.7 is the atomic kernel (for neutral atoms $z_o = 1$) and " δ_n " ($\delta = n - n^*$ or in terms of the angular moment $\delta = l - l^*$) is the quantum defect in the n^{th} principal quantum number.

In the light of WBEPM theory, Rydberg states energies can be calculated by:

$$T = T_{limit} + E \quad (8)$$

Using Eqs. (6), (7), and (8) converts into:

$$T = T_{limit} - R \left(\frac{z_{net}}{n - \delta_n} \right)^2 \quad (9)$$

herein, T_{limit} is the ionization limit. The above Eq. (5) is valid only for unperturbed series, i.e., no mixing of foreign energy levels [1].

Zheng took the foreign energy levels mixing into consideration by combining Martin's equation with Langer's equation [33], and Eq. (5) is converted to Eq (10).

$$\delta_n = \sum_{i=1}^4 \frac{a_i}{m^{2(i-1)}} + \sum_{j=1}^N \frac{b_j}{m^{-2+\varepsilon_j}} \quad (10)$$

In the above Eq.10, " $m = n - \delta_o$ " and " $\varepsilon_j = \frac{T_{limit} - T_{perturb}}{R(z_{net}^2)}$ ", " N " is the number of perturbing levels and " $T_{perturb}$ " is the energy of those corresponding perturbing levels. " b_j " are coefficients of the extended Martin's equation, and the number of coefficients depends on the number of perturbing-levels.

Further, the solution of Eq. (4) gives the radial wave function R as:

$$R = C \exp \left(-\frac{Z^* r}{n^*} \right) r^{l^*} L_{n^*-l^*-1}^{2l^*+1} \left(\frac{2Z^* r}{n^*} \right) \quad (11)$$

$$\text{where } C = \left(\frac{2Z^*}{n^*} \right)^{l^*+\frac{3}{2}} \sqrt{\frac{(n^*-l^*-1)!}{2n^* \Gamma(n^*+l^*+1)}}.$$

The transition probability A_{fi} of a transition for spontaneous emission between levels (n_f, l_f) & (n_i, l_i) , is given as:

$$A_{fi} = 20261 \times 10^{-6} \frac{(E_f - E_i)^3}{2l_i + 1} S \quad (12)$$

Where, $E_f > E_i$ and are the energies of upper and lower states, respectively. S is the electric dipole line strength. Since the LS coupling dominates in lighter atoms, thus line strength may be:

$$S_{LS} = [J_f, J_i, L_f, L_i] \left(\begin{Bmatrix} L_f & S & J_f \\ J_i & 1 & L_i \end{Bmatrix} \begin{Bmatrix} L_f & l_f & L_c \\ 1 & L_i & l_i \end{Bmatrix} P_{l_i l_f}^{(1)} \right)^2 \quad (13)$$

The terms contain two 6J symbols in brackets and a matrix element, $P_{l_i l_f}^{(1)}$, which is given as:

$$P_{l_i l_f}^{(1)} = l_{>} < n_i, l_i | r | n_f, l_f > = l_{>} \int_0^\infty r^3 R_{n_i l_i} R_{n_f l_f} dr \quad (14)$$

3. Results and Discussion

A significant gap exists in the available high- n transition-rate data for Mg X, as discussed in the Introduction of this article. The application of quantum defect theory (QDT) enables the determination of high- n transition-rate data without the need for foreign-level configuration mixing. Such mixing arises when energy levels possess the same total angular momentum quantum number j and parity, causing them to act as perturbers of the corresponding series. This interaction leads to perturbations in the energy structure. In such cases, perturbation theory (PT), combined with QDT, must be applied to analyze the perturbed series, provided that the energies of the perturbers are known. However, for Mg X, the perturber energies are not available in the NIST database [1].

Therefore, in the present work, a new approach is introduced in which hypothetical foreign levels are employed in place of unavailable real perturber values to determine the energies of the perturbed levels. These hypothetical levels were obtained using QDT. Energy levels that deviate from NIST [1] reference values are considered perturbers of the series. The perturbations arise from the strong core potential in lithium-like Mg X. Due to the enhanced shielding effect, the behavior differs from that observed in alkali-metal systems such as lithium, allowing series with the same spectral term but different j values to mix with one another.

Consequently, the Extended-Martin formula was employed to determine the energies of the perturbed series (see Eq. 10). The method was applied to the $1s^2 nl$ configurations ($l = s, p, d$) corresponding to the $^2P_{1/2}^o$, $^2P_{3/2}^o$, $^2D_{3/2}$, and $^2D_{5/2}$ series in Mg X. The calculated energies showed good agreement with the NIST [1] energy data, as presented in Tables 2- 6, with the deviations approaching zero in many cases.

In total, 68 Rydberg levels and 935 values of transition probabilities, oscillator strengths, and line strengths were computed. Among these, 29 Rydberg levels and 751 corresponding transition parameters are currently unavailable in the NIST database [1] (see Tables 2-6). In these tables, the second column lists the configurations of the upper and lower levels, while the third, fourth, and fifth columns present the transition probabilities, line strengths, and oscillator

strengths, respectively. Each of these columns is further divided into three subcolumns containing values from the NIST database, QDT calculations, and the hypothetical perturber technique (HPT), respectively. Comparison of the results indicates that most transition probabilities deviate by less than 2%, while only a few exhibit deviations approaching 5%.

3.1 Hypothetical Perturbers

The energies of the five configurations $1s^2nl$ ($l = s, p, d$), corresponding to the $^2S_{1/2}$, $^2P_{1/2}^o$, $^2P_{3/2}^o$, $^2D_{3/2}$, $^2D_{5/2}$ series, were determined using QDT. The energy levels that exhibited deviations when compared with the NIST data [1] were identified as perturber energies. These deviated levels were subsequently treated as hypothetical

perturbers (HPs). The quantum defects were then recalculated using the Extended-Martin formula by incorporating the hypothetical perturber energies, and the corrected energy levels were obtained using Eq. (8). After incorporating the hypothetical perturbers, the deviations in the calculated energies were effectively eliminated.

The coefficients a_i , b_j , and δ_0 were evaluated using Eqs. (8), (9), and (10), and their values are listed in Table 1. In Table 1, the coefficient a_0 indicates whether a series is perturbed, weakly perturbed, or unperturbed, based on whether the quantum defects converge toward a_0 . Meanwhile, the coefficient a_1 reflects the core nature of the series.

TABLE 1. Shows the coefficients of Quantum defect theory and Perturbation theory with the core-nature of series.

Coefficients	$1s^2ns^2S_{1/2}$	$1s^2np^2P_{1/2}^o$	$1s^2np^2P_{3/2}^o$	$1s^2nd^2D_{3/2}$	$1s^2nd^2D_{5/2}$
a_0	0.07760	0.02390	0.02198	-0.00060	0.00761
a_1	-0.02744	-0.00271	0.08268	0.30058	-0.17608
a_2	0.02533	-0.18500	-1.23561	-6.06023	1.41791
a_3	-0.00006	0.63504	3.46977	31.73088	-3.30930
b_1		5.14E-09	-4.41E-08	-9.23E-08	4.61E-08
b_2		1.61E-09	-2.53E-08		
b_3		2.96E-09	-5.59E-08		
b_4		1.47E-09			
δ_0	0.07587	0.07587	0.02031	0.00148	0.00100
Core-Nature ³⁵	Polarization	Polarization	Penetration	Penetration	Polarization

The perturbed and unperturbed Rydberg energy series converge to the same ionization limit ($E_{limit} = 2964000 \text{ cm}^{-1}$). The transition probabilities, oscillator strengths, and line strengths were calculated for all five aforementioned series up to principal quantum

number $n = 15$, as listed in Tables 2-6. The computed values show good agreement with the experimental data available in the NIST database [1]. Detailed results are provided in the supplementary material.

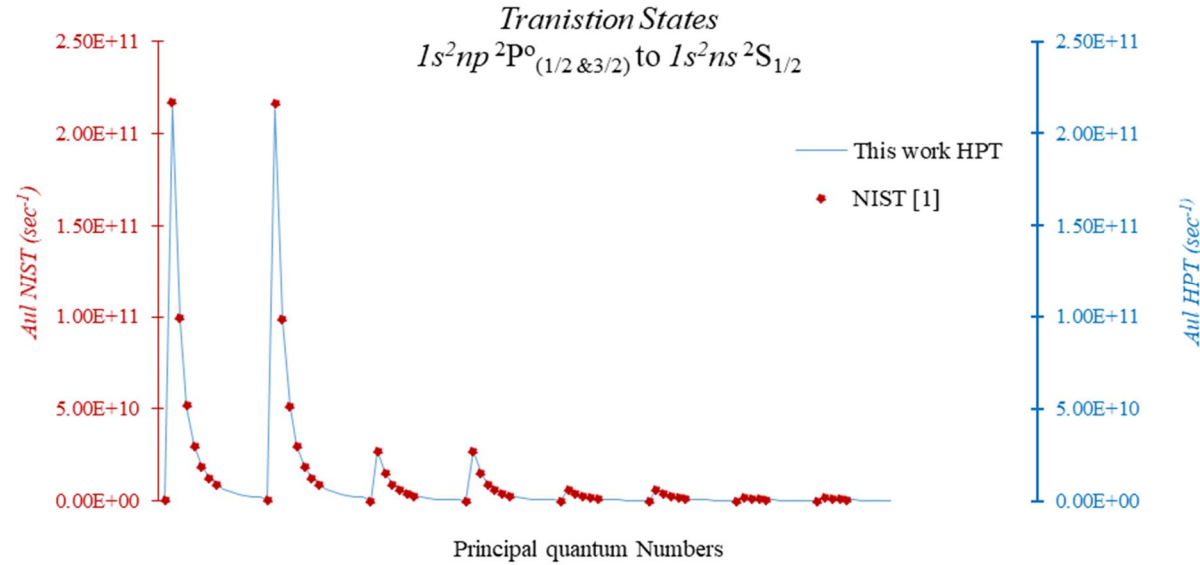
TABLE 2. Computed and compared values of A_{ul} , S_{lu} and f_{lu} transitions from $1s^2(2 \leq n \leq 15) np \ ^2P^o_{(1/2 \text{ \& } 3/2)}$ to $1s^2(2 \leq n \leq 5) ns \ ^2S_{1/2}$.

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E _u)			Lower State (E _l)	A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}	
	1s ² n ^l (2s+1)L _(l±s)	1s ² n' ^{l'} (2s+1)L _(l'±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
1	2 p ² P _{1/2} ^o	1s ² 2s ² S _{1/2}	160015.00	160015.00	160015.00	0.00	7.10E+08	6.95E+08	4.15E-02	4.07E-02	1.71E-01	1.67E-01
	3 p ² P _{1/2} ^o		1726520.00	1726520.00	1726520.00		2.14E+11	2.17E+11	1.07E-01	1.09E-01	4.09E-02	4.16E-02
	4 p ² P _{1/2} ^o		2270150.00	2270150.00	2270150.00		9.77E+10	9.93E+10	2.83E-02	2.89E-02	8.19E-03	8.38E-03
	5 p ² P _{1/2} ^o		2520900.00	2520900.00	2520900.00		5.09E+10	5.17E+10	1.19E-02	1.22E-02	3.11E-03	3.19E-03
	6 p ² P _{1/2} ^o		2656759.04	2656500.00	2656500.00		2.97E+10	2.97E+10	6.28E-03	6.30E-03	1.55E-03	1.56E-03
	7 p ² P _{1/2} ^o		2738522.02	2738400.00	2738400.00		1.87E+10	1.87E+10	3.73E-03	3.74E-03	8.95E-04	8.99E-04
	8 p ² P _{1/2} ^o		2791513.32	2791200.00	2791200.00		1.26E+10	1.26E+10	2.42E-03	2.42E-03	5.70E-04	5.71E-04
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00		8.88E+09	8.80E+09	1.66E-03	1.65E-03	3.85E-04	3.84E-04
	10 p ² P _{1/2} ^o		2853739.09	2853740.00			6.43E+09		1.18E-03		2.71E-04	
	11 p ² P _{1/2} ^o		2872914.62	2872910.00			4.83E+09		8.73E-04		2.00E-04	
	12 p ² P _{1/2} ^o		2887490.55	2887490.00			3.72E+09		6.65E-04		1.51E-04	
	13 p ² P _{1/2} ^o		2898828.40	2898830.00			2.93E+09		5.19E-04		1.18E-04	
	14 p ² P _{1/2} ^o		2907820.82	2907820.00			2.34E+09		4.13E-04		9.34E-05	
	15 p ² P _{1/2} ^o		2915072.79	2915070.00			1.91E+09		3.34E-04		7.54E-05	
	2		2 p ² P _{1/2} ^o	1s ² 2s ² S _{1/2}	160015.00		163990.00	160015.00	0.00	7.65E+08	7.51E+08	8.52E-02
3 p ² P _{1/2} ^o		1726520.00	1727830.00		1726520.00	2.13E+11	2.16E+11	2.13E-01		2.16E-01	8.12E-02	8.25E-02
4 p ² P _{1/2} ^o		2270150.00	2270150.00		2270150.00	9.77E+10	9.88E+10	5.66E-02		5.75E-02	1.64E-02	1.67E-02
5 p ² P _{1/2} ^o		2520900.00	2520900.00		2520900.00	5.09E+10	5.15E+10	2.39E-02		2.43E-02	6.23E-03	6.35E-03
6 p ² P _{1/2} ^o		2656759.04	2656500.00		2656500.00	2.97E+10	2.97E+10	1.26E-02		1.26E-02	3.11E-03	3.12E-03
7 p ² P _{1/2} ^o		2738522.02	2738400.00		2738400.00	1.87E+10	1.87E+10	7.46E-03		7.48E-03	1.79E-03	1.80E-03
8 p ² P _{1/2} ^o		2791513.32	2791200.00		2791200.00	1.26E+10	1.26E+10	4.84E-03		4.84E-03	1.14E-03	1.14E-03
9 p ² P _{1/2} ^o		2827803.68	2827600.00		2827600.00	8.81E+09	8.83E+09	3.28E-03		3.31E-03	7.63E-04	7.71E-04
10 p ² P _{1/2} ^o		2853739.09	2853760.00			6.42E+09		2.35E-03			5.42E-04	
11 p ² P _{1/2} ^o		2872914.62	2872940.00			4.82E+09		1.74E-03			3.99E-04	
12 p ² P _{1/2} ^o		2887490.55	2887510.00			3.72E+09		1.33E-03			3.02E-04	
13 p ² P _{1/2} ^o		2898828.40	2898840.00			2.92E+09		1.04E-03			2.35E-04	
14 p ² P _{1/2} ^o		2907820.82	2907830.00			2.34E+09		8.25E-04			1.87E-04	
15 p ² P _{1/2} ^o		2915072.79	2915080.00			1.90E+09		6.68E-04			1.51E-04	
3		3 p ² P _{1/2} ^o	1s ² 3s ² S _{1/2}		1726520.00	1726520.00	1726520.00	1682700.00		8.88E+07	8.85E+07	6.92E-02
	4 p ² P _{1/2} ^o	2270150.00		2270150.00	2270150.00	2.69E+10	2.69E+10		1.17E-01	1.17E-01	1.31E-01	1.31E-01
	5 p ² P _{1/2} ^o	2520900.00		2520900.00	2520900.00	1.52E+10	1.52E+10		3.24E-02	3.24E-02	2.54E-02	2.55E-02
	6 p ² P _{1/2} ^o	2656759.04		2656500.00	2656500.00	9.12E+09	9.05E+09		1.44E-02	1.43E-02	9.71E-03	9.67E-03
	7 p ² P _{1/2} ^o	2738522.02		2738400.00	2738400.00	5.80E+09	5.76E+09		7.77E-03	7.75E-03	4.84E-03	4.83E-03

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S.No	Transition Array		Rydberg Energies (cm ⁻¹)			Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)	A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}	
	1s ² n ^l ^(2s+1) L _(l±s)	1s ² n' ^{l'} ^(2s'+1) L' _(l'±s')	This work QDT	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
4	8 p ² P _{1/2} ^o	1s ² 3s ² S _{1/2}	2791513.32	2791200.00	2791200.00	3.95E+09	3.87E+09	4.80E-03	4.72E-03	2.85E-03	2.80E-03
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00	2.78E+09	2.73E+09	3.17E-03	3.12E-03	1.82E-03	1.79E-03
	10 p ² P _{1/2} ^o		2853739.09	2853740.00		2.00E+09		2.18E-03		1.23E-03	
	11 p ² P _{1/2} ^o		2872914.62	2872910.00		1.51E+09		1.59E-03		8.78E-04	
	12 p ² P _{1/2} ^o		2887490.55	2887490.00		1.16E+09		1.19E-03		6.52E-04	
	13 p ² P _{1/2} ^o		2898828.40	2898830.00		9.13E+08		9.23E-04		4.99E-04	
	14 p ² P _{1/2} ^o		2907820.82	2907820.00		7.32E+08		7.28E-04		3.91E-04	
	15 p ² P _{1/2} ^o		2915072.79	2915070.00		5.95E+08		5.86E-04		3.13E-04	
	3 p ² P _{1/2} ^o		1726520.00	1727830.00	1726520.00	9.71E+07	9.67E+07	1.43E-01	1.42E-01	2.08E+00	2.08E+00
	4 p ² P _{1/2} ^o		2270150.00	2270150.00	2270150.00	2.69E+10	2.68E+10	2.34E-01	2.33E-01	2.62E-01	2.61E-01
	5 p ² P _{1/2} ^o		2520900.00	2520900.00	2520900.00	1.52E+10	1.52E+10	6.48E-02	6.48E-02	5.09E-02	5.09E-02
	6 p ² P _{1/2} ^o		2656759.04	2656500.00	2656500.00	9.12E+09	9.01E+09	2.87E-02	2.85E-02	1.94E-02	1.93E-02
	7 p ² P _{1/2} ^o		2738522.02	2738400.00	2738400.00	5.80E+09	5.76E+09	1.55E-02	1.55E-02	9.69E-03	9.67E-03
	8 p ² P _{1/2} ^o		2791513.32	2791200.00	2791200.00	3.95E+09	3.87E+09	9.60E-03	9.45E-03	5.70E-03	5.61E-03
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00	2.73E+09	2.73E+09	6.23E-03	6.24E-03	3.58E-03	3.59E-03
	10 p ² P _{1/2} ^o		2853739.09	2853760.00		2.00E+09		4.35E-03		2.45E-03	
	11 p ² P _{1/2} ^o		2872914.62	2872940.00		1.50E+09		3.17E-03		1.75E-03	
	12 p ² P _{1/2} ^o		2887490.55	2887510.00		1.16E+09		2.38E-03		1.30E-03	
	13 p ² P _{1/2} ^o		2898828.40	2898840.00		9.11E+08		1.84E-03		9.96E-04	
	14 p ² P _{1/2} ^o		2907820.82	2907830.00		7.30E+08		1.45E-03		7.81E-04	
	15 p ² P _{1/2} ^o		2915072.79	2915080.00		5.94E+08		1.17E-03		6.24E-04	
5	4 p ² P _{1/2} ^o	1s ² 4s ² S _{1/2}	2270150.00	2270150.00	2270150.00	1.91E+07	1.91E+07	9.30E-02	9.31E-02	3.49E+00	3.49E+00
	5 p ² P _{1/2} ^o		2520900.00	2520900.00	2520900.00	6.21E+09	6.15E+09	1.29E-01	1.28E-01	3.17E-01	3.14E-01
	6 p ² P _{1/2} ^o		2656759.04	2656500.00	2656500.00	4.01E+09	3.94E+09	3.68E-02	3.62E-02	5.99E-02	5.90E-02
	7 p ² P _{1/2} ^o		2738522.02	2738400.00	2738400.00	2.59E+09	2.55E+09	1.64E-02	1.62E-02	2.22E-02	2.20E-02
	8 p ² P _{1/2} ^o		2791513.32	2791200.00	2791200.00	1.78E+09	1.73E+09	9.18E-03	8.96E-03	1.12E-02	1.10E-02
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00	1.26E+09	1.23E+09	5.68E-03	5.56E-03	6.50E-03	6.37E-03
	10 p ² P _{1/2} ^o		2853739.09	2853740.00		9.02E+08		3.73E-03		4.09E-03	
	11 p ² P _{1/2} ^o		2872914.62	2872910.00		6.80E+08		2.64E-03		2.80E-03	
	12 p ² P _{1/2} ^o		2887490.55	2887490.00		5.24E+08		1.94E-03		2.02E-03	
	13 p ² P _{1/2} ^o		2898828.40	2898830.00		4.12E+08		1.48E-03		1.50E-03	
	14 p ² P _{1/2} ^o		2907820.82	2907820.00		3.31E+08		1.15E-03		1.16E-03	
	15 p ² P _{1/2} ^o		2915072.79	2915070.00		2.69E+08		9.17E-04		9.11E-04	

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength		
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)	A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}			
	1s ² n/ ^(2s+1) L _(l±s)	1s ² n'l ^(2s+1) L _(l±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]	
6	4 p ² P _{1/2} ^o	1s ² 4s ² S _{1/2}	2270150.00	2270150.00	2270150.00	2252600.00	1.91E+07	1.91E+07	1.86E-01	1.86E-01	6.98E+00	6.98E+00	
	5 p ² P _{1/2} ^o		2520900.00	2520900.00	2520900.00		6.21E+09	6.15E+09	2.58E-01	2.56E-01	6.34E-01	6.28E-01	
	6 p ² P _{1/2} ^o		2656759.04	2656500.00	2656500.00		4.01E+09	3.94E+09	7.35E-02	7.24E-02	1.20E-01	1.18E-01	
	7 p ² P _{1/2} ^o		2738522.02	2738400.00	2738400.00		2.59E+09	2.56E+09	3.28E-02	3.25E-02	4.45E-02	4.40E-02	
	8 p ² P _{1/2} ^o		2791513.32	2791200.00	2791200.00		1.78E+09	1.73E+09	1.84E-02	1.79E-02	2.24E-02	2.19E-02	
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00		1.23E+09	1.22E+09	1.11E-02	1.11E-02	1.27E-02	1.27E-02	
	10 p ² P _{1/2} ^o		2853739.09	2853760.00			9.00E+08		7.45E-03		8.15E-03		
	11 p ² P _{1/2} ^o		2872914.62	2872940.00			6.77E+08		5.26E-03		5.58E-03		
	12 p ² P _{1/2} ^o		2887490.55	2887510.00			5.22E+08		3.87E-03		4.01E-03		
	13 p ² P _{1/2} ^o		2898828.40	2898840.00			4.11E+08		2.95E-03		3.00E-03		
	14 p ² P _{1/2} ^o		2907820.82	2907830.00			3.30E+08		2.30E-03		2.31E-03		
	15 p ² P _{1/2} ^o		2915072.79	2915080.00			2.68E+08		1.83E-03		1.82E-03		
	7	5 p ² P _{1/2} ^o	1s ² 5s ² S _{1/2}	2520900.00	2520900.00	2520900.00	2511600.00	7.15E+06	7.15E+06	1.24E-01	1.24E-01	8.76E+00	8.78E+00
		6 p ² P _{1/2} ^o		2656759.04	2656500.00	2656500.00		1.99E+09	1.97E+09	1.42E-01	1.41E-01	6.46E-01	6.41E-01
		7 p ² P _{1/2} ^o		2738522.02	2738400.00	2738400.00		1.38E+09	1.38E+09	4.00E-02	4.01E-02	1.16E-01	1.16E-01
8 p ² P _{1/2} ^o			2791513.32	2791200.00	2791200.00		9.65E+08	9.39E+08	1.85E-02	1.80E-02	4.35E-02	4.24E-02	
9 p ² P _{1/2} ^o			2827803.68	2827600.00	2827600.00		6.83E+08	6.73E+08	1.02E-02	1.01E-02	2.13E-02	2.10E-02	
10 p ² P _{1/2} ^o			2853739.09	2853740.00			4.90E+08		6.26E-03		1.20E-02		
11 p ² P _{1/2} ^o			2872914.62	2872910.00			3.70E+08		4.23E-03		7.71E-03		
12 p ² P _{1/2} ^o			2887490.55	2887490.00			2.85E+08		3.02E-03		5.28E-03		
13 p ² P _{1/2} ^o			2898828.40	2898830.00			2.24E+08		2.24E-03		3.80E-03		
14 p ² P _{1/2} ^o			2907820.82	2907820.00			1.80E+08		1.71E-03		2.85E-03		
15 p ² P _{1/2} ^o			2915072.79	2915070.00			1.46E+08		1.35E-03		2.20E-03		
8		5 p ² P _{1/2} ^o	1s ² 5s ² S _{1/2}	2520900.00	2520900.00	2520900.00	2511600.00	7.15E+06	7.15E+06	2.47E-01	2.48E-01	1.75E+01	1.76E+01
		6 p ² P _{1/2} ^o		2656759.04	2656500.00	2656500.00		1.99E+09	1.98E+09	2.84E-01	2.83E-01	1.29E+00	1.29E+00
		7 p ² P _{1/2} ^o		2738522.02	2738400.00	2738400.00		1.38E+09	1.38E+09	8.00E-02	8.02E-02	2.32E-01	2.33E-01
		8 p ² P _{1/2} ^o		2791513.32	2791200.00	2791200.00		9.65E+08	9.41E+08	3.69E-02	3.61E-02	8.70E-02	8.50E-02
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00		6.66E+08	6.69E+08	1.99E-02	2.01E-02	4.15E-02	4.19E-02	
	10 p ² P _{1/2} ^o		2853739.09	2853760.00			4.88E+08		1.25E-02		2.40E-02		
	11 p ² P _{1/2} ^o		2872914.62	2872940.00			3.67E+08		8.42E-03		1.53E-02		
	12 p ² P _{1/2} ^o		2887490.55	2887510.00			2.84E+08		6.01E-03		1.05E-02		
	13 p ² P _{1/2} ^o		2898828.40	2898840.00			2.24E+08		4.46E-03		7.59E-03		
	14 p ² P _{1/2} ^o		2907820.82	2907830.00			1.79E+08		3.42E-03		5.67E-03		
	15 p ² P _{1/2} ^o		2915072.79	2915080.00			1.46E+08		2.68E-03		4.37E-03		

FIG. 1. Comparison of NIST values of transition probabilities with this work of HPT in sec^{-1} for $1s^2(2 \leq n \leq 15) np^2P^o_{(1/2 \text{ \& } 3/2)}$ to $1s^2(2 \leq n \leq 5) ns^2S_{1/2}$.TABLE 3. Computed and compared values of A_{ul} , S_{lu} and f_{lu} transitions from $1s^2(3 \leq n \leq 15) ns^2S_{1/2}$ & $1s^2(3 \leq n \leq 15) nd^2D_{3/2}$ to $1s^2(2 \leq n \leq 7) np^2P^o_{1/2}$.

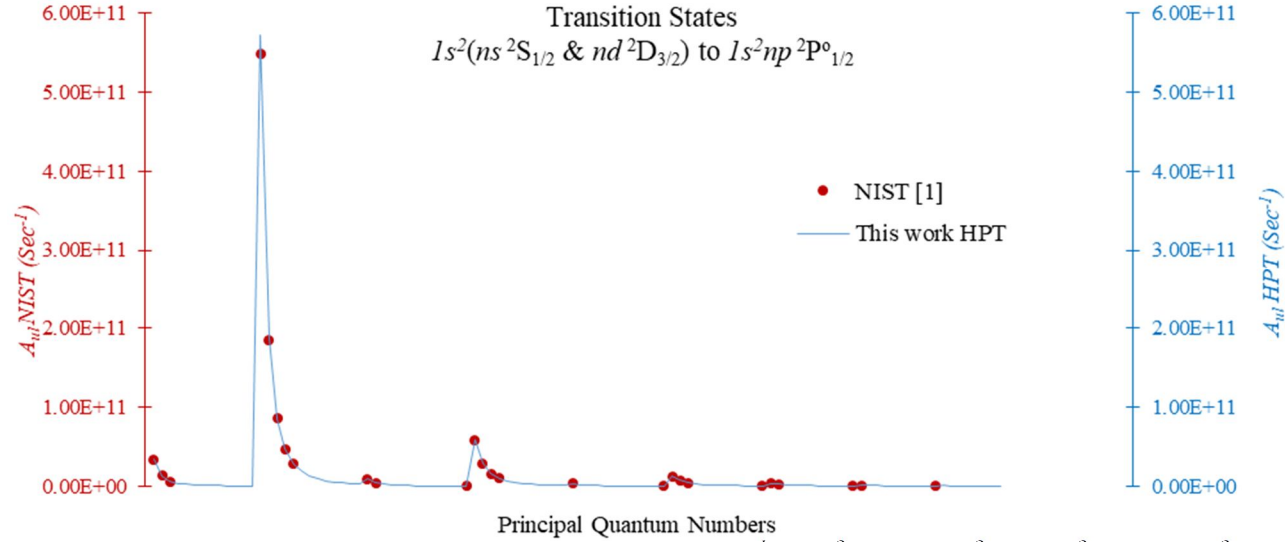
S.No	Transition Array		Rydberg Energies (cm ⁻¹)			Transition Probabilities		Line strength		Oscillator strength		
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)	A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}		
	1s ² n'l (2s+1)L _(l±s)	1s ² n'l (2s+1)L _(l±s)	This work QDT	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]	
9	3 s ² S _{1/2}	1s ² 2p ² P ^o _{1/2}	1682700.00	1682700.00	1682700.00	160015.00	3.46E+10	3.39E+10	2.23E-02	2.19E-02	9.63E-03	9.49E-03
	4 s ² S _{1/2}		2252600.00	2252600.00	2252600.00		1.35E+10	1.34E+10	4.60E-03	4.58E-03	1.45E-03	1.44E-03
	5 s ² S _{1/2}		2511600.00	2511600.00	2511600.00		6.75E+09	6.27E+09	1.82E-03	1.70E-03	5.09E-04	4.76E-04
	6 s ² S _{1/2}		2651171.97	2651171.97			3.87E+09		9.31E-04		2.46E-04	
	7 s ² S _{1/2}		2734901.82	2734901.82			2.43E+09		5.46E-04		1.39E-04	
	8 s ² S _{1/2}		2789037.53	2789037.53			1.62E+09		3.50E-04		8.75E-05	
	9 s ² S _{1/2}		2826038.66	2826038.66			1.14E+09		2.39E-04		5.88E-05	
	10 s ² S _{1/2}		2852438.14	2852438.14			8.27E+08		1.70E-04		4.15E-05	
	11 s ² S _{1/2}		2871929.09	2871929.09			6.21E+08		1.26E-04		3.05E-05	
	12 s ² S _{1/2}		2886726.62	2886726.62			4.78E+08		9.58E-05		2.31E-05	
	13 s ² S _{1/2}		2898224.59	2898224.59			3.75E+08		7.46E-05		1.79E-05	
	14 s ² S _{1/2}		2907335.49	2907335.49			3.00E+08		5.93E-05		1.42E-05	

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)		A_{ul} (Sec ⁻¹)		S_{lu} (a.u.)		f_{lu}	
	$1s^2n'l_{(2s+1)L_{(l\pm s)}}$	$1s^2n'l_{(2s+1)L_{(l\pm s)}}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
10	$15s^2S_{1/2}$	$1s^22p^2P^o_{1/2}$	2914676.97	2914676.97			2.44E+08		4.79E-05		1.14E-05	
	$3d^2D_{3/2}$		1743500.00	1743500.00	1743500.00	160015.00	5.72E+11	5.48E+11	6.81E-01	6.55E-01	2.83E-01	2.72E-01
	$4d^2D_{3/2}$		2277380.00	2277380.00	2277380.00		1.87E+11	1.85E+11	1.24E-01	1.24E-01	3.87E-02	3.84E-02
	$5d^2D_{3/2}$		2524400.00	2524400.00	2524400.00		8.59E+10	8.61E+10	4.59E-02	4.62E-02	1.27E-02	1.29E-02
	$6d^2D_{3/2}$		2658800.00	2658800.00	2658800.00		4.71E+10	4.62E+10	2.25E-02	2.22E-02	5.92E-03	5.85E-03
	$7d^2D_{3/2}$		2739842.15	2739600.00	2739600.00		2.85E+10	2.82E+10	1.28E-02	1.27E-02	3.25E-03	3.24E-03
	$8d^2D_{3/2}$		2792422.53	2792420.00			1.89E+10		8.14E-03		2.03E-03	
	$9d^2D_{3/2}$		2828457.65	2828460.00			1.31E+10		5.50E-03		1.35E-03	
	$10d^2D_{3/2}$		2854225.21	2854220.00			9.48E+09		3.89E-03		9.50E-04	
	$11d^2D_{3/2}$		2873285.63	2873280.00			7.07E+09		2.87E-03		6.94E-04	
	$12d^2D_{3/2}$		2887779.96	2887780.00			5.43E+09		2.18E-03		5.24E-04	
	$13d^2D_{3/2}$		2899058.38	2899060.00			4.26E+09		1.69E-03		4.06E-04	
	$14d^2D_{3/2}$		2908006.50	2908010.00			3.40E+09		1.34E-03		3.21E-04	
	$15d^2D_{3/2}$		2915224.81	2915220.00			2.75E+09		1.08E-03		2.58E-04	
11	$4s^2S_{1/2}$	$1s^23p^2P^o_{1/2}$	2252600.00	2252600.00	2252600.00	1726520.00	9.32E+09	9.47E+09	5.04E-02	5.13E-02	6.30E-02	6.42E-02
	$5s^2S_{1/2}$		2511600.00	2511600.00	2511600.00		4.50E+09	4.44E+09	1.09E-02	1.08E-02	9.15E-03	9.06E-03
	$6s^2S_{1/2}$		2651171.97	2651171.97			2.52E+09		4.41E-03		3.14E-03	
	$7s^2S_{1/2}$		2734901.82	2734901.82			1.56E+09		2.29E-03		1.49E-03	
	$8s^2S_{1/2}$		2789037.53	2789037.53			1.03E+09		1.36E-03		8.45E-04	
	$9s^2S_{1/2}$		2826038.66	2826038.66			7.17E+08		8.87E-04		5.31E-04	
	$10s^2S_{1/2}$		2852438.14	2852438.14			5.20E+08		6.13E-04		3.58E-04	
	$11s^2S_{1/2}$		2871929.09	2871929.09			3.89E+08		4.43E-04		2.54E-04	
	$12s^2S_{1/2}$		2886726.62	2886726.62			2.98E+08		3.31E-04		1.88E-04	
	$13s^2S_{1/2}$		2898224.59	2898224.59			2.34E+08		2.54E-04		1.43E-04	
	$14s^2S_{1/2}$		2907335.49	2907335.49			1.87E+08		2.00E-04		1.11E-04	
	$15s^2S_{1/2}$		2914676.97	2914676.97			1.51E+08		1.60E-04		8.88E-05	
12	$3d^2D_{3/2}$	$1s^23p^2P^o_{1/2}$	1743500.00	1743500.00	1743500.00	1726520.00	3.37E+06	3.38E+06	3.50E-02	3.51E-02	1.36E+00	1.36E+00
	$4d^2D_{3/2}$		2277380.00	2277380.00	2277380.00		5.91E+10	5.83E+10	5.83E-01	5.76E-01	6.96E-01	6.88E-01
	$5d^2D_{3/2}$		2524400.00	2524400.00	2524400.00		2.92E+10	2.89E+10	1.37E-01	1.36E-01	1.13E-01	1.12E-01
	$6d^2D_{3/2}$		2658800.00	2658800.00	2658800.00		1.63E+10	1.61E+10	5.62E-02	5.56E-02	3.97E-02	3.93E-02
	$7d^2D_{3/2}$		2739842.15	2739600.00	2739600.00		1.00E+10	9.93E+09	2.92E-02	2.90E-02	1.89E-02	1.88E-02
	$8d^2D_{3/2}$		2792422.53	2792420.00			6.63E+09		1.75E-02		1.08E-02	
	$9d^2D_{3/2}$		2828457.65	2828460.00			4.61E+09		1.14E-02		6.78E-03	
	$10d^2D_{3/2}$		2854225.21	2854220.00			3.34E+09		7.84E-03		4.58E-03	

Treatment of Perturbation in Energy Levels using Hypothetical Perturbers: A Case Study of Mg X

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)		A_{ul} (Sec ⁻¹)		S_{lu} (a.u.)		f_{lu}	
	$1s^2n'l_{(2s+1)L_{(l\pm s)}}$	$1s^2n'l_{(2s+1)L_{(l\pm s)}}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
	11 $d^2D_{3/2}$		2873285.63	2873280.00			2.49E+09		5.67E-03		3.25E-03	
	12 $d^2D_{3/2}$		2887779.96	2887780.00			1.91E+09		4.24E-03		2.40E-03	
	13 $d^2D_{3/2}$		2899058.38	2899060.00			1.50E+09		3.26E-03		1.83E-03	
	14 $d^2D_{3/2}$		2908006.50	2908010.00			1.20E+09		2.57E-03		1.43E-03	
	15 $d^2D_{3/2}$		2915224.81	2915220.00			9.71E+08		2.05E-03		1.14E-03	
13	5 $s^2S_{1/2}$	$1s^24p^2P^o_{1/2}$	2511600.00	2511600.00	2511600.00	2270150.00	3.19E+09	3.15E+09	8.19E-02	8.09E-02	2.23E-01	2.21E-01
	6 $s^2S_{1/2}$		2651171.97	2651171.97			1.73E+09		1.78E-02		3.08E-02	
	7 $s^2S_{1/2}$		2734901.82	2734901.82			1.05E+09		7.24E-03		1.03E-02	
	8 $s^2S_{1/2}$		2789037.53	2789037.53			6.83E+08		3.79E-03		4.81E-03	
	9 $s^2S_{1/2}$		2826038.66	2826038.66			4.71E+08		2.28E-03		2.70E-03	
	10 $s^2S_{1/2}$		2852438.14	2852438.14			3.39E+08		1.49E-03		1.69E-03	
	11 $s^2S_{1/2}$		2871929.09	2871929.09			2.52E+08		1.04E-03		1.14E-03	
	12 $s^2S_{1/2}$		2886726.62	2886726.62			1.93E+08		7.58E-04		8.10E-04	
	13 $s^2S_{1/2}$		2898224.59	2898224.59			1.51E+08		5.72E-04		5.99E-04	
	14 $s^2S_{1/2}$		2907335.49	2907335.49			1.20E+08		4.43E-04		4.57E-04	
	15 $s^2S_{1/2}$		2914676.97	2914676.97			9.73E+07		3.50E-04		3.58E-04	
14	4 $d^2D_{3/2}$	$1s^24p^2P^o_{1/2}$	2277380.00	2277380.00	2277380.00	2270150.00	1.11E+06	1.11E+06	6.35E-02	6.35E-02	5.78E+00	5.78E+00
	5 $d^2D_{3/2}$		2524400.00	2524400.00	2524400.00		1.22E+10	1.21E+10	5.63E-01	5.60E-01	1.46E+00	1.45E+00
	6 $d^2D_{3/2}$		2658800.00	2658800.00	2658800.00		7.23E+09	7.15E+09	1.43E-01	1.42E-01	2.43E-01	2.41E-01
	7 $d^2D_{3/2}$		2739842.15	2739600.00	2739600.00		4.53E+09	4.48E+09	6.15E-02	6.09E-02	8.62E-02	8.54E-02
	8 $d^2D_{3/2}$		2792422.53	2792420.00			3.00E+09		3.30E-02		4.15E-02	
	9 $d^2D_{3/2}$		2828457.65	2828460.00			2.10E+09		2.01E-02		2.37E-02	
	10 $d^2D_{3/2}$		2854225.21	2854220.00			1.52E+09		1.33E-02		1.50E-02	
	11 $d^2D_{3/2}$		2873285.63	2873280.00			1.14E+09		9.34E-03		1.02E-02	
	12 $d^2D_{3/2}$		2887779.96	2887780.00			8.72E+08		6.84E-03		7.29E-03	
	13 $d^2D_{3/2}$		2899058.38	2899060.00			6.84E+08		5.18E-03		5.42E-03	
	14 $d^2D_{3/2}$		2908006.50	2908010.00			5.47E+08		4.02E-03		4.15E-03	
	15 $d^2D_{3/2}$		2915224.81	2915220.00			4.44E+08		3.19E-03		3.25E-03	
15	5 $d^2D_{3/2}$	$1s^25p^2P^o_{1/2}$	2524400.00	2524400.00	2524400.00	2520900	3.43E+05	3.44E+05	8.38E-02	8.41E-02	1.58E+01	1.58E+01
	6 $d^2D_{3/2}$		2658800.00	2658800.00	2658800.00		3.61E+09	3.60E+09	5.68E-01	5.67E-01	2.71E+00	2.71E+00
	7 $d^2D_{3/2}$		2739842.15	2739600.00	2739600.00		2.40E+09	2.38E+09	1.50E-01	1.49E-01	4.53E-01	4.49E-01
	8 $d^2D_{3/2}$		2792422.53	2792420.00			1.61E+09		6.53E-02		1.58E-01	
	9 $d^2D_{3/2}$		2828457.65	2828460.00			1.13E+09		3.56E-02		7.63E-02	
	10 $d^2D_{3/2}$		2854225.21	2854220.00			8.19E+08		2.21E-02		4.36E-02	

S.No	Transition Array		Rydberg Energies (cm ⁻¹)			Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)	A_{ul} (Sec ⁻¹)		S_{lu} (a.u.)		f_{lu}	
	$1s^2n'l_{(2s+1)L_{(l\pm s)}}$	$1s^2n'l_{(2s+1)L_{(l\pm s)}}$	This work QDT	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
	11 $d^2D_{3/2}$		2873285.63	2873280.00		6.13E+08		1.48E-02		2.76E-02	
	12 $d^2D_{3/2}$		2887779.96	2887780.00		4.71E+08		1.05E-02		1.88E-02	
	13 $d^2D_{3/2}$		2899058.38	2899060.00		3.70E+08		7.74E-03		1.35E-02	
	14 $d^2D_{3/2}$		2908006.50	2908010.00		2.96E+08		5.90E-03		1.00E-02	
	15 $d^2D_{3/2}$		2915224.81	2915220.00		2.40E+08		4.62E-03		7.71E-03	
16	6 $d^2D_{3/2}$	$1s^26p^2P^o_{1/2}$	2658800.00	2658800.00	2658800.00	2.14E+05	2.13E+05	1.21E-01	1.21E-01	3.46E+01	3.46E+01
	7 $d^2D_{3/2}$		2739842.15	2739600.00	2739600.00	1.36E+09	1.35E+09	5.88E-01	5.87E-01	4.66E+00	4.65E+00
	8 $d^2D_{3/2}$		2792422.53	2792420.00		9.53E+08		1.54E-01		7.48E-01	
	9 $d^2D_{3/2}$		2828457.65	2828460.00		6.76E+08		6.85E-02		2.62E-01	
	10 $d^2D_{3/2}$		2854225.21	2854220.00		4.94E+08		3.78E-02		1.26E-01	
	11 $d^2D_{3/2}$		2873285.63	2873280.00		3.71E+08		2.36E-02		7.17E-02	
	12 $d^2D_{3/2}$		2887779.96	2887780.00		2.85E+08		1.59E-02		4.54E-02	
	13 $d^2D_{3/2}$		2899058.38	2899060.00		2.24E+08		1.14E-02		3.09E-02	
	14 $d^2D_{3/2}$		2908006.50	2908010.00		1.79E+08		8.46E-03		2.21E-02	
	15 $d^2D_{3/2}$		2915224.81	2915220.00		1.45E+08		6.50E-03		1.65E-02	
17	7 $d^2D_{3/2}$	$1s^27p^2P^o_{1/2}$	2739842.15	2739600.00	2739600.00	5.79E+04	5.81E+04	1.20E-01	1.21E-01	6.61E+01	6.64E+01
	8 $d^2D_{3/2}$		2792422.53	2792420.00		5.86E+08		6.01E-01		7.33E+00	
	9 $d^2D_{3/2}$		2828457.65	2828460.00		4.38E+08		1.62E-01		1.18E+00	
	10 $d^2D_{3/2}$		2854225.21	2854220.00		3.23E+08		7.21E-02		4.10E-01	
	11 $d^2D_{3/2}$		2873285.63	2873280.00		2.43E+08		4.00E-02		1.95E-01	
	12 $d^2D_{3/2}$		2887779.96	2887780.00		1.87E+08		2.51E-02		1.11E-01	
	13 $d^2D_{3/2}$		2899058.38	2899060.00		1.47E+08		1.70E-02		6.98E-02	
	14 $d^2D_{3/2}$		2908006.50	2908010.00		1.17E+08		1.22E-02		4.74E-02	
	15 $d^2D_{3/2}$		2915224.81	2915220.00		9.55E+07		9.14E-03		3.40E-02	

FIG. 2. Comparison of NIST values of transition probabilities with this work of HPT in sec^{-1} for $1s^2(3 \leq n \leq 15)ns \ ^2S_{1/2}$ & $1s^2(3 \leq n \leq 15)nd \ ^2D_{3/2}$ to $1s^2(2 \leq n \leq 7)np \ ^2P^{\circ}_{1/2}$.TABLE 4. Computed and compared values of A_{ul} , S_{lu} and f_{lu} transitions from $1s^2(3 \leq n \leq 15)ns \ ^2S_{1/2}$ & $1s^2(3 \leq n \leq 15)nd \ ^2D_{(3/2 \ \& \ 5/2)}$ to $1s^2(2 \leq n \leq 7)np \ ^2P^{\circ}_{3/2}$.

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E _u)			Lower State (E _l)	A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}	
	1s ² n'l ^(2s+1) L _(l±s)	1s ² n'l ^(2s+1) L _(l±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
18	3 s ² S _{1/2}	1s ² 2p ² P ^o _{3/2}	1682700.00	1682700.00	1682700.00	163990.00	6.96E+10	6.81E+10	2.25E-02	2.21E-02	1.95E-02	1.92E-02
	4 s ² S _{1/2}		2252600.00	2252600.00	2252600.00		2.72E+10	2.67E+10	4.65E-03	4.59E-03	2.92E-03	2.89E-03
	5 s ² S _{1/2}		2511600.00	2511600.00	2511600.00		1.36E+10	1.32E+10	1.84E-03	1.80E-03	1.03E-03	1.01E-03
	6 s ² S _{1/2}		2651171.97	2651171.97			7.78E+09		9.38E-04		4.96E-04	
	7 s ² S _{1/2}		2734901.82	2734901.82			4.88E+09		5.50E-04		2.81E-04	
	8 s ² S _{1/2}		2789037.53	2789037.53			3.26E+09		3.53E-04		1.77E-04	
	9 s ² S _{1/2}		2826038.66	2826038.66			2.28E+09		2.40E-04		1.19E-04	
	10 s ² S _{1/2}		2852438.14	2852438.14			1.66E+09		1.71E-04		8.38E-05	
	11 s ² S _{1/2}		2871929.09	2871929.09			1.25E+09		1.27E-04		6.15E-05	
	12 s ² S _{1/2}		2886726.62	2886726.62			9.59E+08		9.65E-05		4.66E-05	
	13 s ² S _{1/2}		2898224.59	2898224.59			7.53E+08		7.51E-05		3.61E-05	
	14 s ² S _{1/2}		2907335.49	2907335.49			6.03E+08		5.97E-05		2.86E-05	
	15 s ² S _{1/2}		2914676.97	2914676.97			4.90E+08		4.82E-05		2.31E-05	

S.No	Transition Array		Rydberg Energies (cm ⁻¹)			Transition Probabilities			Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)	A_{ul} (Sec ⁻¹)		S_{lu} (a.u)	f_{lu}			
	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
19	$3 d^2D_{3/2}$	$1s^22p^2P^o_{3/2}$	1743500.00	1743500.00	1743500.00	163990.00	1.14E+11	1.09E+11	6.82E-02	6.56E-02	5.68E-02	5.47E-02
	$4 d^2D_{3/2}$		2277380.00	2277380.00	2277380.00		3.72E+10	3.66E+10	1.24E-02	1.23E-02	7.73E-03	7.66E-03
	$5 d^2D_{3/2}$		2524400.00	2524400.00	2524400.00		1.71E+10	1.71E+10	4.57E-03	4.60E-03	2.55E-03	2.57E-03
	$6 d^2D_{3/2}$		2658800.00	2658800.00	2658800.00		9.36E+09	9.22E+09	2.24E-03	2.22E-03	1.18E-03	1.17E-03
	$7 d^2D_{3/2}$		2739842.15	2739600.00	2739600.00		5.65E+09	5.62E+09	1.27E-03	1.27E-03	6.49E-04	6.49E-04
	$8 d^2D_{3/2}$		2792422.53	2792420.00			3.76E+09		8.11E-04		4.06E-04	
	$9 d^2D_{3/2}$		2828457.65	2828460.00			2.61E+09		5.48E-04		2.70E-04	
	$10 d^2D_{3/2}$		2854225.21	2854220.00			1.88E+09		3.88E-04		1.90E-04	
	$11 d^2D_{3/2}$		2873285.63	2873280.00			1.41E+09		2.86E-04		1.39E-04	
	$12 d^2D_{3/2}$		2887779.96	2887780.00			1.08E+09		2.17E-04		1.05E-04	
	$13 d^2D_{3/2}$		2899058.38	2899060.00			8.46E+08		1.69E-04		8.11E-05	
	$14 d^2D_{3/2}$		2908006.50	2908010.00			6.76E+08		1.34E-04		6.41E-05	
	$15 d^2D_{3/2}$		2915224.81	2915220.00			5.46E+08		1.08E-04		5.14E-05	
20	$3 d^2D_{5/2}$	$1s^22p^2P^o_{3/2}$	1743890.00	1743890.00	1743890.00	163990.00	6.84E+11	6.55E+11	6.14E-01	5.90E-01	5.11E-01	4.92E-01
	$4 d^2D_{5/2}$		2277700.00	2277700.00	2277700.00		2.24E+11	2.21E+11	1.12E-01	1.11E-01	6.97E-02	6.94E-02
	$5 d^2D_{5/2}$		2524600.00	2524600.00	2524600.00		1.03E+11	1.03E+11	4.13E-02	4.16E-02	2.30E-02	2.32E-02
	$6 d^2D_{5/2}$		2658800.00	2658800.00	2658800.00		5.62E+10	5.54E+10	2.02E-02	2.00E-02	1.06E-02	1.06E-02
	$7 d^2D_{5/2}$		2739757.27	2739600.00	2739600.00		3.39E+10	3.36E+10	1.14E-02	1.14E-02	5.84E-03	5.83E-03
	$8 d^2D_{5/2}$		2792315.61	2792320.00			2.24E+10		7.25E-03		3.63E-03	
	$9 d^2D_{5/2}$		2828353.82	2828350.00			1.55E+10		4.88E-03		2.41E-03	
	$10 d^2D_{5/2}$		2854132.74	2854130.00			1.12E+10		3.45E-03		1.68E-03	
	$11 d^2D_{5/2}$		2873206.08	2873210.00			8.32E+09		2.54E-03		1.23E-03	
	$12 d^2D_{5/2}$		2887712.45	2887710.00			6.36E+09		1.92E-03		9.26E-04	
	$13 d^2D_{5/2}$		2899001.30	2899000.00			4.98E+09		1.49E-03		7.16E-04	
	$14 d^2D_{5/2}$		2907958.19	2907960.00			3.98E+09		1.18E-03		5.66E-04	
	$15 d^2D_{5/2}$		2915183.77	2915180.00			3.21E+09		9.50E-04		4.54E-04	
21	$4 s^2S_{1/2}$	$1s^23p^2P^o_{3/2}$	2252600.00	2252600.00	2252600.00	1727830.00	1.88E+10	1.88E+10	5.10E-02	5.12E-02	1.28E-01	1.29E-01
	$5 s^2S_{1/2}$		2511600.00	2511600.00	2511600.00		9.05E+09	8.77E+09	1.10E-02	1.07E-02	1.85E-02	1.80E-02
	$6 s^2S_{1/2}$		2651171.97	2651171.97			5.07E+09		4.44E-03		6.33E-03	
	$7 s^2S_{1/2}$		2734901.82	2734901.82			3.13E+09		2.31E-03		3.01E-03	
	$8 s^2S_{1/2}$		2789037.53	2789037.53			2.07E+09		1.37E-03		1.70E-03	
	$9 s^2S_{1/2}$		2826038.66	2826038.66			1.44E+09		8.93E-04		1.07E-03	
	$10 s^2S_{1/2}$		2852438.14	2852438.14			1.04E+09		6.17E-04		7.22E-04	
	$11 s^2S_{1/2}$		2871929.09	2871929.09			7.81E+08		4.46E-04		5.13E-04	

Treatment of Perturbation in Energy Levels using Hypothetical Perturbers: A Case Study of Mg X

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength			
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)		A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}			
	1s ² n'l ^(2s+1) L _(l±s)	1s ² n'l ^(2s+1) L _(l±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]		
22	12 s ² S _{1/2}	1s ² 3p ² P _{3/2}	2886726.62	2886726.62			5.99E+08		3.33E-04		3.78E-04			
	13 s ² S _{1/2}		2898224.59	2898224.59			4.70E+08		2.56E-04		2.88E-04			
	14 s ² S _{1/2}		2907335.49	2907335.49			3.75E+08		2.02E-04		2.25E-04			
	15 s ² S _{1/2}		2914676.97	2914676.97			3.04E+08		1.61E-04		1.79E-04			
	3 d ² D _{3/2}		1743500.00	1743500.00	1743500.00	1727830.00	5.30E+05	5.31E+05	3.23E-03	3.24E-03	2.71E-01	2.72E-01		
	4 d ² D _{3/2}		2277380.00	2277380.00	2277380.00		1.18E+10	1.16E+10	5.85E-02	5.75E-02	1.40E-01	1.38E-01		
	5 d ² D _{3/2}		2524400.00	2524400.00	2524400.00		5.83E+09	5.76E+09	1.37E-02	1.36E-02	2.27E-02	2.25E-02		
	6 d ² D _{3/2}		2658800.00	2658800.00	2658800.00		3.26E+09	3.21E+09	5.62E-03	5.55E-03	7.94E-03	7.85E-03		
	7 d ² D _{3/2}		2739842.15	2739600.00	2739600.00		2.00E+09	1.98E+09	2.91E-03	2.90E-03	3.79E-03	3.77E-03		
	8 d ² D _{3/2}		2792422.53	2792420.00			1.32E+09		1.74E-03		2.16E-03			
	9 d ² D _{3/2}		2828457.65	2828460.00			9.19E+08		1.13E-03		1.36E-03			
	10 d ² D _{3/2}		2854225.21	2854220.00			6.65E+08		7.83E-04		9.15E-04			
	11 d ² D _{3/2}		2873285.63	2873280.00			4.97E+08		5.66E-04		6.50E-04			
	12 d ² D _{3/2}		2887779.96	2887780.00			3.81E+08		4.23E-04		4.80E-04			
	13 d ² D _{3/2}		2899058.38	2899060.00			2.99E+08		3.26E-04		3.66E-04			
14 d ² D _{3/2}	2908006.50	2908010.00			2.39E+08		2.56E-04		2.86E-04					
15 d ² D _{3/2}	2915224.81	2915220.00			1.94E+08		2.05E-04		2.27E-04					
23	3 d ² D _{5/2}	1s ² 3p ² P _{3/2}	1743890.00	1743890.00	1743890.00	1727830.00	3.42E+06	3.43E+06	2.98E-02	2.99E-02	2.44E+00	2.45E+00		
	4 d ² D _{5/2}		2277700.00	2277700.00	2277700.00		7.08E+10	6.96E+10	5.26E-01	5.18E-01	1.26E+00	1.24E+00		
	5 d ² D _{5/2}		2524600.00	2524600.00	2524600.00		3.50E+10	3.44E+10	1.24E-01	1.22E-01	2.04E-01	2.02E-01		
	6 d ² D _{5/2}		2658800.00	2658800.00	2658800.00		1.95E+10	1.93E+10	5.06E-02	5.00E-02	7.15E-02	7.07E-02		
	7 d ² D _{5/2}		2739757.27	2739600.00	2739600.00		1.20E+10	1.19E+10	2.62E-02	2.61E-02	3.41E-02	3.40E-02		
	8 d ² D _{5/2}		2792315.61	2792320.00			7.91E+09		1.57E-02		1.93E-02			
	9 d ² D _{5/2}		2828353.82	2828350.00			5.49E+09		1.02E-02		1.21E-02			
	10 d ² D _{5/2}		2854132.74	2854130.00			3.97E+09		7.01E-03		8.19E-03			
	11 d ² D _{5/2}		2873206.08	2873210.00			2.96E+09		5.06E-03		5.81E-03			
	12 d ² D _{5/2}		2887712.45	2887710.00			2.27E+09		3.78E-03		4.29E-03			
	13 d ² D _{5/2}		2899001.30	2899000.00			1.78E+09		2.90E-03		3.26E-03			
	14 d ² D _{5/2}		2907958.19	2907960.00			1.42E+09		2.28E-03		2.55E-03			
	15 d ² D _{5/2}		2915183.77	2915180.00			1.15E+09		1.83E-03		2.02E-03			
	24		5 s ² S _{1/2}	1s ² 4p ² P _{3/2}	2511600.00	2511600.00	2511600.00	2270150.00	6.38E+09	6.29E+09	8.19E-02	8.09E-02	4.47E-01	4.41E-01
			6 s ² S _{1/2}		2651171.97	2651171.97			3.46E+09		1.78E-02		6.16E-02	
7 s ² S _{1/2}		2734901.82	2734901.82				2.09E+09		7.24E-03		2.05E-02			
8 s ² S _{1/2}		2789037.53	2789037.53				1.37E+09		3.79E-03		9.62E-03			

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)		A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}	
	1s ² n'l ^(2s+1) L _(l±s)	1s ² n'l ^(2s+1) L _(l±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
	9 s ² S _{1/2}		2826038.66	2826038.66			9.42E+08		2.28E-03		5.40E-03	
	10 s ² S _{1/2}		2852438.14	2852438.14			6.78E+08		1.49E-03		3.38E-03	
	11 s ² S _{1/2}		2871929.09	2871929.09			5.04E+08		1.04E-03		2.28E-03	
	12 s ² S _{1/2}		2886726.62	2886726.62			3.86E+08		7.58E-04		1.62E-03	
	13 s ² S _{1/2}		2898224.59	2898224.59			3.02E+08		5.72E-04		1.20E-03	
	14 s ² S _{1/2}		2907335.49	2907335.49			2.40E+08		4.43E-04		9.14E-04	
	15 s ² S _{1/2}		2914676.97	2914676.97			1.95E+08		3.50E-04		7.15E-04	
25	4 d ² D _{3/2}	1s ² 4p ² P _{3/2} ^o	2277380.00	2277380.00	2277380.00	2270150.00	2.22E+05	2.21E+05	6.35E-03	6.35E-03	1.16E+00	1.16E+00
	5 d ² D _{3/2}		2524400.00	2524400.00	2524400.00		2.43E+09	2.41E+09	5.63E-02	5.60E-02	2.91E-01	2.90E-01
	6 d ² D _{3/2}		2658800.00	2658800.00	2658800.00		1.45E+09	1.43E+09	1.43E-02	1.42E-02	4.85E-02	4.81E-02
	7 d ² D _{3/2}		2739842.15	2739600.00	2739600.00		9.06E+08	8.95E+08	6.15E-03	6.09E-03	1.72E-02	1.71E-02
	8 d ² D _{3/2}		2792422.53	2792420.00			6.01E+08		3.30E-03		8.31E-03	
	9 d ² D _{3/2}		2828457.65	2828460.00			4.19E+08		2.01E-03		4.74E-03	
	10 d ² D _{3/2}		2854225.21	2854220.00			3.04E+08		1.33E-03		3.00E-03	
	11 d ² D _{3/2}		2873285.63	2873280.00			2.27E+08		9.34E-04		2.04E-03	
	12 d ² D _{3/2}		2887779.96	2887780.00			1.74E+08		6.84E-04		1.46E-03	
	13 d ² D _{3/2}		2899058.38	2899060.00			1.37E+08		5.18E-04		1.08E-03	
	14 d ² D _{3/2}		2908006.50	2908010.00			1.09E+08		4.02E-04		8.30E-04	
	15 d ² D _{3/2}		2915224.81	2915220.00			8.87E+07		3.19E-04		6.51E-04	
26	4 d ² D _{5/2}	1s ² 4p ² P _{3/2} ^o	2277700.00	2277700.00	2277700.00	2270150.00	1.52E+06	1.51E+06	5.97E-02	5.96E-02	1.04E+01	1.04E+01
	5 d ² D _{5/2}		2524600.00	2524600.00	2524600.00		1.45E+10	1.45E+10	5.04E-01	5.04E-01	2.61E+00	2.61E+00
	6 d ² D _{5/2}		2658800.00	2658800.00	2658800.00		8.68E+09	8.60E+09	1.29E-01	1.28E-01	4.37E-01	4.34E-01
	7 d ² D _{5/2}		2739757.27	2739600.00	2739600.00		5.44E+09	5.37E+09	5.53E-02	5.48E-02	1.55E-01	1.54E-01
	8 d ² D _{5/2}		2792315.61	2792320.00			3.61E+09		2.97E-02		7.48E-02	
	9 d ² D _{5/2}		2828353.82	2828350.00			2.51E+09		1.81E-02		4.27E-02	
	10 d ² D _{5/2}		2854132.74	2854130.00			1.82E+09		1.20E-02		2.70E-02	
	11 d ² D _{5/2}		2873206.08	2873210.00			1.36E+09		8.39E-03		1.83E-02	
	12 d ² D _{5/2}		2887712.45	2887710.00			1.04E+09		6.14E-03		1.31E-02	
	13 d ² D _{5/2}		2899001.30	2899000.00			8.18E+08		4.64E-03		9.71E-03	
	14 d ² D _{5/2}		2907958.19	2907960.00			6.54E+08		3.60E-03		7.44E-03	
	15 d ² D _{5/2}		2915183.77	2915180.00			5.30E+08		2.86E-03		5.83E-03	
27	5 d ² D _{3/2}	1s ² 5p ² P _{3/2} ^o	2524400.00	2524400.00	2524400.00	2520900.00	6.86E+04	6.87E+04	8.38E-03	8.41E-03	3.15E+00	3.16E+00
	6 d ² D _{3/2}		2658800.00	2658800.00	2658800.00		7.22E+08	7.19E+08	5.68E-02	5.67E-02	5.42E-01	5.41E-01
	7 d ² D _{3/2}		2739842.15	2739600.00	2739600.00		4.81E+08	4.75E+08	1.50E-02	1.49E-02	9.05E-02	8.97E-02

Treatment of Perturbation in Energy Levels using Hypothetical Perturbers: A Case Study of Mg X

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)		A_{ul} (Sec ⁻¹)		S_{lu} (a.u)		f_{lu}	
	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
28	$8 d^2D_{3/2}$	$1s^25p^2P^o_{3/2}$	2792422.53	2792420.00			3.22E+08		6.53E-03		3.17E-02	
	$9 d^2D_{3/2}$		2828457.65	2828460.00			2.25E+08		3.56E-03		1.53E-02	
	$10 d^2D_{3/2}$		2854225.21	2854220.00			1.64E+08		2.21E-03		8.71E-03	
	$11 d^2D_{3/2}$		2873285.63	2873280.00			1.23E+08		1.48E-03		5.52E-03	
	$12 d^2D_{3/2}$		2887779.96	2887780.00			9.42E+07		1.05E-03		3.76E-03	
	$13 d^2D_{3/2}$		2899058.38	2899060.00			7.40E+07		7.74E-04		2.69E-03	
	$14 d^2D_{3/2}$		2908006.50	2908010.00			5.91E+07		5.90E-04		2.01E-03	
	$15 d^2D_{3/2}$		2915224.81	2915220.00			4.80E+07		4.62E-04		1.54E-03	
	$5 d^2D_{3/2}$		2524600.00	2524600.00	2524600.00	2520900.00	4.87E+05	4.87E+05	7.98E-02	8.00E-02	2.84E+01	2.85E+01
	$6 d^2D_{3/2}$		2658800.00	2658800.00	2658800.00		4.33E+09	4.31E+09	5.11E-01	5.10E-01	4.88E+00	4.87E+00
	$7 d^2D_{3/2}$		2739757.27	2739600.00	2739600.00		2.88E+09	2.85E+09	1.35E-01	1.34E-01	8.15E-01	8.07E-01
	$8 d^2D_{3/2}$		2792315.61	2792320.00			1.93E+09		5.89E-02		2.86E-01	
	$9 d^2D_{3/2}$		2828353.82	2828350.00			1.36E+09		3.22E-02		1.38E-01	
	$10 d^2D_{5/2}$		2854132.74	2854130.00			9.85E+08		1.99E-02		7.86E-02	
	$11 d^2D_{5/2}$		2873206.08	2873210.00			7.37E+08		1.33E-02		4.98E-02	
	$12 d^2D_{5/2}$		2887712.45	2887710.00			5.66E+08		9.44E-03		3.39E-02	
	$13 d^2D_{5/2}$		2899001.30	2899000.00			4.44E+08		6.97E-03		2.43E-02	
	$14 d^2D_{5/2}$		2907958.19	2907960.00			3.55E+08		5.31E-03		1.81E-02	
	$15 d^2D_{5/2}$		2915183.77	2915180.00			2.88E+08		4.15E-03		1.39E-02	
29	$6 d^2D_{3/2}$	$1s^26p^2P^o_{3/2}$	2658800.00	2658800.00	2658800.00	2656500.00	4.27E+04	4.27E+04	1.21E-02	1.21E-02	6.92E+00	6.93E+00
	$7 d^2D_{3/2}$		2739842.15	2739600.00	2739600.00		2.71E+08	2.70E+08	5.88E-02	5.87E-02	9.31E-01	9.30E-01
	$8 d^2D_{3/2}$		2792422.53	2792420.00			1.91E+08		1.54E-02		1.50E-01	
	$9 d^2D_{3/2}$		2828457.65	2828460.00			1.35E+08		6.85E-03		5.24E-02	
	$10 d^2D_{3/2}$		2854225.21	2854220.00			9.88E+07		3.78E-03		2.52E-02	
	$11 d^2D_{3/2}$		2873285.63	2873280.00			7.42E+07		2.36E-03		1.43E-02	
	$12 d^2D_{3/2}$		2887779.96	2887780.00			5.70E+07		1.59E-03		9.08E-03	
	$13 d^2D_{3/2}$		2899058.38	2899060.00			4.48E+07		1.14E-03		6.18E-03	
	$14 d^2D_{3/2}$		2908006.50	2908010.00			3.58E+07		8.46E-04		4.43E-03	
	$15 d^2D_{3/2}$		2915224.81	2915220.00			2.91E+07		6.50E-04		3.31E-03	
30	$6 d^2D_{3/2}$	$1s^26p^2P^o_{3/2}$	2658800.00	2658800.00	2658800.00	2656500.00	2.56E+05	2.56E+05	1.09E-01	1.09E-01	6.23E+01	6.24E+01
	$7 d^2D_{3/2}$		2739757.27	2739600.00	2739600.00		1.63E+09	1.62E+09	5.29E-01	5.28E-01	8.38E+00	8.37E+00
	$8 d^2D_{3/2}$		2792315.61	2792320.00			1.15E+09		1.40E-01		1.36E+00	
	$9 d^2D_{3/2}$		2828353.82	2828350.00			8.16E+08		6.20E-02		4.75E-01	
	$10 d^2D_{5/2}$		2854132.74	2854130.00			5.96E+08		3.42E-02		2.28E-01	

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)		A_{ul} (Sec ⁻¹)		S_{lu} (a.u)		f_{lu}	
	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
	11 $d^2D_{5/2}$		2873206.08	2873210.00			4.47E+08		2.14E-02		1.30E-01	
	12 $d^2D_{5/2}$		2887712.45	2887710.00			3.44E+08		1.44E-02		8.21E-02	
	13 $d^2D_{5/2}$		2899001.30	2899000.00			2.70E+08		1.03E-02		5.59E-02	
	14 $d^2D_{5/2}$		2907958.19	2907960.00			2.15E+08		7.65E-03		4.00E-02	
	15 $d^2D_{5/2}$		2915183.77	2915180.00			1.75E+08		5.87E-03		2.99E-02	
31	7 $d^2D_{3/2}$	$1s^27p^2P^o_{3/2}$	2739842.15	2739600.00	2739600.00	2738400.00	1.16E+04	1.16E+04	1.20E-02	1.21E-02	1.32E+01	1.33E+01
	8 $d^2D_{3/2}$		2792422.53	2792420.00			1.17E+08		6.01E-02		1.47E+00	
	9 $d^2D_{3/2}$		2828457.65	2828460.00			8.75E+07		1.62E-02		2.36E-01	
	10 $d^2D_{3/2}$		2854225.21	2854220.00			6.46E+07		7.21E-03		8.19E-02	
	11 $d^2D_{3/2}$		2873285.63	2873280.00			4.86E+07		4.00E-03		3.91E-02	
	12 $d^2D_{3/2}$		2887779.96	2887780.00			3.74E+07		2.51E-03		2.21E-02	
	13 $d^2D_{3/2}$		2899058.38	2899060.00			2.94E+07		1.70E-03		1.40E-02	
	14 $d^2D_{3/2}$		2908006.50	2908010.00			2.35E+07		1.22E-03		9.48E-03	
	15 $d^2D_{3/2}$		2915224.81	2915220.00			1.91E+07		9.14E-04		6.80E-03	
32	7 $d^2D_{5/2}$	$1s^27p^2P^o_{3/2}$	2739757.27	2739600.00	2739600.00	2738400.00	6.95E+04	6.98E+04	1.08E-01	1.09E-01	1.19E+02	1.20E+02
	8 $d^2D_{5/2}$		2792315.61	2792320.00			7.10E+08		5.48E-01		1.34E+01	
	9 $d^2D_{5/2}$		2828353.82	2828350.00			5.30E+08		1.47E-01		2.15E+00	
	10 $d^2D_{5/2}$		2854132.74	2854130.00			3.91E+08		6.55E-02		7.45E-01	
	11 $d^2D_{5/2}$		2873206.08	2873210.00			2.94E+08		3.63E-02		3.54E-01	
	12 $d^2D_{5/2}$		2887712.45	2887710.00			2.26E+08		2.28E-02		2.01E-01	
	13 $d^2D_{5/2}$		2899001.30	2899000.00			1.78E+08		1.55E-02		1.27E-01	
	14 $d^2D_{5/2}$		2907958.19	2907960.00			1.42E+08		1.11E-02		8.60E-02	
	15 $d^2D_{5/2}$		2915183.77	2915180.00			1.15E+08		8.28E-03		6.16E-02	

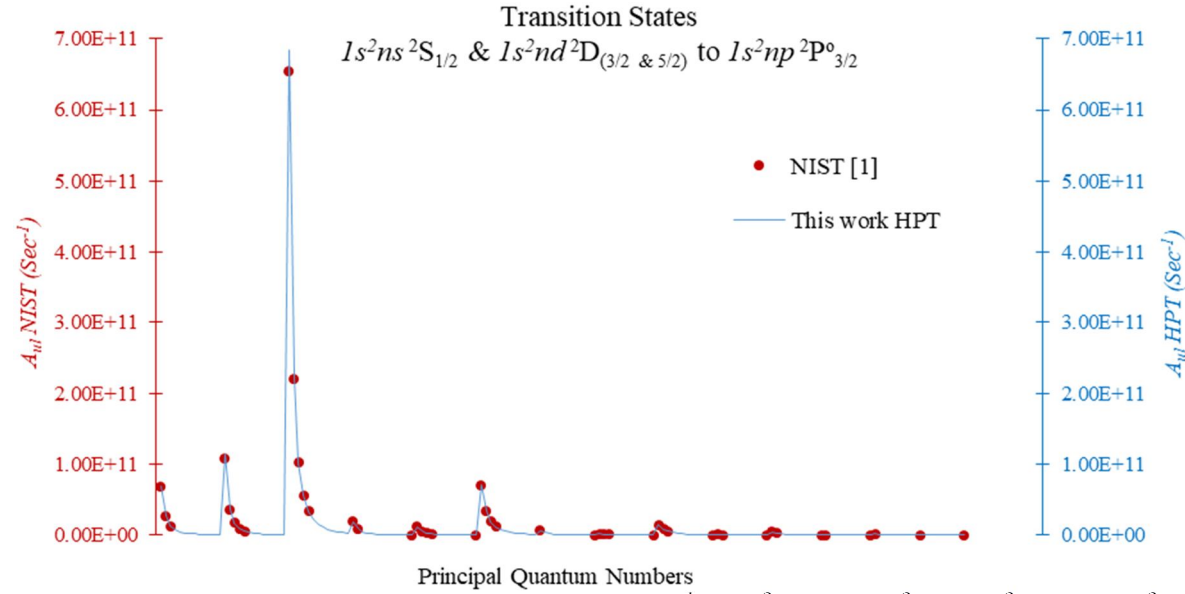


FIG. 3. Comparison of NIST values of transition probabilities with this work of HPT in sec^{-1} for $1s^2(3 \leq n \leq 15)ns^2S_{1/2}$ & $1s^2(3 \leq n \leq 15)nd^2D_{(3/2 \text{ \& } 5/2)}$ to $1s^2(2 \leq n \leq 7)np^2P_{3/2}^o$.

TABLE 5. Computed and compared values of A_{ul} , S_{lu} and f_{lu} transitions from $1s^2(3 \leq n \leq 15)np^2P_{(1/2 \text{ \& } 3/2)}^o$ to $1s^2(3 \leq n \leq 7)nd^2D_{3/2}$.

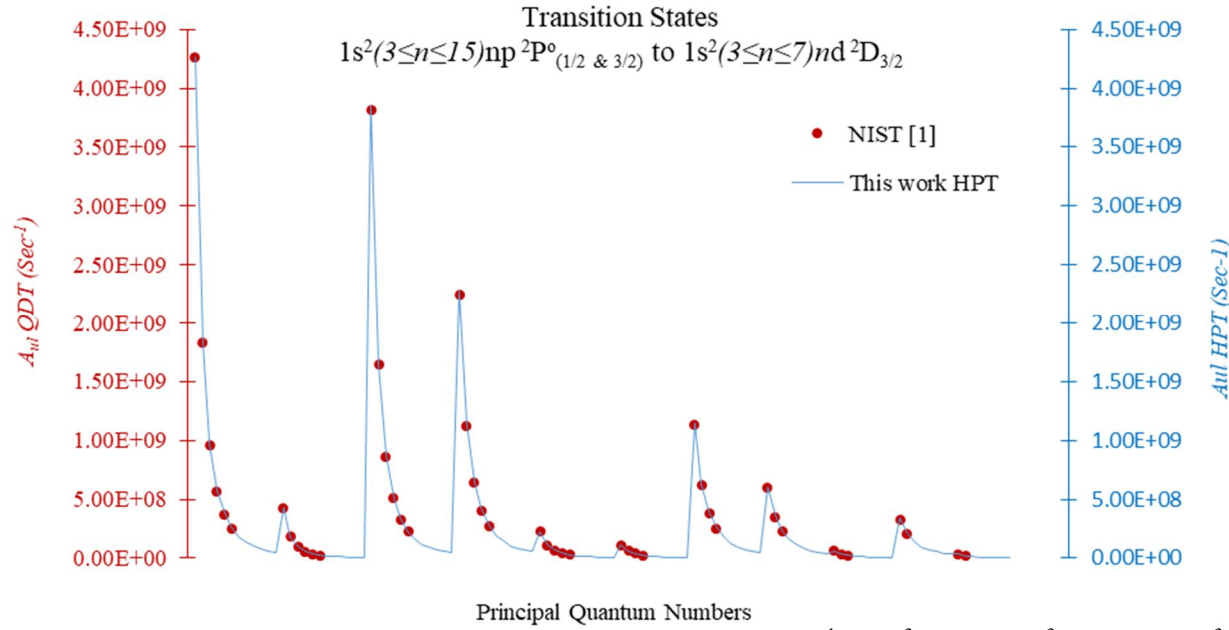
S.No	Transition Array		Rydberg Energies (cm^{-1})			Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)	$A_{ul} (\text{Sec}^{-1})$		$S_{lu} (a.u)$		f_{lu}	
	$1s^2n'l^{(2s+1)}L_{(l \pm s)}$	$1s^2n'l^{(2s+1)}L_{(l \pm s)}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	This work HPT	NIST [1]	This work HPT	NIST [1]
33	$4p^2P_{1/2}^o$	$1s^23d^2D_{3/2}$	2270150.00	2270150.00	2270150.00	1743500.00	4.27E+09	4.26E+09	1.15E-02	1.15E-02	2.88E-02
	$5p^2P_{1/2}^o$		2520900.00	2520900.00	2520900.00		1.82E+09	1.83E+09	2.26E-03	2.27E-03	3.82E-03
	$6p^2P_{1/2}^o$		2656759.04	2656500.00	2656500.00		9.71E+08	9.54E+08	8.71E-04	8.58E-04	1.26E-03
	$7p^2P_{1/2}^o$		2738522.02	2738400.00	2738400.00		5.72E+08	5.66E+08	4.32E-04	4.29E-04	5.71E-04
	$8p^2P_{1/2}^o$		2791513.32	2791200.00	2791200.00		3.84E+08	3.65E+08	2.61E-04	2.49E-04	3.28E-04
	$9p^2P_{1/2}^o$		2827803.68	2827600.00	2827600.00		2.61E+08	2.49E+08	1.66E-04	1.59E-04	2.01E-04
	$10p^2P_{1/2}^o$		2853739.09	2853740.00			1.77E+08		1.07E-04		1.27E-04
	$11p^2P_{1/2}^o$		2872914.62	2872910.00			1.32E+08		7.71E-05		8.98E-05
	$12p^2P_{1/2}^o$		2887490.55	2887490.00			1.00E+08		5.71E-05		6.57E-05
	$13p^2P_{1/2}^o$		2898828.40	2898830.00			7.79E+07		4.36E-05		4.97E-05
	$14p^2P_{1/2}^o$		2907820.82	2907820.00			6.21E+07		3.42E-05		3.87E-05
	$15p^2P_{1/2}^o$		2915072.79	2915070.00			5.03E+07		2.74E-05		3.08E-05

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength			
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)	A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}				
	1s ² n <i>l</i> ^(2s+1) L _(l±s)	1s ² n' <i>l'</i> ^(2s+1) L _(l'±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]		
34	4 <i>p</i> ² P _{3/2} ^o	1s ² 3 <i>d</i> ² D _{3/2}	2270150.00	2270150.00	2270150.00	1743500.00	4.27E+08	4.26E+08	2.30E-03	2.30E-03	5.76E-03	5.75E-03		
	5 <i>p</i> ² P _{3/2} ^o		2520900.00	2520900.00	2520900.00		1.82E+08	1.83E+08	4.51E-04	4.53E-04	7.64E-04	7.67E-04		
	6 <i>p</i> ² P _{3/2} ^o		2656789.55	2656500.00	2656500.00		9.71E+07	9.56E+07	1.74E-04	1.72E-04	2.51E-04	2.48E-04		
	7 <i>p</i> ² P _{3/2} ^o		2738560.41	2738400.00	2738400.00		5.72E+07	5.66E+07	8.64E-05	8.58E-05	1.14E-04	1.14E-04		
	8 <i>p</i> ² P _{3/2} ^o		2791549.57	2791200.00	2791200.00		3.84E+07	3.65E+07	5.22E-05	4.98E-05	6.56E-05	6.26E-05		
	9 <i>p</i> ² P _{3/2} ^o		2827834.94	2827830.00	2827600.00		2.46E+07	2.49E+07	3.13E-05	3.18E-05	3.80E-05	3.86E-05		
	10 <i>p</i> ² P _{3/2} ^o		2853765.17	2853760.00			1.76E+07		2.13E-05		2.53E-05			
	11 <i>p</i> ² P _{3/2} ^o		2872936.15	2872940.00			1.30E+07		1.52E-05		1.77E-05			
	12 <i>p</i> ² P _{3/2} ^o		2887508.32	2887510.00			9.88E+06		1.13E-05		1.30E-05			
	13 <i>p</i> ² P _{3/2} ^o		2898843.13	2898840.00			7.73E+06		8.65E-06		9.86E-06			
	14 <i>p</i> ² P _{3/2} ^o		2907833.10	2907830.00			6.15E+06		6.78E-06		7.66E-06			
	15 <i>p</i> ² P _{3/2} ^o		2915083.10	2915080.00			4.97E+06		5.41E-06		6.08E-06			
	35		4 <i>p</i> ² P _{3/2} ^o	1s ² 3 <i>d</i> ² D _{5/2}	2270150.00	2270150.00	2270150.00	1743890.00	3.86E+09	3.82E+09	1.39E-02	1.38E-02	5.21E-02	5.18E-02
			5 <i>p</i> ² P _{3/2} ^o		2520900.00	2520900.00	2520900.00		1.65E+09	1.64E+09	2.72E-03	2.72E-03	6.90E-03	6.91E-03
			6 <i>p</i> ² P _{3/2} ^o		2656789.55	2656500.00	2656500.00		8.77E+08	8.58E+08	1.05E-03	1.03E-03	2.27E-03	2.23E-03
7 <i>p</i> ² P _{3/2} ^o		2738560.41	2738400.00		2738400.00		5.16E+08	5.10E+08	5.20E-04	5.15E-04	1.03E-03	1.02E-03		
8 <i>p</i> ² P _{3/2} ^o		2791549.57	2791200.00		2791200.00		3.46E+08	3.28E+08	3.14E-04	2.99E-04	5.93E-04	5.64E-04		
9 <i>p</i> ² P _{3/2} ^o		2827834.94	2827830.00		2827600.00		2.22E+08	2.24E+08	1.89E-04	1.91E-04	3.43E-04	3.48E-04		
10 <i>p</i> ² P _{3/2} ^o		2853765.17	2853760.00				1.59E+08		1.28E-04		2.28E-04			
11 <i>p</i> ² P _{3/2} ^o		2872936.15	2872940.00				1.17E+08		9.15E-05		1.60E-04			
12 <i>p</i> ² P _{3/2} ^o		2887508.32	2887510.00				8.92E+07		6.79E-05		1.17E-04			
13 <i>p</i> ² P _{3/2} ^o		2898843.13	2898840.00				6.98E+07		5.21E-05		8.90E-05			
14 <i>p</i> ² P _{3/2} ^o		2907833.10	2907830.00				5.55E+07		4.08E-05		6.92E-05			
15 <i>p</i> ² P _{3/2} ^o		2915083.10	2915080.00				4.49E+07		3.26E-05		5.49E-05			
36		5 <i>p</i> ² P _{1/2} ^o	1s ² 4 <i>d</i> ² D _{3/2}		2520900.00	2520900.00	2520900.00	2277380.00	2.27E+09	2.25E+09	2.86E-02	2.85E-02	1.55E-01	1.54E-01
		6 <i>p</i> ² P _{1/2} ^o			2656759.04	2656500.00	2656500.00		1.14E+09	1.12E+09	5.95E-03	5.84E-03	2.07E-02	2.03E-02
		7 <i>p</i> ² P _{1/2} ^o			2738522.02	2738400.00	2738400.00		6.49E+08	6.41E+08	2.28E-03	2.26E-03	6.52E-03	6.46E-03
	8 <i>p</i> ² P _{1/2} ^o	2791513.32		2791200.00	2791200.00		4.24E+08	4.05E+08	1.20E-03	1.15E-03	3.08E-03	2.95E-03		
	9 <i>p</i> ² P _{1/2} ^o	2827803.68		2827600.00	2827600.00		2.84E+08	2.72E+08	7.02E-04	6.74E-04	1.68E-03	1.61E-03		
	10 <i>p</i> ² P _{1/2} ^o	2853739.09		2853740.00			1.92E+08		4.32E-04		9.87E-04			
	11 <i>p</i> ² P _{1/2} ^o	2872914.62		2872910.00			1.41E+08		2.98E-04		6.59E-04			
	12 <i>p</i> ² P _{1/2} ^o	2887490.55		2887490.00			1.07E+08		2.15E-04		4.63E-04			
	13 <i>p</i> ² P _{1/2} ^o	2898828.40		2898830.00			8.29E+07		1.60E-04		3.40E-04			
	14 <i>p</i> ² P _{1/2} ^o	2907820.82		2907820.00			6.58E+07		1.24E-04		2.59E-04			

Treatment of Perturbation in Energy Levels using Hypothetical Perturbers: A Case Study of Mg X

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)		A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}	
	1s ² n'l ^(2s+1) L _(l±s)	1s ² n'l ^(2s+1) L _(l±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
37	15 p ² P ^o _{1/2}	1s ² 4 d ² D _{3/2}	2915072.79	2915070.00			5.32E+07		9.78E-05		2.02E-04	
	5 p ² P ^o _{3/2}		2520900.00	2520900.00	2520900.00	2277380.00	2.27E+08	2.25E+08	5.72E-03	5.69E-03	3.09E-02	3.08E-02
	6 p ² P ^o _{3/2}		2656789.55	2656500.00	2656500.00		1.14E+08	1.12E+08	1.19E-03	1.17E-03	4.13E-03	4.06E-03
	7 p ² P ^o _{3/2}		2738560.41	2738400.00	2738400.00		6.49E+07	6.41E+07	4.57E-04	4.52E-04	1.30E-03	1.29E-03
	8 p ² P ^o _{3/2}		2791549.57	2791200.00	2791200.00		4.24E+07	4.05E+07	2.40E-04	2.30E-04	6.16E-04	5.89E-04
	9 p ² P ^o _{3/2}		2827834.94	2827830.00	2827600.00		2.70E+07	2.73E+07	1.33E-04	1.35E-04	3.18E-04	3.23E-04
	10 p ² P ^o _{3/2}		2853765.17	2853760.00			1.91E+07		8.58E-05		1.96E-04	
	11 p ² P ^o _{3/2}		2872936.15	2872940.00			1.40E+07		5.88E-05		1.30E-04	
	12 p ² P ^o _{3/2}		2887508.32	2887510.00			1.06E+07		4.25E-05		9.16E-05	
	13 p ² P ^o _{3/2}		2898843.13	2898840.00			8.23E+06		3.19E-05		6.75E-05	
	14 p ² P ^o _{3/2}		2907833.10	2907830.00			6.52E+06		2.45E-05		5.12E-05	
	15 p ² P ^o _{3/2}		2915083.10	2915080.00			5.26E+06		1.94E-05		3.99E-05	
38	6 p ² P ^o _{3/2}	1s ² 5 d ² D _{3/2}	2656789.55	2656500.00	2656500.00	2524400.00	1.15E+08	1.13E+08	9.89E-03	9.69E-03	9.86E-02	9.66E-02
	7 p ² P ^o _{3/2}		2738560.41	2738400.00	2738400.00		6.28E+07	6.20E+07	2.05E-03	2.03E-03	1.26E-02	1.25E-02
	8 p ² P ^o _{3/2}		2791549.57	2791200.00	2791200.00		3.96E+07	3.79E+07	8.33E-04	7.98E-04	4.11E-03	3.94E-03
	9 p ² P ^o _{3/2}		2827834.94	2827830.00	2827600.00		2.48E+07	2.51E+07	4.03E-04	4.09E-04	1.75E-03	1.78E-03
	10 p ² P ^o _{3/2}		2853765.17	2853760.00			1.73E+07		2.38E-04		9.52E-04	
	11 p ² P ^o _{3/2}		2872936.15	2872940.00			1.25E+07		1.54E-04		5.82E-04	
	12 p ² P ^o _{3/2}		2887508.32	2887510.00			9.42E+06		1.07E-04		3.87E-04	
	13 p ² P ^o _{3/2}		2898843.13	2898840.00			7.29E+06		7.78E-05		2.73E-04	
	14 p ² P ^o _{3/2}		2907833.10	2907830.00			5.75E+06		5.85E-05		2.01E-04	
	15 p ² P ^o _{3/2}		2915083.10	2915080.00			4.62E+06		4.53E-05		1.53E-04	
39	6 p ² P ^o _{1/2}	1s ² 5 d ² D _{3/2}	2656759.04	2656500.00	2656500.00	2524400.00	1.15E+09	1.13E+09	4.95E-02	4.84E-02	4.93E-01	4.82E-01
	7 p ² P ^o _{1/2}		2738522.02	2738400.00	2738400.00		6.28E+08	6.23E+08	1.03E-02	1.02E-02	6.31E-02	6.28E-02
	8 p ² P ^o _{1/2}		2791513.32	2791200.00	2791200.00		3.96E+08	3.79E+08	4.17E-03	3.99E-03	2.06E-02	1.97E-02
	9 p ² P ^o _{1/2}		2827803.68	2827600.00	2827600.00		2.60E+08	2.50E+08	2.12E-03	2.04E-03	9.20E-03	8.86E-03
	10 p ² P ^o _{1/2}		2853739.09	2853740.00			1.74E+08		1.20E-03		4.79E-03	
	11 p ² P ^o _{1/2}		2872914.62	2872910.00			1.27E+08		7.80E-04		2.95E-03	
	12 p ² P ^o _{1/2}		2887490.55	2887490.00			9.52E+07		5.40E-04		1.96E-03	
	13 p ² P ^o _{1/2}		2898828.40	2898830.00			7.34E+07		3.92E-04		1.38E-03	
	14 p ² P ^o _{1/2}		2907820.82	2907820.00			5.80E+07		2.95E-04		1.01E-03	
	15 p ² P ^o _{1/2}		2915072.79	2915070.00			4.67E+07		2.29E-04		7.72E-04	
40	7 p ² P ^o _{1/2}	1s ² 6 d ² D _{3/2}	2738522.02	2738400.00	2738400.00	2658800.00	6.06E+08	5.94E+08	7.15E-02	7.03E-02	1.18E+00	1.16E+00
	8 p ² P ^o _{1/2}		2791513.32	2791200.00	2791200.00		3.69E+08	3.51E+08	1.57E-02	1.50E-02	1.57E-01	1.49E-01

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E _u)		Lower State (E _l)		A _{ul} (Sec ⁻¹)		S _{lu} (a.u)		f _{lu}	
	1s ² n'l ^(2s+1) L _(l±s)	1s ² n'l ^(2s+1) L _(l±s)	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00		2.35E+08	2.26E+08	6.18E-03	5.94E-03	4.82E-02	4.63E-02
	10 p ² P _{1/2} ^o		2853739.09	2853740.00			1.54E+08		3.04E-03		2.05E-02	
	11 p ² P _{1/2} ^o		2872914.62	2872910.00			1.11E+08		1.81E-03		1.12E-02	
	12 p ² P _{1/2} ^o		2887490.55	2887490.00			8.27E+07		1.18E-03		6.81E-03	
	13 p ² P _{1/2} ^o		2898828.40	2898830.00			6.33E+07		8.22E-04		4.51E-03	
	14 p ² P _{1/2} ^o		2907820.82	2907820.00			4.98E+07		6.00E-04		3.18E-03	
	15 p ² P _{1/2} ^o		2915072.79	2915070.00			3.99E+07		4.55E-04		2.34E-03	
41	7 p ² P _{3/2} ^o	1s ² 6 d ² D _{3/2}	2738560.41	2738400.00	2738400.00	2658800.00	6.06E+07	5.96E+07	1.43E-02	1.41E-02	2.37E-01	2.33E-01
	8 p ² P _{3/2} ^o		2791549.57	2791200.00	2791200.00		3.69E+07	3.50E+07	3.15E-03	2.99E-03	3.13E-02	2.97E-02
	9 p ² P _{3/2} ^o		2827834.94	2827830.00	2827600.00		2.24E+07	2.26E+07	1.18E-03	1.19E-03	9.16E-03	9.28E-03
	10 p ² P _{3/2} ^o		2853765.17	2853760.00			1.54E+07		6.04E-04		4.08E-03	
	11 p ² P _{3/2} ^o		2872936.15	2872940.00			1.10E+07		3.59E-04		2.20E-03	
	12 p ² P _{3/2} ^o		2887508.32	2887510.00			8.19E+06		2.34E-04		1.35E-03	
	13 p ² P _{3/2} ^o		2898843.13	2898840.00			6.29E+06		1.63E-04		8.96E-04	
	14 p ² P _{3/2} ^o		2907833.10	2907830.00			4.94E+06		1.19E-04		6.30E-04	
42	15 p ² P _{3/2} ^o		2915083.10	2915080.00			3.95E+06		9.01E-05		4.63E-04	
	8 p ² P _{1/2} ^o	1s ² 7 d ² D _{3/2}	2791513.32	2791200.00	2791200.00	2739600.00	3.45E+08	3.31E+08	9.69E-02	9.31E-02	2.47E+00	2.38E+00
	9 p ² P _{1/2} ^o		2827803.68	2827600.00	2827600.00		2.13E+08	2.09E+08	2.06E-02	2.02E-02	3.09E-01	3.02E-01
	10 p ² P _{1/2} ^o		2853739.09	2853740.00			1.37E+08		7.84E-03		9.04E-02	
	11 p ² P _{1/2} ^o		2872914.62	2872910.00			9.65E+07		4.06E-03		4.01E-02	
	12 p ² P _{1/2} ^o		2887490.55	2887490.00			7.09E+07		2.43E-03		2.16E-02	
	13 p ² P _{1/2} ^o		2898828.40	2898830.00			5.38E+07		1.59E-03		1.31E-02	
	14 p ² P _{1/2} ^o		2907820.82	2907820.00			4.20E+07		1.11E-03		8.70E-03	
43	15 p ² P _{1/2} ^o		2915072.79	2915070.00			3.35E+07		8.15E-04		6.11E-03	
	8 p ² P _{3/2} ^o	1s ² 7 d ² D _{3/2}	2791549.57	2791200.00	2791200.00	2739600.00	3.45E+07	3.30E+07	1.94E-02	1.86E-02	4.95E-01	4.75E-01
	9 p ² P _{3/2} ^o		2827834.94	2827830.00	2827600.00		2.03E+07	2.09E+07	3.91E-03	4.04E-03	5.84E-02	6.05E-02
	10 p ² P _{3/2} ^o		2853765.17	2853760.00			1.36E+07		1.56E-03		1.80E-02	
	11 p ² P _{3/2} ^o		2872936.15	2872940.00			9.55E+06		8.03E-04		7.93E-03	
	12 p ² P _{3/2} ^o		2887508.32	2887510.00			7.03E+06		4.81E-04		4.28E-03	
	13 p ² P _{3/2} ^o		2898843.13	2898840.00			5.35E+06		3.16E-04		2.61E-03	
	14 p ² P _{3/2} ^o		2907833.10	2907830.00			4.17E+06		2.21E-04		1.73E-03	
44	15 p ² P _{3/2} ^o		2915083.10	2915080.00			3.32E+06		1.61E-04		1.21E-03	

FIG. 4. Comparison of NIST values of transition probabilities with this work of HPT in sec^{-1} for $1s^2(3 \leq n \leq 15)np^2P^o_{(1/2 \text{ \& } 3/2)}$ to $1s^2(3 \leq n \leq 7)nd^2D_{3/2}$.TABLE 6. Computed and compared values of A_{ul} , S_{lu} and f_{lu} transitions from $1s^2(5 \leq n \leq 15)np^2P^o_{3/2}$ to $1s^2(4 \leq n \leq 7)nd^2D_{5/2}$.

S.No	Transition Array		Rydberg Energies (cm^{-1})			Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)		Lower State (E_l)	$A_{ul} (\text{Sec}^{-1})$		$S_{lu} (a.u)$		f_{lu}	
	$1s^2nl^{(2s+1)}L_{(l \pm s)}$	$1s^2n'l^{(2s+1)}L_{(l \pm s)}$	This work QDT	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
44	$5p^2P^o_{3/2}$	$1s^24d^2D_{5/2}$	2520900.00	2520900.00	2520900.00	2.05E+09	2.02E+09	3.46E-02	3.41E-02	2.81E-01	2.77E-01
	$6p^2P^o_{3/2}$		2656789.55	2656500.00	2656500.00	1.03E+09	1.00E+09	7.19E-03	7.00E-03	3.75E-02	3.65E-02
	$7p^2P^o_{3/2}$		2738560.41	2738400.00	2738400.00	5.87E+08	5.75E+08	2.76E-03	2.71E-03	1.18E-02	1.16E-02
	$8p^2P^o_{3/2}$		2791549.57	2791200.00	2791200.00	3.84E+08	3.64E+08	1.45E-03	1.38E-03	5.58E-03	5.31E-03
	$9p^2P^o_{3/2}$		2827834.94	2827600.00	2827600.00	2.44E+08	2.45E+08	8.04E-04	8.09E-04	2.88E-03	2.91E-03
	$10p^2P^o_{3/2}$		2853765.17	2853760.00		1.73E+08		5.18E-04		1.78E-03	
	$11p^2P^o_{3/2}$		2872936.15	2872940.00		1.26E+08		3.55E-04		1.18E-03	
	$12p^2P^o_{3/2}$		2887508.32	2887510.00		9.57E+07		2.56E-04		8.30E-04	
	$13p^2P^o_{3/2}$		2898843.13	2898840.00		7.45E+07		1.92E-04		6.12E-04	
	$14p^2P^o_{3/2}$		2907833.10	2907830.00		5.90E+07		1.48E-04		4.64E-04	
	$15p^2P^o_{3/2}$		2915083.10	2915080.00		4.76E+07		1.17E-04		3.62E-04	

S.No	Transition Array		Rydberg Energies (cm ⁻¹)				Transition Probabilities		Line strength		Oscillator strength	
	Upper State	Lower State	Upper State (E_u)			Lower State (E_l)	A_{ul} (Sec ⁻¹)		S_{lu} (a.u)		f_{lu}	
	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	$1s^2n'l^{(2s+1)}L_{(l\pm s)}$	This work QDT	This work HPT	NIST [1]	This work	This work HPT	NIST [1]	This work HPT	NIST [1]	This work HPT	NIST [1]
45	$6p\ ^2P^o_{3/2}$	$1s^25d\ ^2D_{5/2}$	2656789.55	2656500.00	2656500.00	2524600.00	1.05E+09	1.01E+09	5.99E-02	5.80E-02	8.97E-01	8.69E-01
	$7p\ ^2P^o_{3/2}$		2738560.41	2738400.00	2738400.00		5.68E+08	5.58E+08	1.24E-02	1.22E-02	1.15E-01	1.13E-01
	$8p\ ^2P^o_{3/2}$		2791549.57	2791200.00	2791200.00		3.59E+08	3.41E+08	5.04E-03	4.79E-03	3.73E-02	3.55E-02
	$9p\ ^2P^o_{3/2}$		2827834.94	2827600.00	2827600.00		2.24E+08	2.25E+08	2.43E-03	2.45E-03	1.58E-02	1.60E-02
	$10p\ ^2P^o_{3/2}$		2853765.17	2853760.00			1.56E+08		1.44E-03		8.64E-03	
	$11p\ ^2P^o_{3/2}$		2872936.15	2872940.00			1.13E+08		9.32E-04		5.28E-03	
	$12p\ ^2P^o_{3/2}$		2887508.32	2887510.00			8.53E+07		6.46E-04		3.51E-03	
	$13p\ ^2P^o_{3/2}$		2898843.13	2898840.00			6.60E+07		4.70E-04		2.48E-03	
	$14p\ ^2P^o_{3/2}$		2907833.10	2907830.00			5.21E+07		3.54E-04		1.82E-03	
	$15p\ ^2P^o_{3/2}$		2915083.10	2915080.00			4.19E+07		2.74E-04		1.39E-03	
46	$7p\ ^2P^o_{3/2}$	$1s^26d\ ^2D_{5/2}$	2738560.41	2738400.00	2738400.00	2658800.00	5.45E+08	5.34E+08	8.59E-02	8.43E-02	2.13E+00	2.09E+00
	$8p\ ^2P^o_{3/2}$		2791549.57	2791200.00	2791200.00		3.32E+08	3.16E+08	1.89E-02	1.80E-02	2.82E-01	2.69E-01
	$9p\ ^2P^o_{3/2}$		2827834.94	2827600.00	2827600.00		2.02E+08	2.03E+08	7.05E-03	7.13E-03	8.24E-02	8.34E-02
	$10p\ ^2P^o_{3/2}$		2853765.17	2853760.00			1.38E+08		3.63E-03		3.67E-02	
	$11p\ ^2P^o_{3/2}$		2872936.15	2872940.00			9.89E+07		2.15E-03		1.98E-02	
	$12p\ ^2P^o_{3/2}$		2887508.32	2887510.00			7.37E+07		1.41E-03		1.21E-02	
	$13p\ ^2P^o_{3/2}$		2898843.13	2898840.00			5.66E+07		9.80E-04		8.07E-03	
	$14p\ ^2P^o_{3/2}$		2907833.10	2907830.00			4.45E+07		7.15E-04		5.67E-03	
	$15p\ ^2P^o_{3/2}$		2915083.10	2915080.00			3.56E+07		5.40E-04		4.16E-03	
47	$8p\ ^2P^o_{3/2}$	$1s^27d\ ^2D_{5/2}$	2791549.57	2791200.00	2791200.00	2739600.00	3.10E+08	2.98E+08	1.16E-01	1.12E-01	4.45E+00	4.29E+00
	$9p\ ^2P^o_{3/2}$		2827834.94	2827600.00	2827600.00		1.83E+08	1.88E+08	2.35E-02	2.42E-02	5.25E-01	5.43E-01
	$10p\ ^2P^o_{3/2}$		2853765.17	2853760.00			1.22E+08		9.35E-03		1.62E-01	
	$11p\ ^2P^o_{3/2}$		2872936.15	2872940.00			8.59E+07		4.82E-03		7.14E-02	
	$12p\ ^2P^o_{3/2}$		2887508.32	2887510.00			6.32E+07		2.88E-03		3.85E-02	
	$13p\ ^2P^o_{3/2}$		2898843.13	2898840.00			4.82E+07		1.89E-03		2.35E-02	
	$14p\ ^2P^o_{3/2}$		2907833.10	2907830.00			3.75E+07		1.32E-03		1.55E-02	
	$15p\ ^2P^o_{3/2}$		2915083.10	2915080.00			2.99E+07		9.68E-04		1.09E-02	

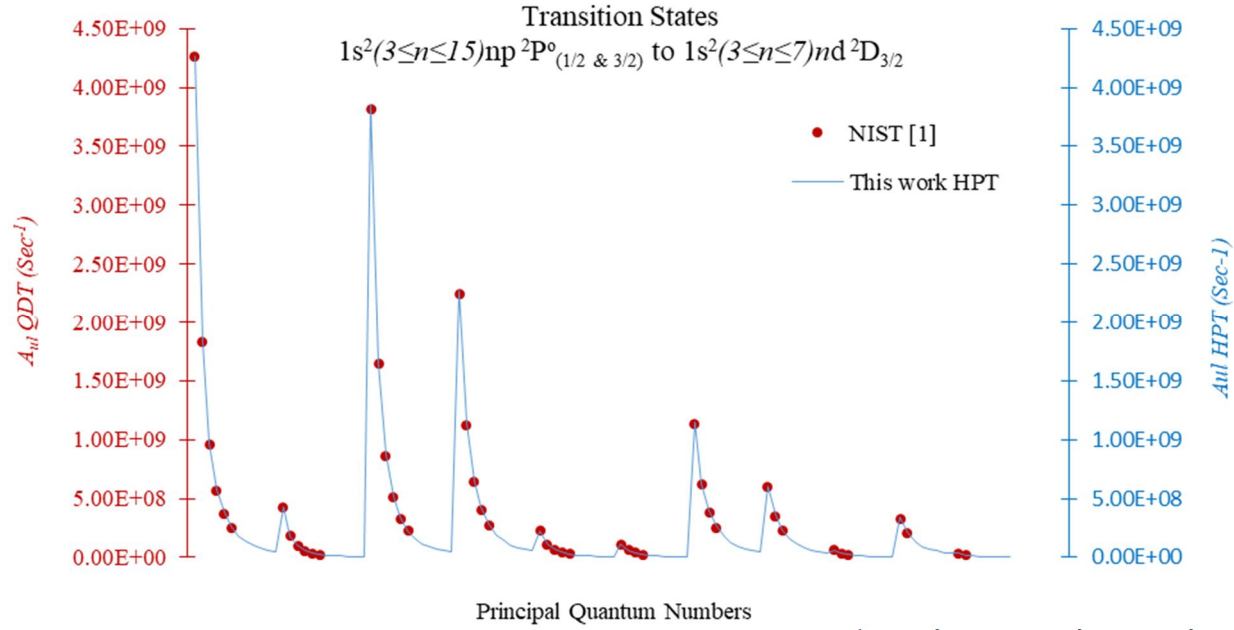


FIG. 5. Comparison of NIST values of transition probabilities with this work of HPT in sec^{-1} for $1s^2(5 \leq n \leq 15)np^2P^o_{3/2}$ to $1s^2(4 \leq n \leq 7)nd^2D_{5/2}$.

4. Conclusion

A new approach for solving perturbed atomic systems using hypothetical energy levels as perturbers has been introduced. The perturbed system of Mg X was investigated to determine the Rydberg energy levels corresponding to the configurations: $1s^2nl$ ($l=s, p, d$), namely $^2S_{1/2}$, $^2P_{1/2}^o$, $^2P_{3/2}^o$, $^2D_{3/2}$, and $^2D_{5/2}$. The calculated energies were further validated by computing and comparing the transition probabilities, oscillator strengths, and line strengths with the corresponding values from NIST [1] data.

In this case study of Mg X, the Rydberg energy series for the above configurations were calculated up to the principal quantum number $n = 15$. Among these series, $1s^2ns$ $^2S_{1/2}$, $1s^2nl$ ($l=p, d$) ($^2P_{1/2}^o$, $^2D_{5/2}$) exhibit core-polarization characteristics, although the $1s^2ns$ $^2S_{1/2}$ series remains unperturbed. The $1s^2nl$ ($l=p, d$) ($^2P_{3/2}^o$, $^2D_{3/2}$) series are identified as perturbed core-penetration series. The strong agreement between the computed results and the NIST data demonstrates the reliability and effectiveness of the proposed method.

In summary, the Weakest Bound Electron Potential Model (WBEPM), combined with the hypothetical perturber technique (HPT), provides a simple and effective framework for studying the Rydberg energy series in Mg X for both low- and high-lying principal quantum numbers. A total of 68 Rydberg levels and 935 values of transition probabilities, oscillator strengths, and line strengths were computed. Among these, 29 Rydberg levels and 751 corresponding transition parameters are currently unavailable in the National Institute of Standards and Technology (NIST) database [1].

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Data Availability

The data supporting this study's findings are available within the text and in the supplementary material file.

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