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Comparison of Calculated Energy Levels and Electric Quadrupole Transitions Probabilities of the Even-Even $^{144\text{-}150}$Nd Isotopes Using IBM-2, NEF and BM

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Abstract: The energy levels of low-lying bands were calculated using the interacting boson model (IBM-2) [1], new empirical formula (NEF) [2, 3], and Bohr-Mottelson model (BM) [4, 5], while the rest of the levels, including the negative parity bands (NPB) of even-even neodymium isotopes Nd (144-150), were calculated using only the NEF and BM. The electrical quadrupole transition probabilities $B(E2)$ of Nd nuclei were also determined. A comparison between the results and the available experimental data showed a very good agreement. To determine the property of the ground-state band, the energy ratio $r\left(\frac{I+2}{I}\right)$ as a function of the spin (I) [6, 7] was plotted.

Keywords: Neutron-rich deformed nuclei, Neodymium isotopes, Energy level structure and electrical quadrupole transition, Interacting Boson model, Bohr-Mottelson model, New empirical formula.

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Introduction

Recently, the study of the neutron-rich deformed nuclei of the mass region A > 100 has become an interesting topic for theoretical and experimental studies. In this work, the energy-level structure and electrical quadrupole transition properties of neodymium isotopes with even neutron numbers N= 84, 86, 88, and 90 were investigated. The interacting boson model is one of the successful approaches for describing the low-lying collective properties of nuclear energy states, including the ground-state band (GSB) $I^+ = 0_1^+ + 2_1^+ + 4_1^+ + \cdots$, the gamma band $I^+ = 2_2^+ + 3_1^+ + 4_2^+ + \cdots$, and the beta band $I^+ = 0_2^+ + 2_3^+ + 4_3^+ + \cdots$ [8, 9].

For the same range of Nd isotopes, the Bohr-Mottelson model (BM) and the new empirical formula (NEF) were used to calculate the energies of the positive and the negative parity states band (NPB).

This paper aimed to calculate and compare energy levels of Nd isotopes (144, 146, 148, and 150) and their reduced electrical quadrupole transition probabilities, $B(E2)$, using the interacting boson model IBM-2, the new empirical formula (NEF), and the Bohr-Mottelson energy collective model (BM). The obtained results are compared with the available experimental data from the nuclear data sheets [10].

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Theory and Calculations:

A. Theory:

1. The interacting boson model (IBM-2)

The IBM was developed in 1974 by F. Iachello and A. Arima [1]. It considers pairs of valence nucleons and treat them as bosons. The number of bosons are conventionally taken to be half the number of valence particles or holes. There are several versions of the IBM. The one considered in this work is the IBM-2, which deals with even-even medium and heavy nuclei and distinguishes between proton bosons and neutron bosons. The general form of the IBM-2 Hamiltonian [11, 12] can be written as:

$$H = H_\nu + H_\pi + H_{\nu,\pi} \quad (1)$$

which consists of the neutron boson Hamiltonian (H_ν), proton boson Hamiltonian (H_π), and neutron boson-proton boson interaction Hamiltonian ($H_{\nu,\pi}$).

$$H_\rho = \varepsilon_\rho n_{d\rho} + V_{\rho\rho},\ \rho = \nu,\pi \quad (2)$$

Here, ε_ρ is the average energy of boson, $n_{d\rho}$ is the number of (d)bosons and $V_{\rho\rho}$ is the interaction between identical bosons, given by:

$$V_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_\rho^{(L)}([d_\rho^\dagger d_\rho^\dagger]^{(L)}.[d_\rho^\sim d_\rho^\sim]^{(L)}) \quad (3)$$

$$H_{\nu,\pi} = k(Q_\nu.Q_\pi) + M_{\nu,\pi} \quad (4)$$

where k is the strength of this interaction, $Q_\nu.Q_\pi$ is the quadrupole-quadrupole interaction, and $M_{\nu,\pi}$ is the Majorana interaction, defined as

$$M_{\nu,\pi} = \frac{1}{2}\xi_2\left(d_\nu^\dagger s_\pi^\dagger - d_\pi^\dagger s_\nu^\dagger\right).(d_\nu^\sim s_\pi - d_\pi^\sim s_\nu) + \sum_{k=1,3}\xi_k\,([d_\nu^\dagger d_\pi^\dagger]^{(k)}.[d_\pi^\sim d_\nu^\sim]^{(k)}) \quad (5)$$

Here, ξ_1, ξ_2, ξ_3 are the Majorana parameters of the interaction between identical bosons ($C_\rho^{(L)}$) and the creation and annihilation operators ($d_\rho^\dagger s_\rho^\dagger$), ($d_\rho s_\rho$).

2. The Bohr-Mottelson (BM)

There are two forms of the BM energy expansion: the GSB and NPB levels with powers of $I(I+1)$ for deformed nuclei [4, 5].

$$E(I)_{GSB} = AI(I+1) - BI^2(I+1)^2 + CI^3(I+1)^3 \quad (6)$$

$$E(I)_{NPB} = E_0 + A'I(I+1) - B'I^2(I+1)^2 + C'I^3(I+1)^3 \quad (7)$$

where E_0 is the band head energy of the negative parity band (NPB) and the coefficients $A > A', B > B'$ and $C > C'$ can be determined from a fit of the available energy levels of this band.

3. The New Empirical formula (NEF):

There are two forms of (NEF):

a- *For the ground band, given by [2, 3]:*

$$E(I) = \frac{A_1\, I(I+1)}{A_2\,(I+1) + I^{A_3}} \quad (8)$$

where A_1, A_2, and A_3 are the fitting parameters.

b- *For negative-state band or any other band, calculated using the following formula:*

$$E(I) = E'_0 + \frac{(A_1+B)I(I+1)}{A_2(I+1) + I^{A_3}} \quad (9)$$

E'_0 and B are parameters of the other bands and can be obtained from the value of $E(I)$ from the negative-state or from any other band.

To define the symmetry for the excited band of even-even nuclei, the following energy ratios were constructed [6, 7]:

$$r\left(\frac{I+2}{I}\right) = \frac{R(I+2/I)_{exp} - R(I+2/I)_{vib}}{R(I+2/I)_{rot} - R(I+2/I)_{vib}} \quad (10)$$

where $R(\frac{I+2}{I})_{exp}$ is the experimental energy value ratio between the ground state band energies of $(I+2)$, and I. The $R(\frac{I+2}{I})_{rot} = \frac{(I+2)(I+3)}{I(I+1)}$ are members of the rotational limit which take the limits $0.6 \le r \le 1.0$.

In the vibrational limit, the energy ratio is $R(\frac{I+2}{I})_{vib} = \frac{I+2}{I}$ in the region $0.1 \le r \le 0.35$, and $0.4 \le r \le 0.6$ for ɣ- unstable nuclei.

4. Transition probability $B(E2)$:

The electric quadropole transitions $(E2)$ are important factors within the collective nuclear structure. The $E2$ transitions in the IBM-2 are governed by the operator [13]:

$$T(E2) = e_\nu Q_\nu + e_\pi Q_\pi \quad (11)$$

$$Q_\rho = d_\rho^\dagger s_\rho + s_\rho^\dagger d_\rho + \chi_\rho[d_\rho^\dagger d_\rho^\sim]^{(2)} \quad (12)$$

In this expression, e_ρ is the effective quadrupole charge and χ_ρ is the quadrupole structure parameter of the neutron and proton bosons.

The electrical quadrupole transitions are also described [14] by the formula:

$$B(E2) = \frac{0.05657}{T_{1l2}^{\gamma}(ps)\, E_\gamma^5\,(MeV)}(e^2b^2) \quad (13)$$

$$T^{\gamma}_{1/2} = T_{1/2}\ (\exp)\ (1 + \alpha_{tot}) \quad (14)$$

where $T_{1/2}$ is a half-life and α_{tot} is total theoretical internal conversion coefficient given in Table 1.

TABLE 1. $T_{1/2}$ and α_{tot} values over I^{+} for $^{144-150}$Nd isotopes.

	^{144}Nd			^{146}Nd		
I^{+}	$T_{1/2}$ psec	α_{tot}	$T^{8}_{1/2}$ psec	$T_{1/2}$ psec	α_{tot}	$T^{8}_{1/2}$ psec
2^{+}	2.97	0.005	2.985	20.9	0.015	21.221
4^{+}	7.4	0.007	7.451	3.8	0.008	3.829
6^{+}	20.8	0.013	21.079	--------	--------	--------
	^{148}Nd			^{150}Nd		
I^{+}	$T_{1/2}$ psec	α_{tot}	$T^{8}_{1/2}$ psec	$T_{1/2}$ psec	α_{tot}	$T^{8}_{1/2}$ psec
2^{+}	80	0.052	84.120	1480	0.857	2748.360
4^{+}	6.90	0.016	7.008	60.50	0.092	66.078
6^{+}	2.90	0.010	2.929	12.50	0.036	12.949
8^{+}	1.40	0.034	1.447	4.70	0.021	4.796
10^{+}	---------	---------	---------	2.59	0.014	2.626
12^{+}	---------	---------	---------	1.80	0.011	1.819

B- Calculations:

The energy eigenstates and electric quadrupole transition probabilities generated by the Neutron-Proton Boson code (NPBOS) were calculated by the best-fitting parameters listed in Table 2.

The energy levels of positive and negative bands of $^{144-150}$Nd isotopes were calculated using the BM and NEF implemented in MATLAB® 9.8 software. Table 3 shows the BM and NEF parameters of the ground-state bands of $^{144-150}$Nd isotopes, while Table 4 lists the best-fitting NEF parameters for the other states. Table 5 displays the best-fitting parameters of the other states of BM for $^{144-150}$Nd isotopes. Table 6 shows a comparison of energy levels of the IBM-2, NEF, and BM models with the experimental data for even-even $^{144-150}$Nd isotopes. The reduced transition probabilities were calculated by the NEF and BM using MATLAB® 9.8 software, and the results are shown in Table 7.

TABLE 2. IBM-2 parameters for $^{144-150}$Nd.

X^{A}_{Z}	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd
ε_d	0.98	0.805	0.650	0.555
$\kappa_{\pi\upsilon}$	-0.215	-0.098	-0.070	-0.093
χ_π	-0.3	-1.200	-1.200	-1.200
χ_ν	-1.40	-1.100	-1.100	-1.200
ξ_2	-0.01	-0.00	0.00	0.00
CL_π	0.9 -0.05 -0.125	-0.600 0.0 0.0	0.0 0.0 -0.05	8.0 0.0 0.0
CL_ν	0.0 0.0 0.0	0.900 -0.0 -0.0	0.0 0.0 0.0	1.00 0.00 0.00

Note: All parameters are in MeV except χ_π and χ_ν.

TABLE 3. NEF & BM parameters for Ground State, even-even $^{144-150}Nd$

Isotopes	NEF Parameters			BM Parameters		
	A_1	A_2	A_3	A	B	C
^{144}Nd	0.06	-0.417	0.807	1.116×10^{-1}	2.532×10^{-3}	2.091×10^{-5}
^{146}Nd	0.116	0.327	-0.507	4.982×10^{-2}	2.143×10^{-4}	3.872×10^{-7}
^{148}Nd	0.093	0.239	0.185	4.001×10^{-2}	2.431×10^{-4}	7.350×10^{-7}
^{150}Nd	0.027	0.053	0.112	1.843×10^{-2}	4.107×10^{-5}	6.512×10^{-8}

Note: NEF & BM parameters are in MeV except A_2 and A_3.

TABLE 4: NEF parameters in MeV of other bands for even-even $^{144-146}Nd$.

Isotopes	^{144}Nd Parameters		^{146}Nd Parameters	
	E'_0	B	E'_0	B
Band 2 (Beta Band)	------	------	------	------
Band 3 $(NPB)_1$	1.0167	-0.0198	0.2309	-0.0097
Band 4 (Gama Band)	1.1991	-0.0183	0.9013	-0.0209
Band 5 $(NPB)_2$	2.0996	-0.0304	-0.1102	-0.0046
Band 6	------	------	0.2900	-0.0096
Band 7 $(NPB)_3$	------	------	-0.3666	0.0107
Band 8	------	------	-0.1702	0.0026

TABLE 4 (continued). NEF parameters in MeV of other bands for even-even $^{148-150}Nd$.

Isotopes	^{148}Nd Parameters		^{150}Nd Parameters	
	E'_0	B	E'_0	B
Band 2 (Beta Band)	0.8899	-0.0016	0.6836	0.0053
Band 3 $(NPB)_1$	0.4867	-0.0198	0.7907	-0.0088
Band 4 (Gamma Band)	1.0555	-0.0219	0.9113	0.0045

TABLE 5. BM parameters in MeV of other bands for even-even ^{144}Nd.

Isotopes	^{144}Nd Parameters			
	E_0	A'	B'	C'
Band 2 (Beta Band)	-----	------	------	-----
Band 3 $(NPB)_1$	1.43	1.75×10^{-2}	-1.79×10^{-5}	-8.73×10^{-8}
Band 4 (Gamma Band)	1.70	1.84×10^{-2}	1.49×10^{-5}	4.94×10^{-8}
Band 5 $(NPB)_2$	0.91	3.79×10^{-2}	1.31×10^{-4}	1.99×10^{-7}
Band 6	-----	------	------	------
Band 7 $(NPB)_3$	-----	------	------	------
Band 8	-----	------	------	------

TABLE 5 (cont.). BM parameters in MeV of other bands for even-even ^{146}Nd.

Isotopes	^{146}Nd Parameters			
	E_0	A'	B'	C'
Band 2 (Beta Band)	-----	------	------	------
Band 3 $(NPB)_1$	0.95	1.91×10^{-2}	-1.23×10^{-5}	-8.53×10^{-8}
Band 4 (Gamma Band)	1.35	2.11×10^{-2}	3.07×10^{-5}	3.38×10^{-8}
Band 5 $(NPB)_2$	1.67	1.27×10^{-2}	-5.97×10^{-6}	-2.02×10^{-8}
Band 6	1.58	2.32×10^{-2}	4.7×10^{-5}	4.67×10^{-8}
Band 7 $(NPB)_3$	1.87	8.67×10^{-3}	-4.56×10^{-5}	-9.88×10^{-8}
Band 8	1.41	1.21×10^{-2}	-4.79×10^{-5}	-1.47×10^{-7}

TABLE 5 (cont.). BM parameters in MeV of other bands for even-even ^{148}Nd.

Isotopes	^{148}Nd			
	Parameters			
	E_0	A'	B'	C'
Band 2 (Beta Band)	0.92	4.1×10^{-2}	3.68×10^{-4}	2.04×10^{-6}
Band 3 $(NPB)_1$	0.81	1.48×10^{-2}	-1.54×10^{-6}	-4.99×10^{-8}
Band 4 (Gamma Band)	0.69	1.19×10^{-1}	4.89×10^{-3}	6.77×10^{-5}

TABLE 5 (cont.). BM parameters in MeV of other bands for even-even ^{150}Nd.

Isotopes	^{150}Nd			
	Parameters			
	E_0	A'	B'	C'
Band 2 (Beta Band)	0.68	3.34×10^{-2}	7.13×10^{-4}	9.71×10^{-6}
Band 3 $(NPB)_1$	0.84	6.64×10^{-3}	-1.34×10^{-4}	-1.12×10^{-6}
Band 4 (Gamma Band)	0.70	8.92×10^{-2}	5.64×10^{-3}	1.41×10^{-4}

TABLE 6. Comparison of energy levels in MeV (positive & negative bands) for even-even $^{144\text{-}150}$Nd.

^{144}Nd				
Band 1 (Ground State)				
I^+	E_{exp}	NEF	BM	IBM-2
0^+	0.000	0.000	0.000	0.000
2^+	0.697	0.719	0.583	0.697
4^+	1.315	1.223	1.386	1.387
6^+	1.792	1.888	1.769	1.742
8^+	2.710	2.678	2.713	2.509
10^+	-----	3.591	9.472	-----
12^+	-----	4.628	35.183	-----
14^+	-----	5.795	105.447	-----
16^+	-----	7.098	263.868	-----
Band 3 $(NPB)_1$				
I^-	E_{exp}	NEF	BM	IBM-2
3^-	1.511	1.648	1.640	-----
5^-	2.093	2.046	1.967	-----
7^-	2.613	2.534	2.451	-----
9^-	2.903	3.103	3.088	-----
11^-	3.830	3.755	3.855	-----
13^-	4.743	4.492	4.689	-----
(15^-)	5.473	5.317	5.466	-----
(17^-)	5.967	6.236	5.976	-----
Band 4				
I^+	E_{exp}	NEF	BM	IBM-2
2^+	1.561	1.698	1.815	1.548
3^+	2.179	1.854	1.924	2.509
4^+	2.110	2.047	2.068	2.178
5^+	2.420	2.266	2.245	-----
6^+	2.218	2.508	2.456	-----
8^+	2.972	3.057	2.972	-----
(9^+)	3.234	3.362	3.278	-----
(10^+)	3.673	3.690	3.617	-----
(11^+)	4.046	4.038	3.991	-----
(12^+)	4.355	4.409	4.404	-----
(14^+)	5.379	5.218	5.375	-----

^{146}Nd				
Band 1 (Ground State)				
I^+	E_{exp}	NEF	BM	IBM-2
0^+	0.000	0.000	0.000	0.000
2^+	0.454	0.414	0.291	0.455
4^+	1.043	1.092	0.914	1.049
6^+	1.780	1.815	1.743	1.748
8^+	2.594	2.545	2.621	2.554
10^+	3.320	3.275	3.403	3.458
12^+	3.994	4.003	4.028	------
14^+	4.694	4.729	4.600	------
16^+	5.461	5.453	5.492	------
Band 3 $(NPB)_1$				
I^-	E_{exp}	NEF	BM	IBM-2
3^-	1.190	0.911	1.179	------
5^-	1.518	1.562	1.529	------
7^-	2.029	2.229	2.039	------
9^-	2.706	2.898	2.701	------
11^-	3.501	3.566	3.483	------
13^-	4.295	4.233	4.311	------
15^-	5.058	4.897	5.054	------
Band 4				
I^+	E_{exp}	NEF	BM	IBM-2
2^+	1.303	1.241	1.472	1.083
3^+	1.777	1.510	1.595	1.713
4^+	1.745	1.798	1.756	1.622
(5^+)	2.046	2.093	1.952	------
(6^+)	2.084	2.391	2.180	------
14^+	4.695	4.782	4.729	------
15^+	5.160	5.079	5.101	------
16^+	5.461	5.376	5.485	------
Band 5 $(NPB)_2$				
I^-	E_{exp}	NEF	BM	IBM-2
11^-	3.405	3.386	3.395	------
13^-	4.028	4.085	4.046	------
15^-	4.761	4.782	4.769	------
17^-	5.559	5.477	5.521	------
19^-	6.203	6.170	6.232	------
21^-	6.807	6.863	6.800	------
Band 6				
I^+	E_{exp}	NEF	BM	IBM-2
16^+	5.363	5.292	5.363	------
18^+	5.900	5.955	5.900	------
20^+	6.514	6.616	6.514	------
(22^+)	7.364	7.277	7.364	------
Band 7 $(NPB)_3$				
I^-	E_{exp}	NEF	BM	IBM-2
10^-	3.246	3.210	3.245	------
12^-	3.958	4.005	3.958	------
14^-	4.787	4.798	4.787	------
16^-	5.612	5.589	5.612	------

Band 8				
I^{+}	E_{exp}	NEF	BM	IBM-2
8^{+}	2.475	2.432	2.474	------
10^{+}	3.124	3.178	3.124	------
12^{+}	3.902	3.922	3.902	------
14^{+}	4.696	4.664	4.696	------

^{148}Nd				
Band 1 (Ground State)				
I^{+}	E_{exp}	NEF	BM	IBM-2
0^{+}	0.000	0.000	0.000	0.000
2^{+}	0.302	0.302	0.231	0.302
4^{+}	0.752	0.751	0.709	0.737
6^{+}	1.280	1.280	1.306	1.289
8^{+}	1.856	1.858	1.894	1.940
10^{+}	2.471	2.470	2.437	2.695
12^{+}	3.106	3.107	3.114	------
14^{+}	------	3.762	4.485	------
16^{+}	------	4.431	7.683	------
Band 2 (Beta Band)				
I^{+}	E_{exp}	NEF	BM	IBM-2
0^{+}	0.917	0.890	0.924	0.888
2^{+}	1.171	1.187	1.157	0.201
4^{+}	1.604	1.628	1.613	1.802
6^{+}	2.149	2.147	2.146	------
8^{+}	2.726	2.715	2.726	------
Band 3 $(NPB)_1$				
I^{-}	E_{exp}	NEF	BM	IBM-2
3^{-}	0.999	0.892	0.991	------
5^{-}	1.242	1.281	1.258	------
7^{-}	1.645	1.720	1.640	------
9^{-}	2.132	2.190	2.124	------
11^{-}	2.676	2.683	2.683	------
13^{-}	3.264	3.193	3.263	------
Band 4 (Gamma Band)				
I^{+}	E_{exp}	NEF	BM	IBM-2
2^{+}	1.249	1.287	1.241	------
3^{+}	1.512	1.449	1.532	------
4^{+}	1.683	1.631	1.661	------
(5^{+})	1.688	1.828	1.699	------
6^{+}	2.099	2.036	2.097	------

^{150}Nd				
Band 1 (Ground State)				
I^{+}	E_{exp}	NEF	BM	IBM-2
0^{+}	0.000	0.000	0.000	0.000
2^{+}	0.130	0.132	0.109	0.138
4^{+}	0.381	0.381	0.353	0.382
6^{+}	0.720	0.719	0.707	0.726
8^{+}	1.130	1.130	1.139	1.173
10^{+}	1.599	1.599	1.617	1.727
12^{+}	2.119	2.119	2.123	------
14^{+}	2.682	2.681	2.663	------
16^{+}	3.280	3.280	3.286	------

Band 2 (Beta Band)				
I^+	E_{exp}	NEF	BM	IBM-2
0^+	0.676	0.684	0.676	0.907
2^+	0.851	0.841	0.851	0.961
4^+	1.138	1.138	1.138	1.181
6^+	1.541	1.542	1.541	------
Band 3 $(NPB)_1$				
I^-	E_{exp}	NEF	BM	IBM-2
1^-	0.853	0.824	0.853	------
3^-	0.935	0.956	0.935	------
5^-	1.129	1.157	1.129	------
(7^-)	1.433	1.412	1.433	------
Band 4 (Gamma Band)				
I^+	E_{exp}	NEF	BM	IBM-2
2^+	1.062	1.065	1.062	0.999
$3^{(+)}$	1.201	1.195	1.201	1.179
4^+	1.353	1.355	1.353	1.345

TABLE 7. Comparison of the transitional probability calculated by IBM-2, NEF, and BM to the experimental data for $^{144-150}$Nd nuclei.

	$I_i \rightarrow I_f$	$B(E_2)$ exp e^2b^2	$B(E_2)$ NEF e^2b^2	$B(E_2)$ BM e^2b^2	$B(E_2)$ IBM-2 e^2b^2
^{144}Nd	$2^+_1 \rightarrow 0^+_1$	0.1155	0.0988	0.2821	0.116
	$4^+_1 \rightarrow 2^+_1$	0.0842	0.2334	0.0227	0.152
	$6^+_1 \rightarrow 4^+_1$	0.1089	0.0206	0.3255	0.167
^{146}Nd	$2^+_1 \rightarrow 0^+_1$	0.1385	0.2182	1.2710	0.229
	$4^+_1 \rightarrow 2^+_1$	0.2077	0.1030	0.1580	0.294
	$6^+_1 \rightarrow 4^+_1$	------	------	------	0.287
^{148}Nd	$2^+_1 \rightarrow 0^+_1$	0.26904	0.26585	1.01262	0.310
	$4^+_1 \rightarrow 2^+_1$	0.43454	0.44344	0.32583	0.474
	$6^+_1 \rightarrow 4^+_1$	0.47279	0.46842	0.25449	0.518
	$8^+_1 \rightarrow 6^+_1$	0.61439	0.60509	0.55371	------
^{150}Nd	$2^+_1 \rightarrow 0^+_1$	0.55012	0.51614	1.32955	0.420
	$4^+_1 \rightarrow 2^+_1$	0.86104	0.90046	0.99758	0.618
	$6^+_1 \rightarrow 4^+_1$	0.97436	0.98263	0.78790	0.679
	$8^+_1 \rightarrow 6^+_1$	1.02550	1.01157	0.78368	--------
	$10^+_1 \rightarrow 8^+_1$	0.95025	0.94099	0.85596	--------
	$12^+_1 \rightarrow 10^+_1$	0.81637	0.82102	0.93781	--------

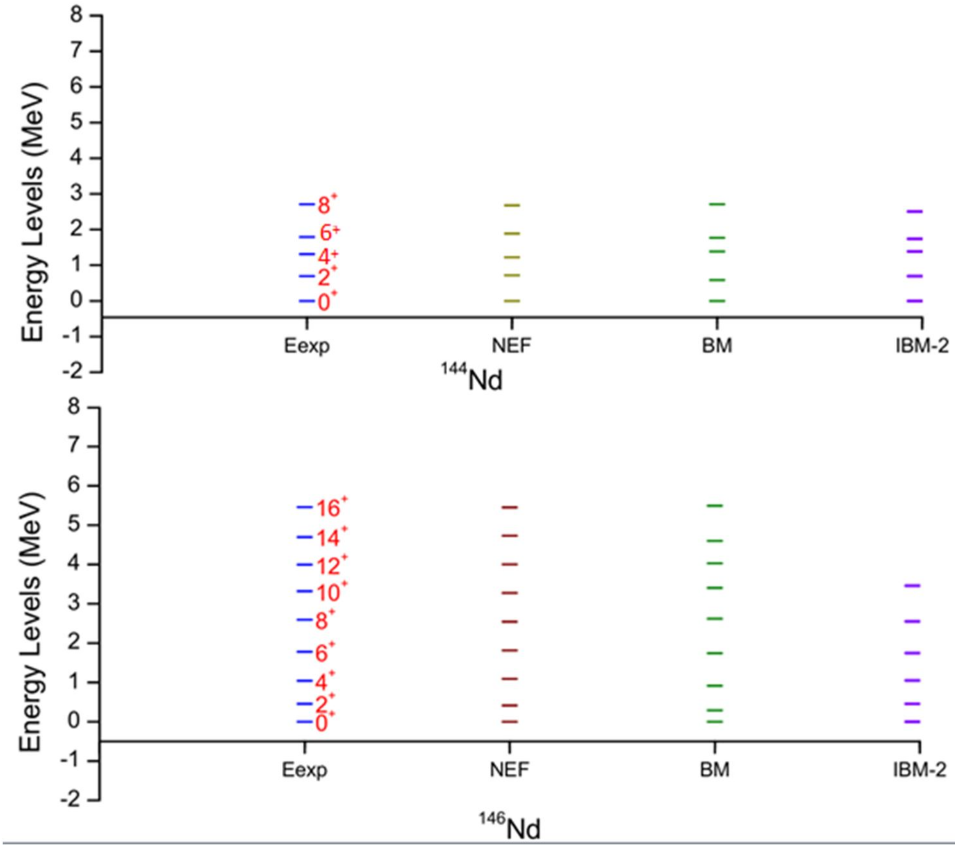

FIG. 1(a). Comparison between energy levels using NEF, BM, and IBM-2 and the experimental data [10] for even-even $^{144-146}$Nd isotopes.

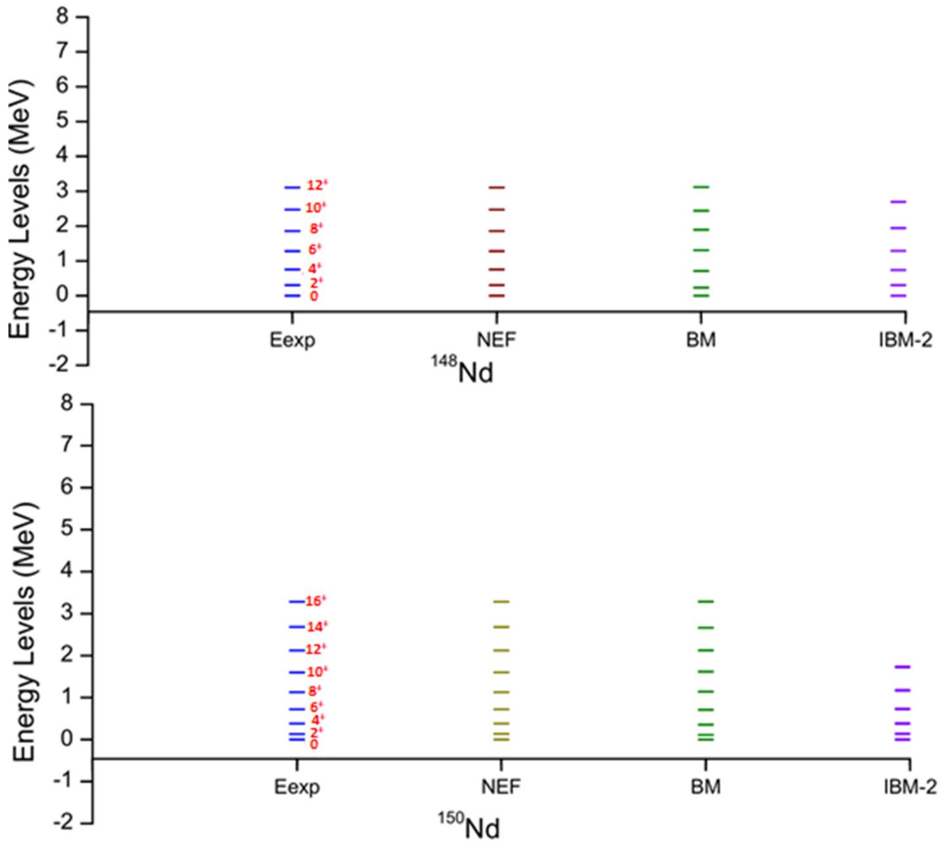

FIG. 1(b). Comparison between energy levels using NEF, BM, and IBM-2 and the experimental data [10] for even-even $^{148-150}$Nd isotopes.

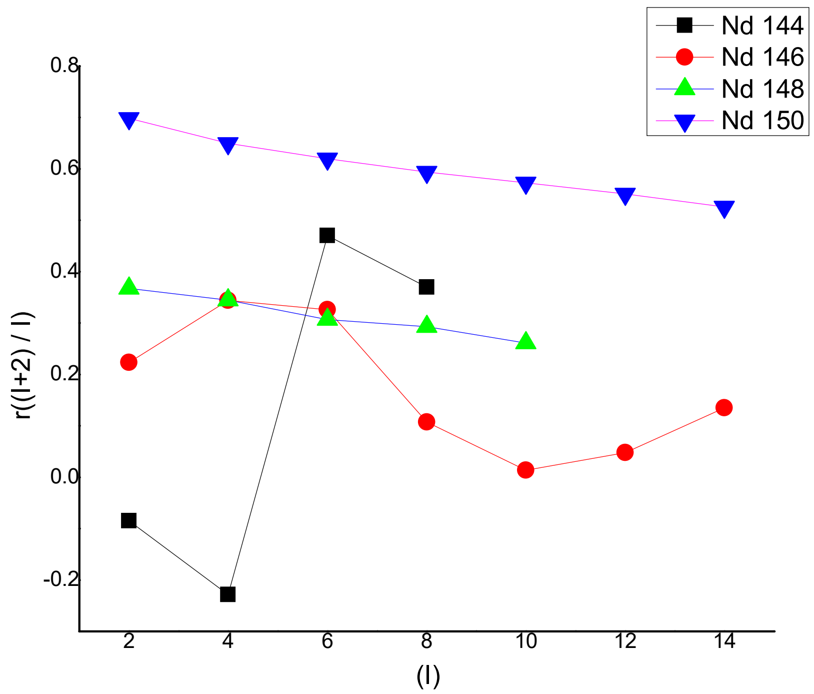

FIG. 2. The energy ratio $r\left(\frac{I+2}{I}\right)$ as function of I for even-even $^{144\text{-}150}$Nd [10].

Results and discussion

In general, the calculated energy levels of the ground-state bands for $^{144\text{-}150}$Nd isotopes are in good agreement with the experimental data, as shown in Figs. 1(a) and 1(b). It is clear from Table 6 that calculations for the ground-state band 1 obtained using the IBM-2 are closer to the experimental data than those calculated by the BM and NEF models. The results for band 2 using the BM are noticeably better than the results obtained by the NEF and IBM-2. The results of negative bands 3 and 5 and positive band 4 using the BM are approximately exact to the experimental data.

It is obvious from the comparison of the BM and NEF calculations of other positive bands (bands 6 and 7) in Table 6 that the results of the BM and NEF are in good agreement with the experimental data.

In Fig. 2, to define the symmetry for the excited band of even-even nuclei, Nd^{144}, the energy ratio started with a small value within the vibrational limit, then rocketed sharply, reaching $r\left(\frac{8}{6}\right)$ at ɣ-unstable nuclei. The energy ratio then dropped again to the vibrational limit at higher I's. The values of the energy ratios for $^{146,\ 148}$Nd were confined in the range of 0.1-0.35 and within the vibrational limit. The calculations of Nd^{150} showed a steady decrease in energy ratio, dropping from the rotational limit into ɣ-unstable nuclei.

As shown in Table 7, the calculated transitional probability using the IBM-2 gave close results for ^{144}Nd with the experimental data, while results obtained using the NEF were in good agreement with the experimental data for ^{148}Nd and ^{150}Nd.

Conclusion

The energy levels and electric quadrupole transition probabilities of the even–even $^{144\text{-}150}$Nd isotopes were investigated using the IBM-2, NEF, and BM models. The positive-parity bands were calculated using all three models (IBM-2, NEF, and BM), while the negative-parity and other excited bands were analyzed using the NEF and BM.

The results of this work show that both the BM and NEF models reproduce the available experimental data with very good agreement. The IBM-2 model also provides satisfactory agreement with experiment, particularly for spin values below 12.

The plot of the $r\left(\frac{I+2}{I}\right)$ as a function of spin I confirms that the $^{144\text{-}148}$Nd isotopes exhibit the U(5)-O(6) property, while ^{150}Nd has the shape phase transition from SU(3)-O(6) property.

The reduced electrical quadrupole transition probabilities, $B(E2)$, were also calculated, and good agreement with experimental data was obtained for ^{148}Nd and ^{150}Nd using the NEF model.

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