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ARTICLE

Role of Solitary Waveform on the Reaction Rate of Slab Reactor Geometry for Different Neutron Absorbers Using GEANT4

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Abstract: Graphite nuclear properties, such as moderating power and absorption cross-section, are not as good as those of heavy water. However, graphite can be prepared in a pure form. Its structural and thermal properties are good, and it has a high thermal conductivity. The thermal neutron in graphite performs an average of 1200 scattering collisions before it is absorbed. This very low absorption cross-section makes graphite an ideal material for applications in nuclear reactors. In the current research, graphite is assumed as a diffusive medium due to its low absorption cross-section (0.0035 barn) and its atomic mass being close to that of the neutron. In this medium, the neutron absorbers boron (^{10}B), cadmium (^{113}Cd), samarium (^{149}Sm), europium (^{151}Eu), hafnium (^{177}Hf), and gadolinium (^{157}Gd) are considered individually. The aim of this paper is to obtain the solitary waveform of the reaction rate in a graphite diffusive medium using these neutron absorbers. This work shows that hafnium has the longest transition time among the materials examined in this research. This means that, for a constant transition length, hafnium requires more time to reach a steady state. The efficiency of hafnium as a neutron absorber is determined not only by the magnitude of its thermal neutron absorption cross-section, but also by its exceptional capacity to absorb higher-energy neutrons over the energy range of their deceleration.

Keywords: Graphite, Neutron absorbers, Soliton theory, KdV equation, Inverse scattering transform, Nuclear reactor.

1. Introduction

A soliton wave is spread in the environment in such a way that its shape and size do not change with time, and it moves with group velocity in a reference frame. In fact, the soliton wave is a nonlinear wave and does not lose its structure due to interaction with other solitons. Each soliton wave packet has a maximum value. Soliton waves have been seen in scientific fields, such as rising water levels and light intensity in fiber optics. If two solitons move in the opposite direction, they pass each other without any change. Solitons can be obtained, for example, by solving the Korteweg de-Vries (KdV) and nonlinear Schrödinger equations. Soliton-type solutions resulting from solving differential equations give useful information about the dynamics of these waves.

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In the 1800s, scientists discovered the existence of soliton waves. John Scott Russell observed solitons for the first time [1]. Russell considered the soliton wave important, while many scientists disagreed with his idea. Airy and Stokes tried to describe soliton-type waves, but unfortunately, they did not find justification. Following them, Bosinsek and Riley, in 1871 and 1876, respectively, were able to provide a theoretical justification for soliton-type waves. However, discussions about soliton-type waves continued until the presentation of the famous KdV equation [2]. Finally, a complete study on soliton waves was presented by Zabusky and Kruskal [3], who analyzed the KdV equation numerically and observed their particle-like behavior.

They found that these waves do not change their velocity and shape after collision and named these waves as solitons [4]. The inverse scattering transformation process (ISTP) about solitons [5] is one of the most important discoveries in physics and mathematics [6-8]. It is worth mentioning that until the ISTP was presented to find multi-soliton solutions, some nonlinear partial differential equations (NLDEs) could not be solved analytically, which emphasizes the importance of ISTP [9-12]. Therefore, the emergence of solitons and ISTP has created a formulation for solving NLDEs. Having soliton solutions for a nonlinear partial differential equation provides exact solutions [13].

The inverse scattering transform or IST acts in precisely the same way as the Fourier transform (FT) does in linear problems; namely, it transforms the dependent variable, which satisfies a given partial differential equation (PDE), to a set of new dependent variables whose evolution in time is described by an infinite sequence of ordinary differential equations (ODEs). For special classes of PDEs, these equations are separable and hence trivially integrable. Compared with the FT, IST differs in two key aspects. First, the basis is no longer fixed (like $e^{\pm ikx}$) but moves in a way that depends on the unknown variable. Second, the spectrum (and here we are considering partial differential equations over infinite spatial intervals) no longer simply consists of the continuum of real wavenumbers k but includes, in addition, a finite number of isolated complex wavenumbers. It is the complex wave-numbers which give rise to the entities known as solitons. They are truly nonlinear quantities and have no linear analogue. Soliton theory has caused a revolution in mathematics and physics, which has led to the creation of new issues in various scientific branches, such as quantum and classical physics.

The credit for the discovery of the soliton concept belongs to John Scott Russell, who observed a special wave in the shallow waters of the British Channel in 1834. A soliton, or a solitary wave, is a type of self-amplifying wave packet that continues to propagate at a constant speed. It retains its originality. Russell noticed that this wave stops due to the obstruction of the wave propagation by the boat, but continues to move at a constant speed without losing its shape. He followed a wave for about 8 miles and found that this wave traveled at a constant speed for up to 2 miles without losing its shape. In the continuation of his research, he stated the properties of transmission waves as: (i) Waves can travel long distances at a constant speed. (ii) Unlike ordinary waves, waves never merge. (iii) The speed of a wave depends on its size, its width, and the depth of the water. (iv) Higher waves travel faster than smaller waves. (v) The speed of these waves is determined by: $V = \sqrt{G(h+A)}$, where G is the acceleration due to gravity, A is the amplitude of the soliton wave, and H is the height of the shallow water channel.

Solitons have numerous applications in both pure and applied mathematics, particularly in areas such as differential equations, Lie groups, Lie algebras, and differential geometry. Through the inverse scattering transform (IST), multi-soliton solutions of many nonlinear partial differential equations (PDEs) that cannot be solved analytically can be obtained. For example, IST has been applied to solve the initial-value problem for the Korteweg–de Vries (KdV) equation, enabling the description of the time evolution of the corresponding scattering data.

The study of solitons, in conjunction with Hamiltonian theory, has revolutionized research in mathematical physics, giving rise to new concepts and theories in diverse fields, including classical mechanics, quantum mechanics, and Lie theory. Beyond mathematics, soliton theory has been applied in fluid dynamics, plasma physics, nonlinear optics, astrophysics, and molecular biology to address important practical problems. A prominent example is in optical fiber communications, where solitons are widely used for the long-distance transmission of digital signals with minimal distortion. Apart from being used in communications, solitons are also used in optical switches. The most active research topics in soliton theory are due to its significant potential applications in information and communication technology. [14-15].

Also, solitons play a special role in the study of plasma, biology, and neutron diffusion in nuclear reactors [16-20]. In this work, we investigate neutron diffusion in diffusive media as solitary waves using diffusion theory. In fission reactors, neutron absorbers (NAs) are used to control reactivity. The reaction rate of neutrons diffusing in a pure absorbing medium is similar to a solitary wave. In the solitary wave, the absorbing material atomic density and the neutron flux spatial shape do not change during burning. The velocity of the burning wave is a function of initial neutron flux per density of NA in the diffusive media. The reason for the generation of soliton waves is the non-linearity of the equations describing the flux and atomic density dependent on space and time in the environment under investigation. Graphite, beryllium, light water, and heavy water are the most important diffusion materials. Graphite nuclear properties, such as moderating power and absorption cross-section, are not as good as those of heavy water. But its pure form can be prepared. Its structural and thermal properties are good, and it has a high thermal conductivity.

The thermal neutron in graphite performs an average of 1200 scattering collisions before it is absorbed. This very low absorption cross-section makes graphite an ideal material for applications in nuclear reactors. In the current research, graphite is assumed as a diffusive medium due to its low absorption cross-section (0.0035 barn) together with its atomic mass being close to that of the neutron. In this medium, boron (^{10}B), cadmium (^{113}Cd), samarium (^{149}Sm), europium (^{151}Eu), hafnium (^{177}Hf), and gadolinium (^{157}Gd) are studied individually as neutron absorbers. The aim of this paper is to obtain the solitary waveform of the reaction rate in a graphite diffusive medium using these neutron absorbers. Therefore, to achieve this goal, the sections that are studied in this article are listed below.

Diffusive media are turbid media where propagation occurs in a diffusive regime. In fact, propagation is dominated by multiple scattering and photon scattering events before being detected. In this article, we study the soliton behavior of neutrons in the neutron diffusive media. The diffusion theory model of neutron transport plays a crucial role in reactor theory. Here, the neutrons are characterized by a single energy or speed, and the model allows preliminary design estimates. The mathematical methods used to analyze such a model are the same as those applied in more sophisticated methods, such as multi-group diffusion theory, and transport theory.

The selection of appropriate materials for neutron absorption and radiation shielding is crucial for both industrial and research applications. One of the main requirements is to protect personnel from radiation, and in most places, the principle is that below the limit, the dose received should be "acceptably low". When using radiation sources for applications in science and technology, the definition of collimated beams is often an important requirement, and this also requires effective absorbers [21].

Recent papers describe important criteria for the selection of neutron absorbers [22, 23]. Elements such as boron, gadolinium, and samarium that may be used in combination with various polymers in terms of neutron absorption and external radiation dose have been evaluated by Castley *et al*. [22]. A review of the wide range of different materials that can be used with instrumentation for neutron scattering is presented by Stone *et al*. [23].

In Section 2, the concept of solitary waves using the KdV equation is introduced. In Section 3, soliton-soliton interaction is reviewed. In Section 4, the properties of selected neutron-absorbing media are described. In Section 5, the solitary wave variables are described. In Section 6, the solving method of the soliton wave equations of the diffusive medium for determining the neutron reaction rate is presented. In Section 7, our selected geometry of a neutron diffusive and absorbing medium for determining the numerical values of neutron reaction rate is given, and in the final Section, the conclusions are presented.

2. KdV Equation

To confirm the creation of solitons, we examine the KdV equation. The KdV equation is not only a scattered waves equation but also a nonlinear evolution equation. In fact, Scott Russell [1] observed that:

i) A soliton wave propagates in a stationary form over a smooth area without breaking its geometry or speed.

Note that a single wave is a wave that does not interact with any other waves. If these waves do not change their shape and speed after impact and remain transparent, they are called solitons [24]. Also, certain sine waves are initially divided into several soliton waves.

ii) A soliton wave continues to travel after a stationary collision, retaining its geometry and momentum.

iii) The soliton wave probably experiences a change in phase and a time delay in the interval of the collision.

The issues mentioned above must be supported by solitons. The equation that so far introduces the properties of solitons well is the KdV equation [9]:

$$\partial_t u + \beta u \partial_x u + \partial_x^3 u = 0, x \in R \tag{1}$$

where β is a real coefficient and βu is known as the velocity of propagation, which depends on the wave mode. The KdV equation accepts several exact solutions for the propagating waves as follows:

$$u = \frac{3c}{\beta} sech^2 \left[\frac{\sqrt{c}}{2}(x - ct)\right] \tag{2}$$

Here, c is the wave propagation velocity. The above equation is known as the d'Alembert solution. The solitary wave solution must preserve soliton properties, which is verified if it obeys the following equation:

$$u = \frac{72}{\beta} \frac{3 + 4cosh(2x - 8t) + cosh(4x - 64t)}{\{3cosh(x - 28t) + cosh(3x - 36t)\}^2} \tag{3}$$

The asymptotic solution for large t is given by:

$$u = \frac{12_{k_i}}{\beta} sech^2\left[k_i\left(x - 4k_i^2 t\right) + \delta_i\right] \tag{4}$$

where i = 1, 2 and $k_1 = 1, k_2 = 2$. Also, δ_i is a constant.

3. Soliton–Soliton Interaction

Research has shown that the interaction between two solitons is a function of their separation distance and relative phase [25]. Therefore, as a result of the interaction between two solitons, a bound state of two solitons may be formed. The main issue that should be mentioned here is that the relative phase of two solitons varies dynamically when the distance between them varies. When two solitons that initially have the same phase slowly get closer to each other, their relative phase varies, and therefore the force between them becomes repulsive.

On the other hand, if two solitons travel away from each other, the force they exert on each other becomes attractive and can complete its cycle. Knowing the concept that varies the relative phase between solitons is necessary to find their coupling properties. It should be noted that as the distance between solitons increases, the number of equilibrium points around which they can oscillate also increases. If the phase changes from 0 to π within half of the oscillation period, the interaction force alternates between attraction and repulsion.

It should also be emphasized that the completely "elastic" collision of solitons implies that there is no net energy exchange between them, and their fundamental parameters, such as amplitude and velocity, remain unchanged. One of the most important parameters in the dynamics of soliton–soliton interactions is the relative phase at the moment of collision. For instance, the maximum amplification amplitude observed during soliton collisions is determined by their phase synchronization. Phase synchronization also plays a crucial role in the formation of high-amplitude waves. Recent studies have analytically investigated phase synchronization in multi-soliton arrays [26–28].

4. Properties of Neutron Absorbing Media

Burnable absorbers or burnable neutron poisons are materials that are placed in the core of a fission reactor, which include fertile nuclei with a large absorption cross-section (σ_a). By absorbing neutrons, these materials significantly reduce the neutron population during the reactor's operational cycle. As the inventory of burnable absorbers decreases through irradiation, their effect on reactivity diminishes.

Burnable absorbers are particularly important for controlling reactivity in long-term fuel cycles [29–34]. They are often employed to reduce peak reactor power levels. Ideally, a burnable absorber at a rate comparable to the fuel consumption rate, thereby maintaining a relatively stable reactivity balance over time. In the operating fuel cycle, if a burnable absorber burns at a high rate, the positive feedback oscillations can exceed the limits of the fission reactor, while if the burnable absorber burns at a low rate, the remaining burnable absorber will result in negative feedback oscillations. Fuel rods containing gadolinium are applied to control the reaction rate of boiling water reactors, while boron solutions are used to control the reaction rate in the pressurized water reactors.

Since neutron absorption cross-section in burnable poisons is a function of energy, burnable absorbers are related to temperature variations. The properties of six different burnable absorber isotopes are briefly presented below. I) Boron (^{10}B): Its neutron absorption cross-section reduces exponentially with neutron energy. II) Gadolinium (^{155}Gd and ^{157}Gd): These isotopes of gadolinium are burnable absorbers with resonance energies higher than 1eV. Depending on the spectrum of the reactor, these two isotopes will have an increase in reaction rate as a function of temperature. Some gadolinium isotopes are stable, and some of them are strong neutron absorbers. ^{157}Gd has a very high thermal absorption cross-section. III) Europium (^{167}Er): This isotope has a large thermal resonance at $1eV$, which leads to increased absorption and, thus, a negative reaction rate in terms of temperature. Note that in fast reactors, B and Gd are two common burnable absorbers. One of the main challenges of isotopes is their separation, which is expensive. IV) Hafnium (Hf): Although hafnium-based materials are not very common, they are good burnable absorber candidates due to their ability to absorb neutrons with more than thermal energy. If a neutron is absorbed by hafnium, another isotope of hafnium is created, which is a neutron absorber. If a layer of hafnium dioxide is created on the surface of the Hf metal, it will show good corrosion resistance against hot water. V) Cadmium: Cd-based burnable absorbers exhibit favorable neutronic behavior in extended fuel cycles. Cd is one of the components of control rods containing Ag-In-Cd in fission reactors. Cadmium-based burnable absorbers are made as cadmium wires or cadmium oxide pellets [35-38]. However, the use of Cd-based burnable absorbers in fission reactors poses significant challenges due to the high cost of production and the need for safe handling and storage, given cadmium's health hazards. VI) Samarium: Natural samarium contains six stable isotopes, of which ^{149}Sm accounts for about 14%. This isotope is a strong neutron absorber with a large cross-section, making it particularly effective in reactivity control. In this paper, we will focus on the above burnable absorbers with their properties summarized in Table 1, where $\sigma_{a,th}$ and $\sigma_{a,fast}$ are the thermal and fast cross-sections at neutron energies of $0.025\ eV$ and $200\ keV$, respectively [39].

TABLE 1. Properties of selected burnable absorbers [39].

Burnable absorber	Symbol	$T_{1/2}$	Abundance, %	$\sigma_{a,th}(b)$	$\sigma_{a,fast}(b)$
Boron	^{10}B	stable	19.9	3844	1.47
Cadmium	^{113}Cd	$8.0 \times 10^{15} y$	12.2	19969	0.28
Samarium	^{149}Sm	stable	13.8	40150	0.70
Europium	^{151}Eu	$5.0 \times 10^{18} y$	47.8	9185	0.30
Hafnium	^{177}Hf	stable	18.5	375	0.55
Gadolinium	^{157}Gd	stable	15.7	252912	0.33

It is worth noting that some isotopes have been omitted from Table 1 because they are either highly unstable or have low neutron absorption capabilities. Figure 1 shows the absorption cross-sections of the selected burnable absorbers as a function of neutron energy [40].

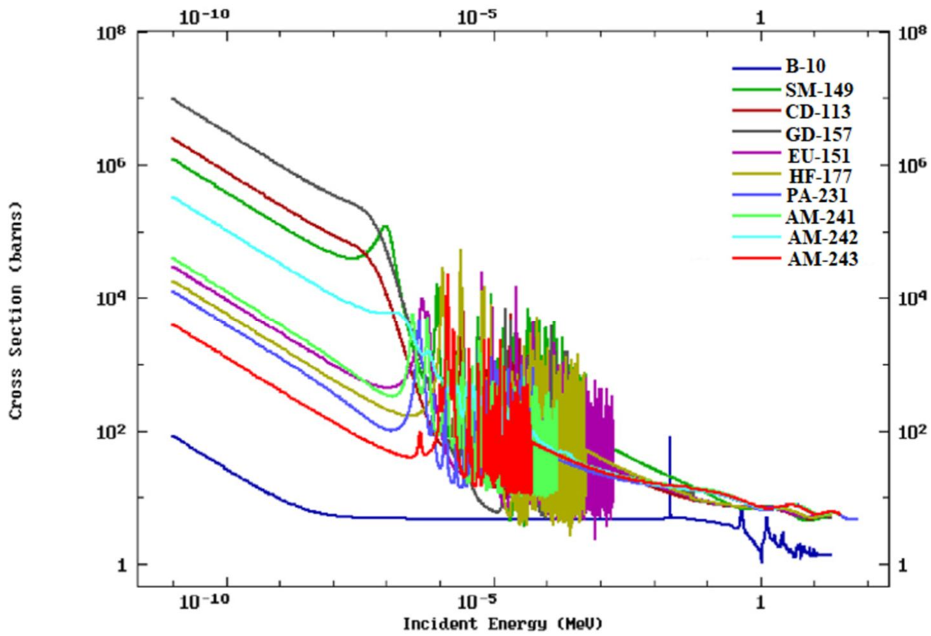


FIG. 1. Comparison of capture cross-sections [40].

5. Description of Variables

Before entering into the discussion about the absorbing and diffusive medium, it is better to get familiar with the following parameters. These parameters are general and can also be used to describe the characteristics of soliton burnup in absorbing and diffusive materials. In soliton systems, these parameters should be designed as optimal parameters.

a. Phase of Transient

1) Transient time duration (TTD): TTD is the time duration of the transient phase. For determining the TTD parameter, the following two criteria can be used: I) The time duration at which a TTD aims 95 percent of its stationary state amount. II) Time duration at which a TTD aims 99 percent of its stationary state amount. In these two cases, the transient phase value varies very fast in the initial states and very slowly as it approaches the stationary state.
2) Transient distance (TD): TD is equal to the distance the wave travels to reach its stationary state shape. Similar to TTD, TDs are known as TD 95 and 99 percent.

b. Phase of Stationary State

1) Wave speed: It is the same as the propagation speed of a burning wave, and this quantity can be interpreted as flux per nuclide density.
2) The width of the reaction rate region:

i) FWHM: The FWHM parameter equals the width of the reaction rate curve measured between half of its maximum value.
ii) 1% of FWHM: This parameter is defined as the width of the reaction rate curve measured between 1% of its maximum value.

It is worth noting that the mean free path of the burnable absorbers presented above depends on σ_a, which represents the absorption cross-section.

6. Solution of Partial Differential Soliton Wave Equations in a Diffusive Medium

In general, the neutron diffusion equation in the fission reactor is as follows:

$$D\frac{\partial^2 \phi(x,t)}{\partial x^2} - \sigma N(x,t)\phi(x,t) = \frac{1}{v}\frac{\partial \phi(x,t)}{\partial t} \qquad (5)$$

where $\phi(x,t), N(x,t), \sigma, D$, and v are the flux of neutrons, the nuclide density of neutron burnable absorber, the cross-section of absorber، the diffusion coefficient, and the neutron speed, respectively. We assume that the scattering matter does not absorb any neutrons. The right-hand side of the above equation is omitted because the time-dependence of $\phi(x,t)$ changes slowly, whereas the v variations are high. Therefore, Eq. (5) can be written in the following form:

$$D\frac{\partial^2 \phi(x,t)}{\partial x^2} - \sigma N(x,t)\phi(x,t) = 0 \qquad (6)$$

The equation of burnup for a neutron burnable absorber is given by:

$$\frac{\partial N(x,t)}{\partial t} = -\sigma N(x,t)\phi(x,t) \quad (7)$$

The required conditions for solving these equations are: $J(0,t) = J_0$, $N(x,t) = N_0$ and $\phi(100,\mathrm{t}) = \phi_0$, where $J(x,t)$ is the current density, and $\phi(x,t)$ must be zero at the boundary. The reaction rate, $R(x,t)$, is determined by the following expression:

$$R(x,t) = \sigma N(x,t)\phi(\mathrm{x},\mathrm{t}) \quad (8)$$

We can solve Eqs. (6) and (7) using the asymptotic solution. Choosing appropriate boundary conditions, we obtain the asymptotic solutions, which are as follows:

$$N_{as}(x,t) = \frac{N_0}{\left(1+\exp\left(-\frac{x-vt}{l_0}\right)\right)^{3.5}} \quad (9)$$

$$\varphi_{as}(x,t) = \frac{v\sqrt{2}}{\sigma l_0}\left(\frac{N}{N_0} - 1 - \ln\left(\frac{N}{N_0}\right)\right)^{\frac{1}{2}} \quad (10)$$

and the asymptotic form of reaction rate density is given by:

$$R_{as}(x,t) = \frac{v\sqrt{2}}{l_0} N_{as}\left(\frac{N_{as}}{N_0} - 1 - \ln\left(\frac{N_{as}}{N_0}\right)\right)^{\frac{1}{2}} \quad (11)$$

Here, l_0, represents the initial diffusion length of neutrons in the medium. The diffusion length is obtained using the following equation:

$$l_0 = \sqrt{\frac{D}{N_0\sigma}} \quad (12)$$

For example, for boron as a neutron absorber, the diffusion length is L = 5 cm. In this study, we consider a graphite diffusive medium with six neutron absorbers: boron (^{10}B), cadmium (^{113}Cd), samarium (^{149}Sm), europium (^{151}Eu), hafnium (^{177}Hf), and gadolinium (^{157}Gd), each studied separately. In order to make a better comparison, we consider the diffusion length of the media under study to be 5 cm, similar to the diffusion length of graphite and boron neutron absorbers, and we will calculate the density of different neutron absorbers according to Table 2. It should be noted that in all environments, the net incoming neutron flow is 10^{14}cm^{-2} s^{-1}.

TABLE 2. Properties of six neutron absorbers in a graphite diffusive medium.

Element	^{10}B	^{113}Cd	^{149}Sm	^{151}Eu	^{177}Hf	^{157}Gd
Burnup wave speed (cm/day)	0.83	4.55	8.80	2.0	0.09	54
Diffusion length (cm)	5	5	5	5	5	5
Initial absorber density (cm^{-3})	1.04×10^{19}	19×10^{17}	9.8×10^{17}	44×10^{17}	11×10^{19}	1.58×10^{17}

7. Geometry of the Neutron Diffusive and Absorbing Medium

It is assumed that the diffusive medium is graphite, and no neutrons are absorbed through this medium. In order to simplify the matter under discussion, the geometrical shape of our system is considered as a finite slab. In this study, the neutron flux is calculated using a one-dimensional diffusion equation along the xxx-direction, with a slab length of 100 cm. This length is chosen so that variations along the reactor core height can be neglected (see Fig. 2). It should be noted that the authors of this article have chosen the geometry presented in Ref. [41].

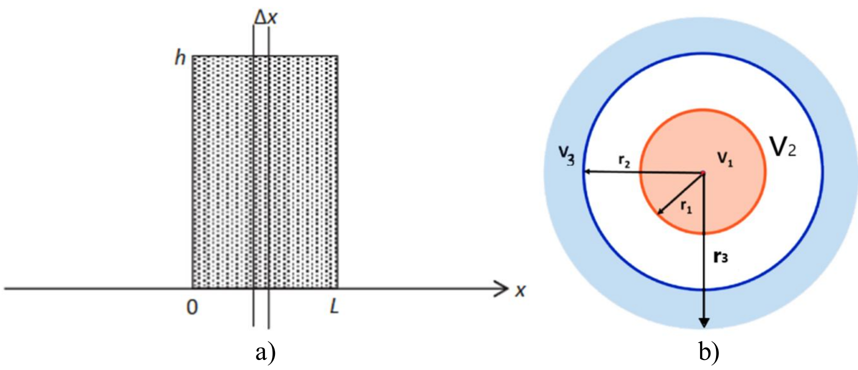


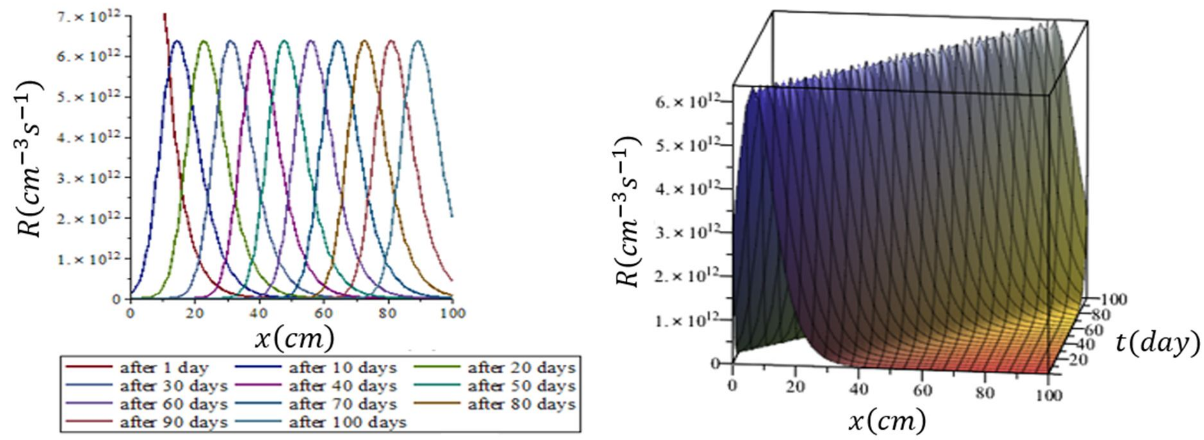
FIG. 2. (a) The selected slab geometry for the reactor core, with height h and width L; (b) the regions of the nuclear fuel cell design.

The nuclear fuel cell geometry is cylindrical and is divided into three regions: fuel, cladding, and coolant, as shown in Fig.1(b). The reactor core design is constructed using several meshes Δx and boundary conditions. The selected parameters for this calculation are: natural absorber impurities in uranium = 4 ppm, natural absorber impurities in graphite = 1.3 ppm, density of graphite in the fueled zone = 1.73 g/cc, and UO_2 density = 10.4 g/cm$^{3.}$

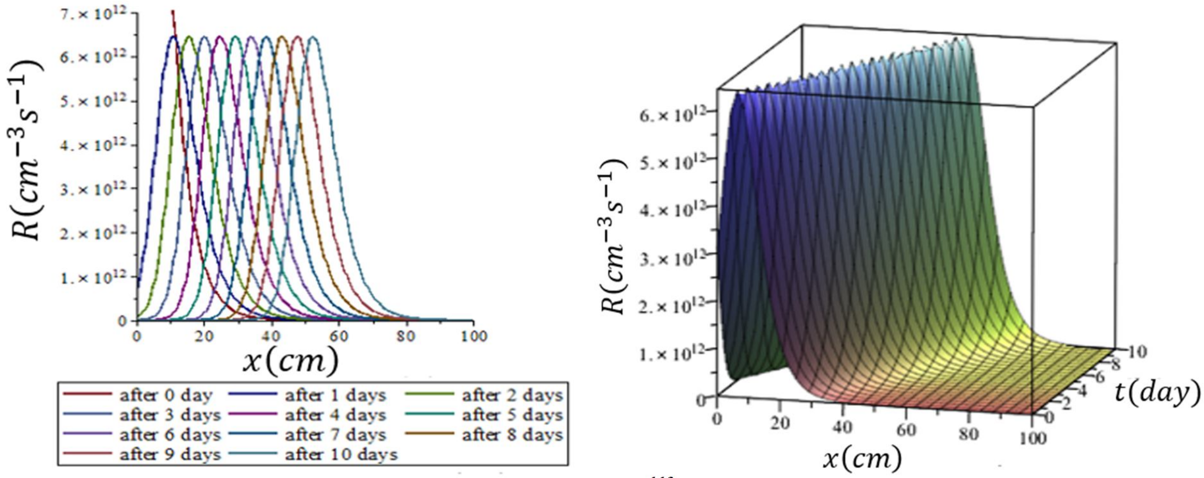
It should be noted that the *GEANT4*-11.2.2. code was used in this work, and the graphs were drawn with the help of MAPLE.20 programming. Also, we used the parameters and specifications of nuclear fuel cell design from Table 1 of Ref. [41]. In Monte Carlo simulation methods, neutrons are tracked individually from emission to their final interaction or removal by any process, including leakage. This method is able to treat complex geometries with high resolution and fidelity. With the need for accurate modeling in the physics and dynamics of reactors and the great innovation in computer technology, the Monte Carlo method is becoming a more powerful tool and attracting increasing attention. That is why we used this method in this article. In general, calculations for thick samples rely on empirical expressions and numerical methods such as Monte Carlo computer simulation codes.

In Fig. 3, we plotted the two- and three-dimensional variations of the reaction rate density in solitary wave form for six selected neutron-absorbing media, as a function of space and time. Figure 3(a) shows boron (^{10}B) plotted at an interval of 10 days and covers a period of 100 days. Figure 3(b) presents cadmium (^{113}Cd) at 1-day intervals over 10 days, while Figs. 3(c) and 3(d) illustrate samarium (^{149}Sm) and europium (^{151}Eu), respectively, both plotted at 1-day intervals over a 10-day period. Hafnium (^{177}Hf) is depicted in Fig. 3(e) at 10-day intervals over 100 days, and gadolinium (^{157}Gd) is shown in Fig. 3(f), plotted at 0.1-day intervals over a 1-day period. According to the applied boundary conditions governing the proposed geometry, the peak reaction rate (R) for all neutron absorbers is $\sim 6.5 \times 10^{12} cm^{-2} s^{-1}$. Our numerical calculations indicate that Hafnium has the longest transition time among the examined materials. This means that, for a constant transition length, Hafnium requires more time to reach a steady state. Its efficiency as a neutron absorber is determined not only by its thermal neutron absorption cross-section but also by its exceptional ability to absorb higher-energy neutrons during their deceleration.

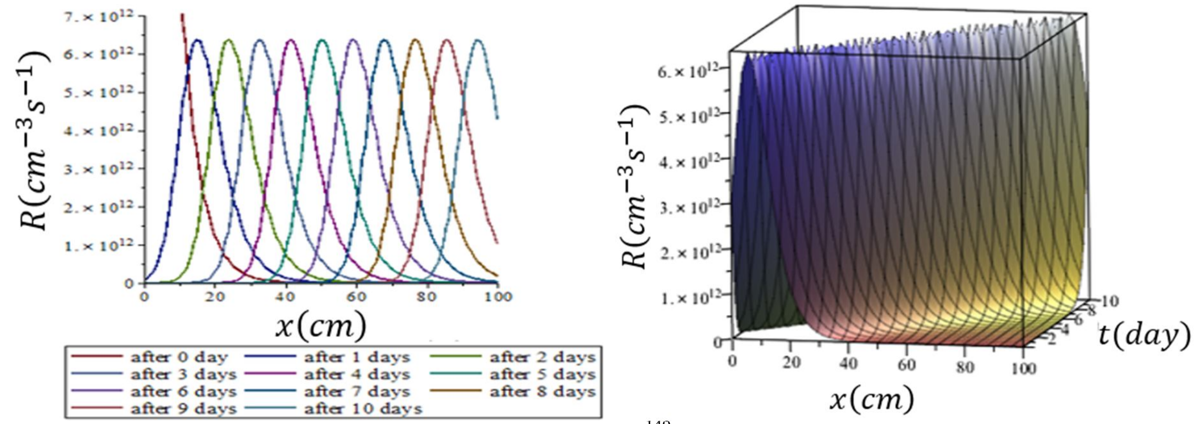
From the comparison of the graphs in Fig. 3, it is evident that all the curves physically follow the transient and stationary-state phases described in Section 5. The figures show the gradual development of the asymptotic (steady-state) burnup wave through the transient phase. During this phase, the shapes of the neutron flux and nuclide density evolve until they attain their steady-state forms. (Note: To better illustrate the role of neutron absorbers, both 2D and 3D diagrams are provided.)



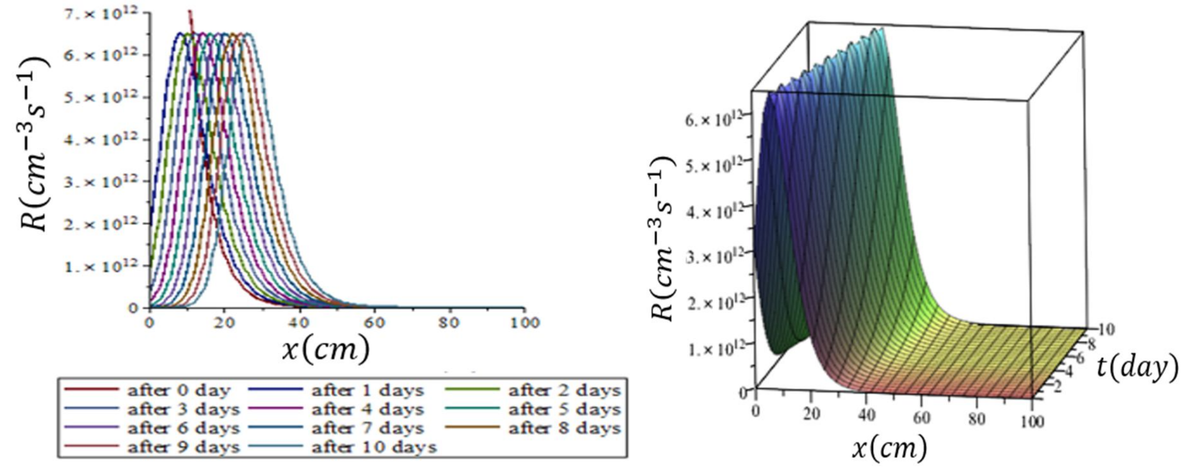
(a) Boron (^{10}B)



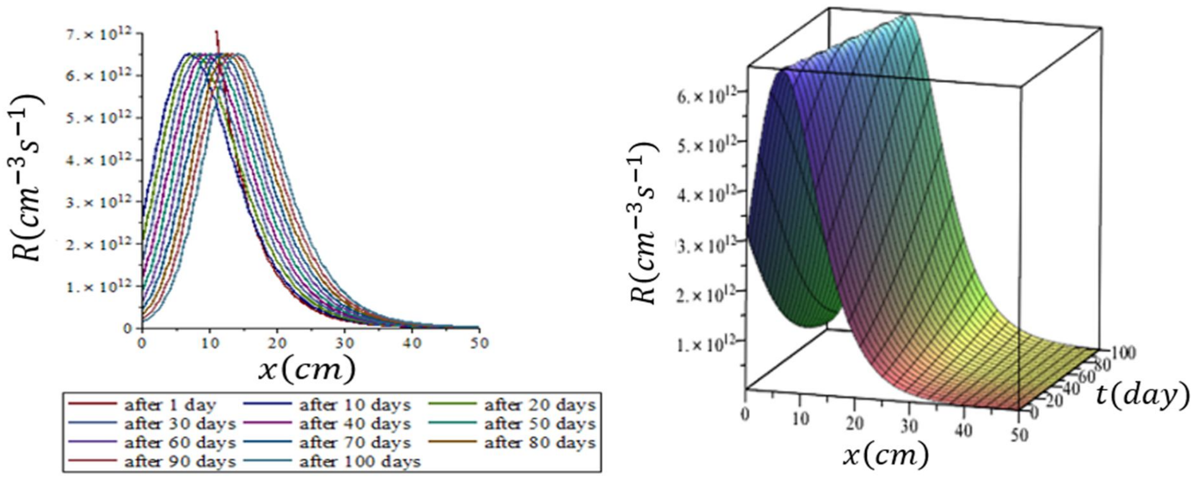
(b) Cadmium (^{113}Cd)



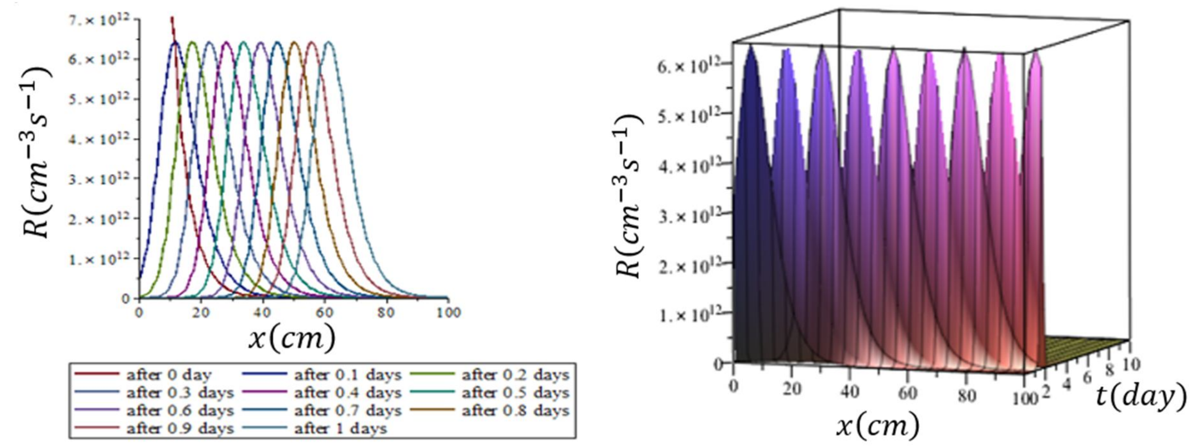
(c) Samarium (^{149}Sm)



(d) Europium(^{151}Eu)



(e) Hafnium(^{177}Hf)



(f) Gadolinium (^{157}Gd)

FIG. 3. Two and three-dimensional variations of the reaction rate for different neutron-absorbing media in terms of x and t.

Our numerical results show that, as neutrons gradually penetrate the medium, the primary nuclides of the absorbing material begin to be consumed, leading to a decrease in the density of the neutron-absorbing nuclides. This action intensifies over time and continues until the end. The transition time depends on the σ_a. As σ_a decreases, the transition time increases. Our numerical calculations show that hafnium has the longest transition time among the materials examined in this research. It means that with the constant length of the transition among these materials, hafnium spends more time reaching a steady state. From the plotted diagrams in Fig. 2, we find that the reaction rate distribution quickly approaches the asymptotic form given in Eq. (11). The transition time for the cadmium, samarium, europium, and gadolinium neutron absorbers is less than that of boron and hafnium. For this reason, we have considered a period of ten days to draw the diagrams for boron and hafnium. From these figures, we calculated the special properties of the solitary burnup wave for selected absorbers in a graphite diffusive medium, and we listed them in Table 3.

TABLE 3. Properties of the solitary burnup wave for the selected absorbers in a graphite diffusive medium.

Element	^{10}B	^{113}Cd	^{149}Sm	^{151}Eu	^{177}Hf	^{157}Gd
Initial absorber density (atoms/cm^3)	1.04×10^{19}	19×10^{17}	9.8×10^{17}	44×10^{17}	11×10^{19}	1.58×10^{17}
Speed simulation (cm/day)	0.83	4.6	8.8	2	0.08	55
Speed analytical	0.83	4.6	8.8	2	0.09	54

Element	^{10}B	^{113}Cd	^{149}Sm	^{151}Eu	^{177}Hf	^{157}Gd
(cm/day)						
Transient time duration (days)	20.7	3.4	1.75	7.49	184.5	0.3
Characteristics time, τ (days)	6	1.11	0.56	2.52	61.4	0.1
Transient time duration / Characteristics time	3.4	3.1	3.1	3	3	3
FWHM (cm)	14	14	14.1	14	13.9	14
FWHM/ Diffusion length	2.8	2.8	2.82	2.8	2.78	2.8

8. Conclusions

In this work, solitary burnup waves in a graphite diffusion medium containing six pure neutron absorbers, namely boron (^{10}B), cadmium (^{113}Cd), samarium (^{149}Sm), europium (^{151}Eu), hafnium (^{177}Hf), and gadolinium (^{157}Gd), were examined separately. The results of our studies show that the speed of the burnup solitary wave in the desired medium is a function of transient time, transient length until it finally reaches its asymptotic form, and the width of the propagation zone (FWHM) of soliton waves in the environment under investigation. For the first time, this article examines the conditions of soliton wave development and its related parameters in reactor design. The results of our calculations about the characteristics of the burnup solitary wave in the graphite diffusion environment and the different mentioned neutron absorbers are classified into two groups:

Group 1. a: The estimated burnup wave solitary speed is independent of the cross-section of the absorber (σ_a), b: when σ_a increases, the transient length increases, c: transient time decreases with the increase of σ_a, and d: the burnup solitary wave FWHM grows with the mean free path of neutron absorption.

Group 2. a: The estimated burnup solitary wave speed is independent of σ_a and D (diffusion coefficient), b: when the D increases, the TD increases; we also found that TD per D does not change, c: TTD grows with increasing D, TTD per characteristic time does not change, and d: the burnup solitary wave FWHM grows with D. Our numerical calculations reveal that hafnium exhibits the longest transition time among the studied materials. With a constant transition length, hafnium requires more time to reach steady state due to its ability to absorb not only thermal neutrons but also higher-energy neutrons during their deceleration. Boron, particularly the ^{10}B isotope, is also effective due to its high thermal absorption cross-section and natural abundance (about 20%), allowing unenriched compounds to be used efficiently as absorbers.

Recent studies have highlighted important criteria for selecting neutron absorbers [22, 23]. Elements such as boron, gadolinium, and samarium, when combined with polymers, offer advantages in neutron absorption and reduction of external radiation dose, as evaluated by Castley *et al.* [22]. For materials dominated by absorption, increasing the cross-section reduces transmission, while scattering and diffraction can create features such as the Bragg slope. Neutron-absorbing polymer composites provide good mechanical and thermal properties, enabling fabrication of complex shapes for diaphragms, baffles, and collimators.

Based on these findings, further research is recommended to study solitary waveforms of reaction rates in other diffusive media using neutron-absorbing polymer composites and alternative reactor core geometries.

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