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ARTICLE

Dust Ion Acoustic Solitary Waves in A Relativistic Ion Plasma with Kappa Described Electrons and Positrons

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Abstract: The propagation features of ion-acoustic solitary waves in a dusty plasma containing relativistic ions, as well as kappa-distributed electrons and positrons, are studied. The negative dust is defined by q-nonextensive distribution. The Korteweg-de Vries (KdV) equation is formulated using the reductive perturbation technique to study the evolution of dust-ion-acoustic solitary waves (DIASW). The variations in amplitude, width, and phase velocity of solitary waves along with spectral index κ , non-extensive parameter q, and relativistic factors are studied. We observed that the persistence of dust particles enhances the strength of solitary structures formed in relativistic plasma. The relativistic factors and the spectral indices of electrons and positrons enhance the amplitude of solitary structures. Our results may be useful for studying the dynamics of solitary structures existing in any astrophysical dusty plasma composed of relativistic streaming ions, such as those found in interplanetary space and the relativistic winds of pulsars.

Keywords: Dust-ion-acoustic solitary waves, Kappa distribution, q-nonextensive distribution, Korteweg-de Vries (KdV) equation.

1. Introduction

Over the past few decades, the studies regarding the nonlinear wave propagation in various kinds of multicomponent astrophysical and space plasma systems have steadily grown, which is also the case for relativistic dusty plasmas. Nonlinear waves such as solitons, shocks, super solitons, and double layers have been investigated in these plasma environments. A soliton is characterized by the balance between nonlinearity and dispersion, which counteract each other equally, allowing the wave to maintain a constant speed and amplitude. In contrast, shocks possess an additional dissipative property.

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The perusal of multicomponent plasmas, such as electron-ion (e-i), electron-positron (e-p), electron-positron-ion (e-p-i), and electron-positron-ion-dust (e-p-i-d) plasmas, is helpful in the understanding of early universe, atmospheres of various stages of stars, interstellar medium, and galactic environments in which they are found [1, 2]. These plasmas can also be generated and analyzed in laboratory settings using high-intensity lasers. Under intense laser-plasma interactions, ions and electrons can be accelerated to relativistic speeds, enabling the study of relativistic astrophysical plasmas [3].

Relativistic magnetized e-p-i plasma is observed in pulsar wind nebulae and black hole magnetospheres [4]. Here, the non-thermal electrons and positrons are highly energetic, occupying the tail region of the Maxwellian velocity distribution. As a result, they are described using the nonthermal kappa distribution function. Shah *et al.* derived the Korteweg-de Vries (KdV) equation for solitons in a non-thermal e-p-i plasma composed of kappa-distributed electrons and positrons, along with weakly relativistic ions. They found that soliton amplitude increases with the cold ion streaming factor and kappa parameters but decreases with positron concentration [5].

By deriving the modified Korteweg-de Vries (mKdV) equation, Das *et al.* studied higher-order DIASWs in weakly relativistic dusty plasma. According to their research, an increase in dust charge causes a corresponding increase in the amplitude of DIASWs [6].

In a recent research, Das and Das studied the impact of various parameters on relativistic DIASWs using the relativistic factor within the framework of the Kaniadakis distribution (KD). Their findings indicate that DIASWs are observed only within a specific range of dust concentrations [7].

The reductive perturbation method has been employed to establish the coexistence of both compressive and rarefactive solitons in electron-positron-ion (e-p-i) plasma consisting of thermal positrons, nonthermal electrons, and highly relativistic thermal ions [8]. It has been observed that variations in ion species temperature significantly influence the fundamental characteristics of solitary wave amplitude and width.

Madhukalya *et al.* investigated ion-acoustic solitary disturbances in a magnetized plasma composed of relativistic electrons and nonthermal ions [9]. By deriving the Sagdeev potential equation, they demonstrated that in a magnetized relativistic plasma, only rarefactive solitons exist under both subsonic and supersonic conditions.

Another significant study on e-p-i plasma, which includes highly relativistic thermal ions, nonthermal electrons, and thermal positrons, also confirmed the existence of only rarefactive solitons, particularly in the fast ion-acoustic mode [10].

Plasma environments that contain dust grains enable electrons or ions to collide with them, causing the dust to acquire either a negative or a positive charge. Charged dust and dusty plasmas are ubiquitous in the cosmos. Dusty plasmas occur in planetary rings, Earth’s mesosphere, comet tails, the interstellar medium, zodiacal dust clouds, and solar nebulae [11-13]. Dust plays a significant role in astrophysics as it is responsible for the formation of stars and planets.

The existence of electrostatic dust-acoustic (DA) solitons and periodic waves in Saturn’s magnetosphere has been analyzed using data acquired from the Voyager space mission and the Freja satellite [14]. The Korteweg-de Vries-Burgers (KdVB) equation, which governs small-amplitude nonlinear DA waves in a collisionless, unmagnetized, dissipative dusty plasma with superthermal electrons, has been derived and examined by Hanbaly et al. Additionally, the Sagdeev potential method has been used to investigate large-amplitude DA waves in the system [15].

Dust is also observed in relativistic astrophysical environments such as accretion disks around rotating black holes and neutron stars, active galactic nuclei, stellar winds, supernova explosions, and pulsar magnetospheres [16–18]. The Zakharov–Kuznetsov (ZK) equation has recently been analyzed for a relativistic magnetized four-component plasma consisting of thermal ions, nonthermal electrons and positrons, and negatively charged dust. It was found that the Mach number of dust-ion acoustic (DIA) solitons lies within the supersonic range and is influenced by the nonthermal properties of both electrons and positrons, positron density, dust density, and relativistic effects associated with ions [19].

The characteristics of compressive and rarefactive DIA solitons have been examined in a plasma model containing weakly relativistic ions, negative dust, and electrons [20]. Furthermore, Banerjee and Maitra employed Sagdeev's pseudo-potential approach to establish the concurrence of negative and positive potential double layers and solitary waves in a nonthermal, unmagnetized dusty plasma [21]. Their study revealed that this phenomenon occurs within a specific range of the nonthermal parameter at very low positron densities.

DIASWs have been examined using the kappa distribution of electrons and positrons in weakly relativistic complex plasma. The effects of the relativistic factor and superthermal parameters on higher-order phase shifts of the wave have been studied in the context of pulsars [22]. Ion-acoustic solitons in an electron-positron-ion (e-p-i) plasma with weakly relativistic cold ions have been investigated using the Korteweg-de Vries (KdV) equation, revealing that the nonextensive parameter of q-distributed electrons causes the existence of only compressive solitons [23]. By deriving the Burgers equation, the DIA multi-shock waves have been explored in relativistic dusty plasmas containing positrons, q-nonextensive electrons, and stationary dust [24]. Some of the recent investigations by Khater have provided in-depth analyses of solitary waves in various plasma contexts using advanced mathematical methods. His work offers analytical solutions to equations describing solitary waves, which may accurately explain the stability and properties of nonlinear waves in quantum and magnetized multicomponent plasmas, as well as in multi-dimensional settings [25–31].

In this study, we discuss the propagation characteristics of small-amplitude solitary waves in a dusty plasma composed of relativistic ions, kappa-distributed electrons, and positrons by deriving the KdV equation. Our results may contribute to ongoing research on relativistic plasmas in interstellar and space environments.

2. Theoretical Model and Governing Equations

Our investigation involves the mathematical modelling of unmagnetized, collisionless, tetra-component e-p-i-d plasma consisting of superthermal electrons and positrons, weakly relativistic cold ions, and dust grains which are negatively charged. The highly energetic electrons are kappa-distributed, while the dust particles are q-nonextensively distributed.

The quasineutrality condition is written as:

$$n_{i0} + n_{p0} - z_d n_{d0} - n_{e0} = 0 \qquad (1)$$

where n_{i0}, n_{p0}, n_{d0}, and n_{e0} are equilibrium densities of ions, positrons, dust, and electrons, respectively. All normalizations are done using the electron. Normalizing the densities of the plasma species using n_{e0}, we obtain:

$$\frac{n_{i0}}{n_{e0}} = 1 - p + d \qquad (2)$$

where $p = \frac{n_{p0}}{n_{e0}}$ is the normalized positron density and $d = \frac{z_d n_{d0}}{n_{e0}}$ is the normalized dust density.

The normalised fluid equations governing mass and momentum conservations for relativistic ions are given by:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \qquad (3)$$

$$\frac{\partial (\gamma u_i)}{\partial t} + u_i \frac{\partial (\gamma u_i)}{\partial x} + \frac{\partial \Phi}{\partial x} = 0 \qquad (4)$$

The normalized Poisson's equation is:

$$\frac{\partial^2 \Phi}{\partial x^2} = n_e - p n_p - (1 - p + d) n_i + d n_d \qquad (5)$$

In Eqs. (3)–(5), we normalize the electrostatic potential using the quantity T_e/e, where e is the charge of electron. The spatial variable x is normalized using the electron Debye length $\lambda_{De} = \sqrt{\frac{T_e}{4\pi n_{e0} e^2}}$, the ion velocity u_i is normalized using the acoustic speed $c_s = \sqrt{\frac{T_e}{m_e}}$, and the time variable is normalized using the inverse of the electron plasma frequency $\omega_{pe}^{-1} = \sqrt{\frac{m_e}{4\pi n_{e0} e^2}}$. In Eq. (4), the relativistic streaming factor is given by: $\gamma = \left(1 - \frac{u_i^2}{c^2}\right)^{-1/2}$, with c being the speed of light. It is approximated to $\left(1 + \frac{u_i^2}{2c^2}\right)$ for weakly relativistic ions [5].

The normalized kappa distributions for electrons and positrons are given by:

$$n_e = \left[1 - \frac{\Phi}{(\kappa_e - \frac{3}{2})}\right]^{-\kappa_e + \frac{1}{2}} \qquad (6)$$

$$n_p = \left[1 + \frac{\delta\Phi}{(\kappa_p - \frac{3}{2})}\right]^{-\kappa_p + \frac{1}{2}} \tag{7}$$

and the q-nonextensive distribution for dust is:

$$n_d = [1 + \sigma\,(q-1)\,\Phi\,]^{\frac{3q-1}{2(q-1)}} \tag{8}$$

Here, $\delta = T_e/T_p$ and $\sigma = T_e / T_d$, where T_e, T_p, and T_d denote the temperatures of electrons, positrons, and dust, respectively. The q-nonextensive distribution approaches the Maxwellian distribution in the limit $q \to 1$ [32]. Also, within the limit of $\kappa_e = \kappa_p \to \infty$, the kappa distribution reduces to the Maxwellian velocity distribution [33].

3. Derivation of the KdV Equation

Here, we formulate the KdV equation for small-amplitude solitary waves travelling in one dimension using the reductive perturbation method. This involves transforming the space and time coordinates and expanding various parameters [5]. They are given as:

$$\xi = \varepsilon^{\frac{1}{2}}(x - \lambda_0 t), \tau = \varepsilon^{\frac{3}{2}} t \tag{9}$$

and

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + \cdots \tag{10}$$

$$u_i = u_0 + \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \varepsilon^3 u^{(3)} + \cdots \tag{11}$$

$$\Phi = \varepsilon\Phi^{(1)} + \varepsilon^2\Phi^{(2)} + \varepsilon^3\Phi^{(3)} + \cdots \tag{12}$$

Here, λ_0 denotes the Mach number or phase velocity of the soliton, and ε is a small parameter representing the perturbation magnitude in the system. By substituting Eqs. (6)–(12) into Eqs. (3)–(5) and equating the lowest-order terms of ε, we obtain the following relations:

$$u^{(1)} = \frac{\Phi^{(1)}}{\gamma_1(\lambda_0 - u_0)};\ n_i^{(1)} = \frac{\Phi^{(1)}}{\gamma_1\,(\lambda_0 - u_0)^2} \tag{13}$$

The phase velocity of the soliton is given y

$$\lambda_0 = u_0 + \sqrt{\frac{2(2\kappa_e-3)(2\kappa_p-3)(1-p+d)}{\gamma_1(2(2\kappa_e-1)(2\kappa_p-3)+2p\delta(2\kappa_p-1)(2\kappa_e-3)+d\sigma(3q-1)(2\kappa_e-3)(2\kappa_p-3))}} \tag{14}$$

By equating the next highest order of ϵ\epsilonϵ, we derive the following set of equations:

$$\frac{\partial n_i^{(1)}}{\partial\tau} - (\lambda_0 - u_0)\frac{\partial n_i^{(2)}}{\partial\xi} + \frac{\partial u^{(2)}}{\partial\xi} + \frac{\partial}{\partial\xi}\left(n_i^{(1)}u^{(1)}\right) = 0 \tag{15}$$

$$\gamma_1\frac{\partial u^{(1)}}{\partial\tau} - (\lambda_0 - u_0)\gamma_1\frac{\partial u^{(2)}}{\partial\xi} + \left(\gamma_1 - 2\gamma_2\frac{\lambda_0 - u_0}{u_0}\right)u^{(1)}\frac{\partial u^{(1)}}{\partial\xi} + \frac{\partial\Phi^{(2)}}{\partial\xi} = 0 \tag{16}$$

$$\frac{\partial^2\Phi^{(1)}}{\partial\xi^2} - \left\{\frac{2\kappa_e-1}{2\kappa_e-3} + \frac{2\kappa_p-1}{2\kappa_p-3}p\delta + \frac{3q-1}{2}d\sigma\right\}\Phi^{(2)} + \left\{\frac{4\kappa_e^2-1}{(2\kappa_e-3)^2} - \frac{4\kappa_p^2-1}{(2\kappa_p-3)^2}p\delta^2 + \frac{(3q-1)(q+1)}{4}\sigma^2 d\right\}\frac{[\Phi^1]^{(2)}}{2} + (1-p+d)n_i^{(2)} = 0 \tag{17}$$

where $\gamma_1 = 1 + \gamma_2, \gamma_2 = 1.5\beta^2$, and $\beta = u_0/c$, which is the relativistic cold-ion streaming factor.

By substituting Eq. (13) into Eqs. (15)–(17) and simplifying by removing second-order terms we get the KdV equation for DIASW in our e-p-i-d plasma model::

$$\frac{\partial\Phi^{(1)}}{\partial\tau} + A\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + B\,\frac{\partial^3\Phi^{(1)}}{\partial\xi^3} = 0 \tag{18}$$

where

$$A = \frac{1}{\gamma_1(\lambda_0-u_0)} - \frac{\gamma_1(\lambda_0-u_0)^3}{2(1-p+d)}\left\{\frac{4\kappa_e^2-1}{(2\kappa_e-3)^2} - \frac{4\kappa_p^2-1}{(2\kappa_p-3)^2}p\delta^2 + \frac{(3q-1)(q+1)}{4}\sigma^2 d\right\} + \frac{1}{2\gamma_1^2(\lambda_0-u_0)}\left(\gamma_1 - 2\gamma_2\frac{\lambda_0-u_0}{u_0}\right) \tag{19}$$

and

$$B = \gamma_1\frac{(\lambda_0-u_0)^3}{2(1-p+d)} \tag{20}$$

Here, A is the coefficient of nonlinearity, and B is the coefficient of dispersion.

4. Solution of the KdV Equation

The standard solution of the KdV Eq. (20) is given in [5, 34] as:

$$\Phi^{(1)}(\zeta) = \Phi_0\,sech^2\left(\frac{\xi - v\tau}{\Delta}\right) \tag{21}$$

where the new stretched variable is $\zeta = \xi - v\tau$, and $\Phi^0 = \frac{3v}{A}$ is the amplitude of soliton, while $\Delta = \sqrt{\frac{4B}{v}}$ represents the soliton width. Here, v is the constant speed of the soliton.

The amplitude of the soliton is inversely proportional to the nonlinearity coefficient A, and its width has a direct relationship with the dispersive effects.

5. Results and Discussions

We thoroughly execute the numerical analysis of the DIASW solution [Eq. (21)] in our e-p-i-d plasma model.

Figure 1 represents the variation of the solitary profile as a function of the relativistic streaming factor (β) of ions. The parameters used for this analysis are δ = 1, p = 0.2, $\kappa_e = \kappa_p = 3$, d = 0.2, σ = 0.3, and q = 0.2. It can be seen that as the relativistic streaming factor of ions increases, the solitary wave becomes more pronounced. This behavior can be attributed to a decrease in the nonlinearity coefficient A due to the rise in the relativistic streaming factor. Our findings are consistent with the results reported by Saed *et al*. and El-Wakil *et al*. [35, 36].

In Fig. 2, the influence of spectral index κ_e of electron on the solitary profile is studied. Here, we fix $\kappa_p = 3$ and β = 0.1. All other parameters are the same as in Fig.1. The results clearly show that an increase in κ_e enhances both the amplitude and width of the solitary wave. This occurs because a higher κ_e value leads to an increase in electron pressure, which provides the necessary restoring force for the solitary wave. The stronger restoring force, in turn, contributes to the growth of the soliton amplitude.

Similarly, an increase in the positron kappa index (κ_p) also strengthens the soliton, as depicted in Figure 3. Notably, smaller values of the kappa indices indicate a higher presence of suprathermal particles in the plasma. Consequently, suprathermal electrons and positrons significantly influence the solitary wave profile. The solitary waves reach their maximum strength when the plasma approaches a Maxwellian distribution ($\kappa_e = \kappa_p \to \infty$).

Figure 4 depicts the variation in the solitary wave profile with respect to the normalized positron density (p). It is evident that an increase in positron concentration weakens the solitary wave structures. A higher positron concentration reduces the positive ion density in the plasma, thereby diminishing the driving force of the ion acoustic solitary wave. This reduction in driving force ultimately leads to a decrease in the strength of the solitary wave profile [5].

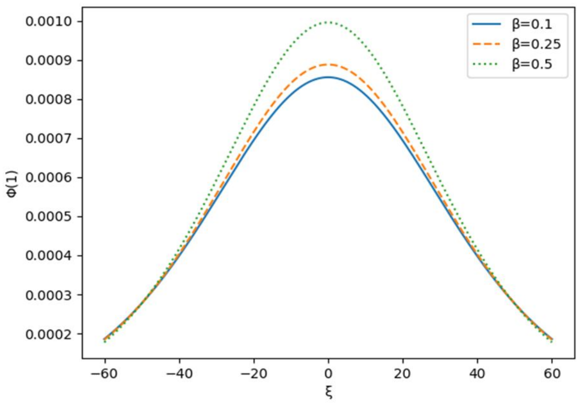

FIG. 1. Change in the soliton profile as a function of the relativistic streaming factor of ions (β).

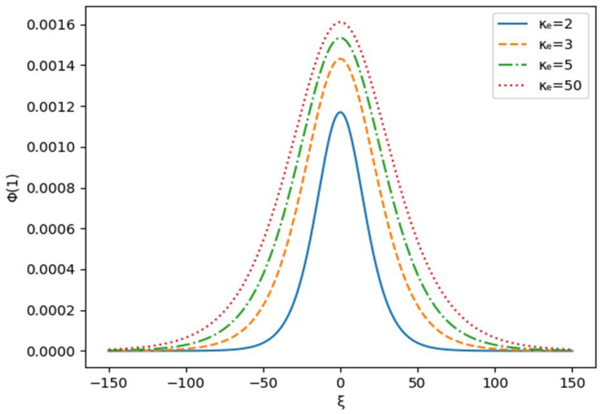

FIG. 2. Change in the soliton profile as a function of spectral indices of electron (κ_e).

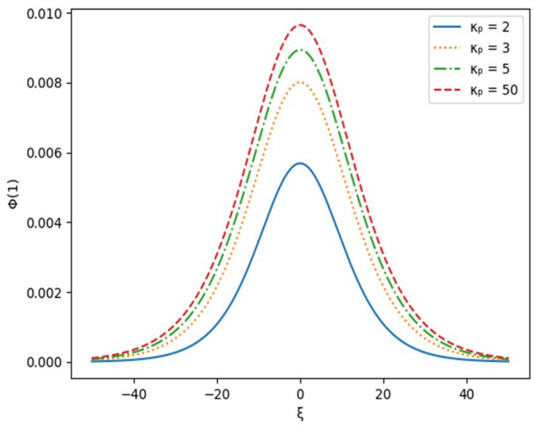

FIG. 3. Change in the soliton profile as a function of spectral indices of positron (κ_p).

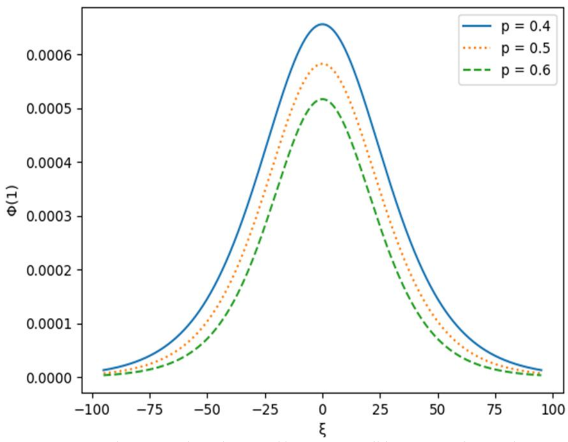

FIG. 4. Change in the soliton profile as a function of normalized positron density p.

Figure 5 shows variations in the solitary profile with respect to the electron-to-positron temperature ratio ($\delta = T_e / T_p$) for a relativistic parameter $\gamma_1 = 0.2$. We observed that the strength of the solitary profile decreases along with increasing electron-to-positron temperature ratios. The solitary structure exhibits its maximum amplitude at lower electron temperatures. This behavior can be explained by the fact that a decrease in electron temperature reduces the system's nonlinearity, as the solitary wave amplitude is inversely related to the nonlinearity coefficient A. This variation is similar to the results obtained by Saed *et al*. and El-Wakil *et al*. [35, 36].

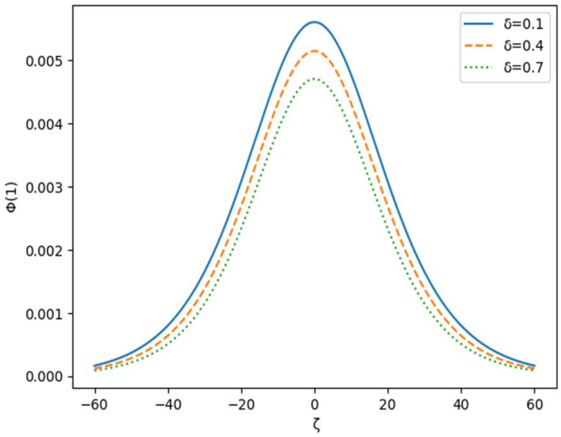

FIG. 5. Change in the solitary profile as a function of electron to positron temperature ratio δ.

Figure 6 illustrates the change in the nonlinearity coefficient A with the relativistic parameter γ_1 as a function of the q-nonextensive parameter of dust particles. We found that the nonlinearity coefficient A increases along with an increase in q and decreases with the relativistic parameter γ_1. This implies that higher relativistic velocities of ions reduce the nonlinearity coefficient A, thereby enhancing the amplitude of solitary waves at higher relativistic limits, as the amplitude is inversely proportional to A.

At larger values of the q-nonextensive parameter ($q \rightarrow 1$), the nonthermal distribution approaches the Maxwellian distribution. This indicates that the presence of nonthermal dust particles significantly influences the amplitude of solitary structures. Conversely, the strength of solitary waves diminishes when the plasma approaches the Maxwellian limit ($q \rightarrow 1$).

Figure 7 further illustrates the variation of A with the normalized dust particle density (d) as a function of the q-nonextensive parameter. An increase in dust concentration decreases the nonlinearity. As the density of dust grains increases in plasma, more charged electrons and ions accumulate on the dust surface, reducing their contribution to nonlinearity.

For zero dust concentration, the nonlinearity coefficient A exhibits a single maximum for different values of q. However, as dust concentration increases, A diverges. These findings clearly indicate that the presence of dust particles enhances the strength of solitary structures in relativistic plasma. These results are consistent with the observations reported by Dev *et al*. [19].

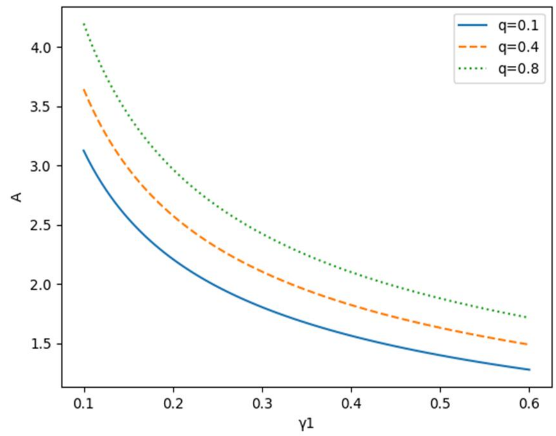

FIG. 6. Variation of the nonlinearity coefficient A with the relativistic parameter γ_1 as a function of q-nonextensive parameter.

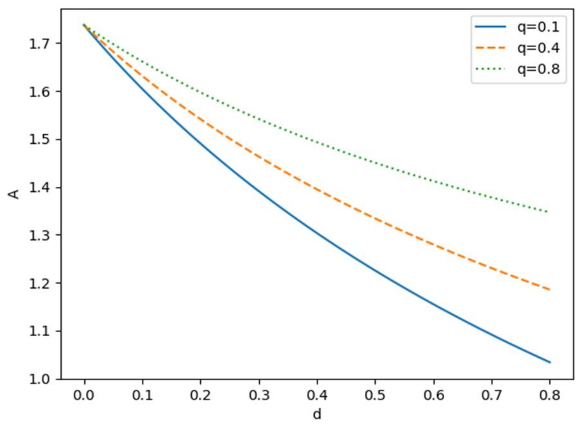

FIG. 7. Variation of the nonlinearity coefficient A with normalized densities of dust as a function of q-nonextensive parameter.

Figure 8 depicts the plot of phase velocity of solitons (λ_0) versus positron concentration (p) as a function of the nonextensive parameter (q). We observed that the Mach number (or phase velocity) of solitary waves decreases with an increase in both positron concentration and the q-nonextensive parameter of dust in the relativistic plasma. The presence of positrons reduces the restoring force of solitary waves. As the population of positrons in the plasma system increases, the corresponding decrease in the restoring force leads to an increase in ion concentration. This, in turn, enhances ion interactions, ultimately reducing the phase velocity of the solitons.

Since q tends to 1 corresponds to the Maxwellian distribution, it can be concluded that the presence of nonthermal dust particles enhances the phase velocity of solitary waves.

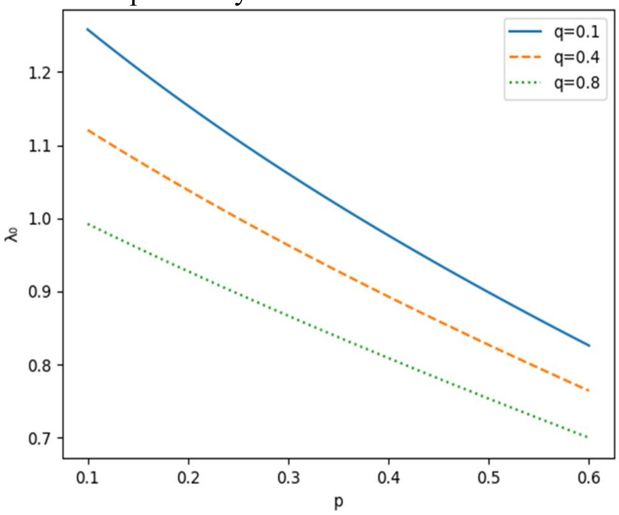

FIG. 8. Change in phase velocity (λ_0) of solitary wave with positron densities (p) as a function of q-nonextensive parameter.

6. Conclusions

Presence of dust is very relevant for laboratory, space and astrophysical plasmas. We have done mathematical modelling of an unmagnetized collisionless relativistic four component e-p-i-d plasma contains kappa distributed electrons and positrons. The dust particle is described by q-nonextensive distribution. By deriving KdV equation, nonlinear dynamics of the DIASW in relativistic astrophysical plasma environment are examined. From our investigation, we find that presence of dust particles enhances strength of solitary structures formed in relativistic plasma. The relativistic factors (β and γ_i), and the spectral indices of electrons and positrons have an enhancing impact on the amplitude of solitary structures; whereas amplitude decreases with an increase in positron concentration, q-nonextensive parameter of dusts and electron-to-positron temperature ratios. We also noticed that phase velocity of solitary waves decreases with an increase in concentration of positron and q–nonextensive parameter of dust in the plasma.

Given the critical significance of e-p-i-d plasma applications, including interstellar medium studies and astrophysics, there exists an imperative to explore relativistic ion dynamics amidst the existence of superthermal/non-Maxwellian electrons and positrons. Consequently, our present investigation aims to explore the dynamics of solitary structures existing in any astrophysical dusty plasma with relativistic streaming ions like interplanetary space and relativistic wind of pulsars.

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