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ARTICLE

# Transition Probabilities, Oscillator Strengths and Line Strengths Result From Radiative Transitions

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**Abstract:** The single-electron approximation and quantum defect theory have been used to study the spectral properties of radiative transitions in beryllium ions (Be II). A total of 6386 transitions of Be II have been investigated. The lifetime of nS, nP, nD, and nF Rydberg states of Be II ($1s^2$ nl), for n up to 30, have been computed and compared with the available data. Most of the lifetimes are new. Transition probabilities, oscillator strengths, and line strengths of 6386 radiative transitions of Be II are being reported here. A Python program was developed to calculate the radial integral, and the same program was used to compute the corresponding expectation values < r >, ensuring the validity of the results. This extensive study provides a substantial dataset on spectral properties of radiative transitions in Be II, which holds valuable applications in various spectroscopic contexts.

**Keywords:** Rydberg ion (atom), Radiative lifetimes, Oscillator strength, Transition probability, Rydberg energy, Beryllium ion.

## 1. Introduction

Transition probability and radiative lifetimes serve as fundamental characteristics of excited states in atoms, ions, and molecules, offering important information about atomic structure. Singly ionized alkaline-earth metal beryllium (Be-II) holds significance in theoretical and experimental atomic physics, astrophysics, space physics, and plasma. Beryllium has vast applications in gyroscopes, computer parts, instruments, nuclear work, and ceramic applications [1, 9]. While ample research has explored a range of spectroscopic characteristics of beryllium, there is a scarcity of literature specifically addressing transition probabilities. Bergström *et al.* employed the beam foil technique and measured some lifetimes of excited terms of Be I and Be II. These researchers used an isotope separator as an accelerator. The available energy at the separator, which ranged from 40 to 80 keV, was compatible with excitations of neutral and singly ionized atoms [2]. T. Andersen *et al.* measured the lifetimes of excited states in neutral and ionic types of beryllium and boron employing the foil-excitation technique. An electromagnetic isotope separator was used to produce 60 - 400 keV ion beams. Wavelengths ranging from 1600 to 7000 Å were examined. They measured the lifetimes of 11 states of Be I, 09 of Be II, and 01 of Be III [3]. Lindgård and Nielsen derived the dipole transition probabilities, oscillator strengths, lifetimes, and branching ratios by employing the numerical Coulomb approximation for experimentally acknowledged and extrapolated states of alkali isoelectronic sequences [4]. J. Bromander *et al.* measured the lifetimes of five of the $n = 3$ terms in Be III. They studied the beam-foil spectra of Be for energies between 200 and 400 keV and wavelengths of 500 - 1200 Å. To find out the strong spectra of doubly excited Be II and Be III and to get the precise magnitude of short lifetimes, a heavy-ion accelerator of 600

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keV was employed [5]. Chou used a relativistic many-body perturbation theory (MBPT) and calculated the transition rates for electric dipole transitions in Be-like ions. The worked-out transition rates were found in good concurrence with experiments for all ions [6]. Neng Wu Zheng *et al.* calculated the transition probabilities between $LS$ multiples of Be and Mg ionic states. They employed the WBEPM for calculation. The effective charges $Z^*$ and effective quantum numbers $n^*$and $l^*$ were concluded through a coupled set of equations. Transition probabilities were estimated between highly excited states through modified hydrogen wavefunctions [7]. Fuhr and Wiese compiled the data of atomic transition probabilities for the spectra of beryllium and boron atoms and ions. They tabulated the data for about 1400 allowed and forbidden transitions by covering all ionization stages [8]. In 2016, Çelik *et al.* employed the WBEPM and quantum defect theory and calculated the lifetimes for singly ionized beryllium (Be II) [9]. Wang *et al.* employed the time-resolved laser-induced fluorescence technique to measure the radiative lifetimes of 05 levels in Be I. They further figured out transition probabilities and oscillator strengths for 90 Be I spectral lines [10]. This work aims to extend the known values of transition probabilities, oscillator strengths, and line strengths, which help to determine other spectroscopic characteristics, e.g., dipole polarizability.

## 2. Theory

In a multi-electron system, the outermost electron is supposed to be the most loosely bound (weakest bound) compared to the rest of the electronic cloud, thus, it can most easily be excited. If examined from an excitation aspect, the behavior of the weakest bound electron differs from its counterparts. Hence, the weakest bound electron can be dealt with alone. This notion simplifies a complex multi-electron problem to a single-electron problem [11]. This idea and quantum defect models were recently combined by Zheng *et al.* in the weakest bound electron potential model theory (WBEPM). The foundation of WBEPM rests on the proposal that in any system, the orbiting electrons may be grouped into the weakest and the non-weakest bound electrons [12-16]. The weakest bound electron is supposed to be in a field created by the nucleus and inner core electrons. When excited to a higher energy level, the weakest electron moves in an enlarged orbit for a longer period. Consequently, the coupling between core and weakest bound electrons diminishes [17]. The single-electron Hamiltonian and corresponding nonrelativistic Schrödinger equation for the weakest bound electron $i$ are given below:

$$H_i = -\frac{1}{2} + \nabla_i^2 + V(r_i) \tag{1}$$

$$\left[-\frac{1}{2} + \nabla_i^2 + V(r_i)\right]\psi_i = \varepsilon_i \psi_i \tag{2}$$

Using the polar coordinates and separating the angular part, we get the radial part as

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + 2\left(E - \frac{A}{r_i} - \frac{B}{r_i^2} - \frac{l(l+1)}{r_i^2}\right) = 0 \tag{3}$$

Here, $A/r_i - B/r_i^2 = V(r_i)$ is the potential observed by the weakest bound (WB) electrons. $A = -z^*$ is the effective nuclear charge where $r_i$ is the distance between the WB electron $i$ and the nucleus.

$B = \frac{d(d+1)+2dl}{2}$ wherein the factor $d$ has been introduced due to quantum defects. It is the modification of integral values of $n$ and $l$ into non-integral values $n^*$ and $l^*$. Hence Eq. (3) becomes:

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + 2\left(E + \frac{Z^*}{r_i} - \frac{l^*(l^*+1)}{r_i^2}\right) = 0 \tag{4}$$

Here, $2B + l(l+1) = l^*(l^*+1)$ and $l^* = l - \delta_n$ , $n^* = n - \delta_n$, are effective orbital- and principal-quantum numbers, respectively, whereas, $\delta_n$ is the quantum defect and is given as a function of $n$:

$$\delta_n = a + \frac{b}{(n-\delta_o)^2} + \frac{c}{(n-\delta_o)^4} + \frac{d}{(n-\delta_o)^6} \tag{5}$$

Here $\delta_o$ is the lowest value of quantum defect. The variables ***a*, *b*, *c*,** and ***d*** can be calculated by fitting the first few energies of Rydberg levels. The energy $E$ of the weakest bound electron is given as

$$E = -\frac{Z^{*2}}{2n^{*2}} = -\frac{Z^{*2}}{2(n-\delta_n)^2} \tag{6}$$

The solution of Eq. (4) gives the radial wave function R as

$$R = C\exp\left(-\frac{Z^*r}{n^*}\right) r^{l^*} L_{n^*-l^*-1}^{2l^*+1}\left(\frac{2Z^*r}{n^*}\right) \tag{7}$$

where $C = \left(\frac{2Z^*}{n^*}\right)^{l^*+\frac{3}{2}} \sqrt{\frac{(n^*-l^*-1)!}{2n^*\Gamma(n^*+l^*+1)}}$

The transition probability $A_{fi}$ of a transition for spontaneous emission between levels $(n_f, l_f)$ & $(n_i, l_i)$, is given as

$$A_{fi} = 20261 \times 10^{-6} \frac{(E_f - E_i)^3}{2l_i + 1} S \tag{8}$$

Where, $E_f > E_i$ and are the energies of upper and lower states, respectively. S is the electric dipole line strength. Since the LS coupling dominates in lighter atoms, thus the line strength may be:

$$S_{LS} = [J_f, J_i, L_f, L_i] \left( \begin{Bmatrix} L_f & S & J_f \\ J_i & 1 & L_i \end{Bmatrix} \begin{Bmatrix} L_f & l_f & L_c \\ 1 & L_i & l_i \end{Bmatrix} P^{(1)}_{l_i l_f} \right)^2 \tag{9}$$

The terms contain two 6J symbols in brackets and a matrix element $P^{(1)}_{l_i l_f}$, which is given as

$$P^{(1)}_{l_i l_f} = l_> < n_i, l_i | r | n_f, l_f > = l_> \int_o^\infty r^3 R_{n_i l_i} R_{n_f l_f} dr \tag{10}$$

The lifetime of a state is the sum of reciprocals of transition probabilities of all possible transitions from the state. The following expression gives the lifetime:

$$\tau_J = \sum_{J'} \frac{1}{A_{JJ'}} \tag{11}$$

## 3. Results and Discussion

Numerical calculations of lifetimes, transition probabilities, oscillator strengths, etc., have been carried out via many electron self-consistent methods and single-electron approximations. While most of the calculations so far have been limited to n < 11, in this study, we extended the calculations up to n < 31. Hydrogen-like atoms or ions were considered within a single electron approximation, and the wavefunctions, expressed in terms of Laguerre polynomials with modified principal and orbital quantum numbers, were utilized. Quantum defects, determining the modification in quantum numbers, were calculated using quantum defect theory. The wavefunctions for ns, np, nd, and nf states up to n = 30 were calculated using Laguerre polynomials given in Eq. (7). Electric dipole matrix elements were computed using these wavefunctions in Eq. (10), and a Python code was developed for the integration. The radial integral [Eq. (10)] was calculated by Python code generated in this study. The reliability of codes is confirmed by calculating the expectation values < *r* > for $ns,\ np,\ nd, and\ nf$ orbits by Eq. (12).

$$< r > = \frac{3n^{*2} - l^*(l^* + 1)}{2Z^*} \tag{12}$$

We used $Z^* = 2$ to calculate quantum defects and Rydberg energies, and the resulting values were compared with the literature [18]. Glukhov *et al.* [18] have also used *Z =2* for the determination of quantum defects. The values of the electric dipole matrix elements were used in Eq. (9) to calculate line strengths, which, in turn, were employed in Eq. (8) to calculate transition probabilities. Finally, lifetimes for nS, nP, nD, and nF states were computed using Eq. (11), totaling 112 lifetimes for 6386 radiative transitions in Be II.

Table 1 presents the lifetimes of 112 states of Be II up to n = 30. The multiplicity due to spin-orbit coupling makes no difference, the counterparts of each state have the same values of lifetimes, and it is immaterial to mention the J values of each state. Therefore, only one element from each multiplicity is shown in the table. The lifetimes are compared with the corresponding works of [18, 19]. The lifetime results of Theodosiou [19] and Glukhov *et al.* [18] are in good agreement with the experimental results [3,5]. The results of this study are in excellent agreement with the results of Çelik *et al.* [9], Theodosiou, Glukhov *et al.* [18], and [30, 31], therefore, Table 1 shows only lifetimes calculated by Glukhov *et al.* [18] for comparison. Both Theodosiou and Glukhov *et al.* calculated lifetimes up to n < 11. In this work, the nS, nP, nD, and nF lifetimes are calculated up to n < 31.

Zheng *et al.* [7] calculated transition probabilities of one hundred transitions of Be II using the weakest bound electron potential model. The results of our calculations are in good agreement with the corresponding findings published by Zheng *et al.* [7]. Table 2 shows the transition probabilities, oscillator strengths, and line strengths for transitions listed on the NIST site [20]. There are almost 6386 transition probabilities calculated in this work, with the majority being new and presented in a supplementary table accompanying the manuscript. In most cases, the deviation from known values is less than 1%, with some cases below 5%, and only a few instances exhibit variations exceeding 5%. Figures 1 and 2 compare calculated values of transition probabilities and oscillator strengths for the transitions np to 2S state and the corresponding values taken from NIST data, encompassing n lines between 1 and 9. These figures illustrate a high level of agreement between calculated values and those taken from the NIST site.

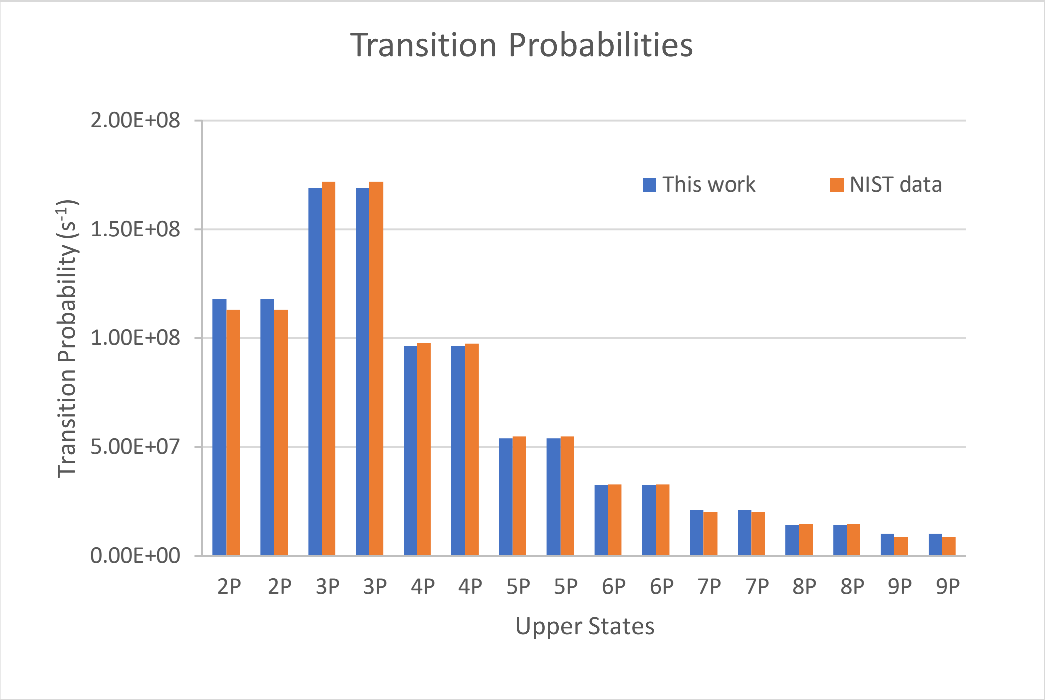

FIG. 1. Comparison of transition probabilities in NIST data and calculated in this work for the transition from nP to 2S state.

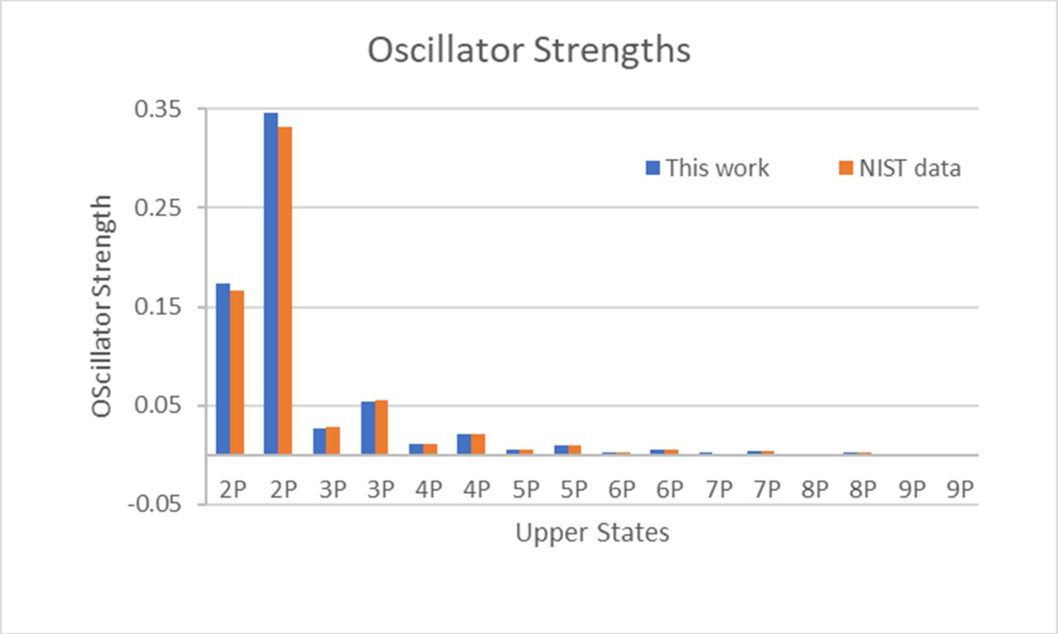

FIG. 2. Comparison of oscillator strengths in NIST data and calculated in this work for the transition from nP to 2S state.

TABLE 1. Radiative Lifetimes of s, p, d, and *f (l ± s)* sequence of Rydberg levels compared with previously published data for Be II.

| | Radiative Lifetimes τ (nSec) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Config. | $^{(2s+1)}) L_j$ | This Work | CAHS [19] | FMP [18] | WBEPMT [9] | QDT [9] | Experimental Results | RMBT [29 | Kazakov [30] | Kurucz [31] | Other Results |
| 1 | *3s* | $^2S_{1/2}$ | 2.46 | 2.473 | 2.461 | 2.46 | 2.46 | 3.3(4)[c], 2.0(2)[d], 2.3(3)[f] | 2.451(1) | 2.5 | 2.545 | 2.51[a], 2.37[e], 2.38[g] |
| | *4s* | $^2S_{1/2}$ | 4.19 | 4.196 | 4.199 | 4.24 | 4.2 | 4.4(3)[c], 4.9(3)[d], 5.0(3)[f,] 4.2(3)[i] | 4.174(1) | 4.3 | 4.486 | 4.254[a] |
| | *5s* | $^2S_{1/2}$ | 7.22 | 7.213 | 7.236 | 7.35 | 7.24 | | 7.914(2) | 7.3 | 7.519 | |
| | *6s* | $^2S_{1/2}$ | 11.73 | 11.7 | 11.76 | 12 | 11.8 | | 11.673(2) | 12 | 12.21 | |
| | *7s* | $^2S_{1/2}$ | 17.99 | 18.02 | 18.02 | 18.4 | 18 | | 17.918(4) | 18.5 | 18.622 | |
| | *8s* | $^2S_{1/2}$ | 26.29 | 26.22 | 26.34 | 25.9 | 26.4 | | 26.17(1) | 26.5 | 27.174 | |
| | *9s* | $^2S_{1/2}$ | 36.93 | 36.81 | 36.99 | 37.9 | 37 | | 36.78(2) | 37.5 | 38.023 | |
| | *10s* | $^2S_{1/2}$ | 50.19 | 50.02 | 50.27 | 51.5 | 50.3 | | 49.89(7) | 51 | 51.546 | |
| | *11s* | $^2S_{1/2}$ | 66.37 | 66.14 | | | | | | | | |
| | *12s* | $^2S_{1/2}$ | 85.78 | 85.44 | | | | | | | | |
| | *13s* | $^2S_{1/2}$ | 108.70 | | | | | | | | | |
| | *14s* | $^2S_{1/2}$ | 135.44 | | | | | | | | | |
| | *15s* | $^2S_{1/2}$ | 166.30 | | | | | | | | | |
| | *16s* | $^2S_{1/2}$ | 201.57 | | | | | | | | | |
| | *17s* | $^2S_{1/2}$ | 241.56 | | | | | | | | | |
| | *18s* | $^2S_{1/2}$ | 286.55 | | | | | | | | | |
| | *19s* | $^2S_{1/2}$ | 336.86 | | | | | | | | | |
| | *20s* | $^2S_{1/2}$ | 392.77 | | | | | | | | | |
| | *21s* | $^2S_{1/2}$ | 454.59 | | | | | | | | | |
| | *22s* | $^2S_{1/2}$ | 522.62 | | | | | | | | | |
| | *23s* | $^2S_{1/2}$ | 597.15 | | | | | | | | | |
| | *24s* | $^2S_{1/2}$ | 678.48 | | | | | | | | | |
| | *25s* | $^2S_{1/2}$ | 766.92 | | | | | | | | | |
| | *26s* | $^2S_{1/2}$ | 862.76 | | | | | | | | | |
| | *27s* | $^2S_{1/2}$ | 966.29 | | | | | | | | | |
| | *28s* | $^2S_{1/2}$ | 1077.83 | | | | | | | | | |
| | *29s* | $^2S_{1/2}$ | 1197.63 | | | | | | | | | |
| | *30s* | $^2S_{1/2}$ | 1326.03 | | | | | | | | | |

|  | Radiative Lifetimes τ (nSec) |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | Config. | $^{(2s+1)}L_j$ | This Work | CAHS [19] | FMP [18] | WBEPMT [9] | QDT [9] | Experimental Results | RMBT [29 | Kazakov [30] | Kurucz [31] | Other Results |
| 2 | *2p* | $^2P^o_{1\pm1/2}$ | 8.48 | 8.93 | 8.504 | 8.64 | 8.5 | 8.1(4)[c], 9.5(2)[f], 8.7(2)[i] | 8.85(2) | 8.9 | 8.621 | 8.939[a], 8.62[e], 8.63[g], 8.85[j] |
|  | *3p* | $^2P^o_{1\pm1/2}$ | 5.51 | 5.431 | 5.522 | 5.18 | 5.52 | 6.7(8)[i] | 5.417(2) | 5.6 | 4.149 | 5.615[a], 5.58[e], 5.55[g] |
|  | *4p* | $^2P^o_{1\pm1/2}$ | 8.21 | 8.03 | 8.232 | 8.07 | 8.23 | 8.1(8)[b], 8.5(9)[i] | 8.086(1) | 8.3 | 6.211 | 8.312[a] |
|  | *5p* | $^2P^o_{1\pm1/2}$ | 13.12 | 12.76 | 13.15 | 13.1 | 13.2 |  | 12.918(4) | 13.5 | 9.901 |  |
|  | *6p* | $^2P^o_{1\pm1/2}$ | 20.34 | 19.71 | 20.39 | 20.6 | 20.4 |  | 20.05(1) | 20.5 | 15.432 |  |
|  | *7p* | $^2P^o_{1\pm1/2}$ | 30.22 | 29.19 | 30.29 | 30.8 | 30.3 |  | 29.78(2) | 30.5 | 22.936 |  |
|  | *8p* | $^2P^o_{1\pm1/2}$ | 43.17 | 41.57 | 43.25 | 44.1 | 44.3 |  | 42.66(4) | 43.5 | 32.68 |  |
|  | *9p* | $^2P^o_{1\pm1/2}$ | 59.58 | 57.25 | 59.69 | 61.1 | 59.1 |  | 58.86(7) | 60.1 | 45.045 |  |
|  | *10p* | $^2P^o_{1\pm1/2}$ | 79.90 | 76.66 | 80.03 | 82.2 | 80.1 |  | 78.6(2) | 80.5 | 60.606 |  |
|  | *11p* | $^2P^o_{1\pm1/2}$ | 104.53 | 100.1 |  |  |  |  |  |  |  |  |
|  | *12p* | $^2P^o_{1\pm1/2}$ | 133.91 | 128.1 |  |  |  |  |  |  |  |  |
|  | *13p* | $^2P^o_{1\pm1/2}$ | 168.46 |  |  |  |  |  |  |  |  |  |
|  | *14p* | $^2P^o_{1\pm1/2}$ | 208.62 |  |  |  |  |  |  |  |  |  |
|  | *15p* | $^2P^o_{1\pm1/2}$ | 254.80 |  |  |  |  |  |  |  |  |  |
|  | *16p* | $^2P^o_{1\pm1/2}$ | 307.44 |  |  |  |  |  |  |  |  |  |
|  | *17p* | $^2P^o_{1\pm1/2}$ | 366.97 |  |  |  |  |  |  |  |  |  |
|  | *18p* | $^2P^o_{1\pm1/2}$ | 433.82 |  |  |  |  |  |  |  |  |  |
|  | *19p* | $^2P^o_{1\pm1/2}$ | 508.41 |  |  |  |  |  |  |  |  |  |
|  | *20p* | $^2P^o_{1\pm1/2}$ | 591.18 |  |  |  |  |  |  |  |  |  |
|  | *21p* | $^2P^o_{1\pm1/2}$ | 682.57 |  |  |  |  |  |  |  |  |  |
|  | *22p* | $^2P^o_{1\pm1/2}$ | 782.99 |  |  |  |  |  |  |  |  |  |
|  | *23p* | $^2P^o_{1\pm1/2}$ | 892.88 |  |  |  |  |  |  |  |  |  |
|  | *24p* | $^2P^o_{1\pm1/2}$ | 1012.68 |  |  |  |  |  |  |  |  |  |
|  | *25p* | $^2P^o_{1\pm1/2}$ | 1142.80 |  |  |  |  |  |  |  |  |  |
|  | *26p* | $^2P^o_{1\pm1/2}$ | 1283.70 |  |  |  |  |  |  |  |  |  |
|  | *27p* | $^2P^o_{1\pm1/2}$ | 1435.78 |  |  |  |  |  |  |  |  |  |
|  | *28p* | $^2P^o_{1\pm1/2}$ | 1599.50 |  |  |  |  |  |  |  |  |  |
|  | *29p* | $^2P^o_{1\pm1/2}$ | 1775.26 |  |  |  |  |  |  |  |  |  |
|  | *30p* | $^2P^o_{1\pm1/2}$ | 1963.34 |  |  |  |  |  |  |  |  |  |

Radiative Lifetimes τ (nSec)

| | Config. | $^{(2s+1)})\ L_j$ | This Work | CAHS [19] | FMP [18] | WBEPMT [9] | QDT [9] | Experimental Results | RMBT [29 | Kazakov [30] | Kurucz [31] | Other Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | *3d* | $^{2}D_{2\pm1/2}$ | 0.86 | 0.911 | 0.862 | 0.892 | 0.862 | 0.83(8)$^{d}$, 0.92(26)$^{f}$, 1.0(1)$^{i}$ | 0.901(1) | 0.9045 | 0.893 | 0.911$^{a}$ |
| | *4d* | $^{2}D_{2\pm1/2}$ | 1.99 | 2.081 | 1.991 | 2.02 | 1.99 | 2.3(2)$^{c}$, 2.12(9)$^{f}$, 2.30(9)$^{f}$ 2.4(2)$^{i}$ | 2.066(1) | 2.1 | 2.075 | 2.069$^{a}$ |
| | *5d* | $^{2}D_{2\pm1/2}$ | 3.81 | 3.971 | 3.817 | 3.83 | 3.82 | 5.1(3)$^{c}$, 3.5(4)$^{i}$, | 3.938(2) | 3.9 | 3.984 | 3.940$^{a}$ |
| | *6d* | $^{2}D_{2\pm1/2}$ | 6.49 | 6.754 | 6.507 | 6.501 | 6.51 | 8.0(8)$^{c}$, 7.3(8)$^{i}$ | 6.700(2) | 6.7 | 6.757 | 6.688$^{a}$ |
| | *7d* | $^{2}D_{2\pm1/2}$ | 10.21 | 10.61 | 10.23 | 10.2 | 10.2 | | 10.518(4) | 10.5 | 10.65 | |
| | *8d* | $^{2}D_{2\pm1/2}$ | 15.13 | 15.71 | 15.16 | 15.1 | 15.2 | | 15.58(1) | 15.5 | 15.773 | |
| | *9d* | $^{2}D_{2\pm1/2}$ | 21.41 | 22.22 | 21.46 | 21.3 | 21.5 | | 22.01(1) | 22 | 22.321 | |
| | *10d* | $^{2}D_{2\pm1/2}$ | 29.24 | 30.33 | | | | | | | | |
| | *11d* | $^{2}D_{2\pm1/2}$ | 38.78 | 40.22 | | | | | | | | |
| | *12d* | $^{2}D_{2\pm1/2}$ | 50.19 | 52.04 | | | | | | | | |
| | *13d* | $^{2}D_{2\pm1/2}$ | 63.65 | | | | | | | | | |
| | *14d* | $^{2}D_{2\pm1/2}$ | 79.34 | | | | | | | | | |
| | *15d* | $^{2}D_{2\pm1/2}$ | 97.41 | | | | | | | | | |
| | *16d* | $^{2}D_{2\pm1/2}$ | 118.04 | | | | | | | | | |
| | *17d* | $^{2}D_{2\pm1/2}$ | 141.39 | | | | | | | | | |
| | *18d* | $^{2}D_{2\pm1/2}$ | 167.65 | | | | | | | | | |
| | *19d* | $^{2}D_{2\pm1/2}$ | 196.97 | | | | | | | | | |
| | *20d* | $^{2}D_{2\pm1/2}$ | 229.53 | | | | | | | | | |
| | *21d* | $^{2}D_{2\pm1/2}$ | 265.50 | | | | | | | | | |
| | *22d* | $^{2}D_{2\pm1/2}$ | 305.05 | | | | | | | | | |
| | *23d* | $^{2}D_{2\pm1/2}$ | 348.34 | | | | | | | | | |
| | *24d* | $^{2}D_{2\pm1/2}$ | 395.55 | | | | | | | | | |
| | *25d* | $^{2}D_{2\pm1/2}$ | 446.85 | | | | | | | | | |
| | *26d* | $^{2}D_{2\pm1/2}$ | 502.41 | | | | | | | | | |
| | *27d* | $^{2}D_{2\pm1/2}$ | 562.39 | | | | | | | | | |
| | *28d* | $^{2}D_{2\pm1/2}$ | 626.98 | | | | | | | | | |
| | *29d* | $^{2}D_{2\pm1/2}$ | 696.32 | | | | | | | | | |
| | *30d* | $^{2}D_{2\pm1/2}$ | 770.55 | | | | | | | | | |

| | | | Radiative Lifetimes τ (nSec) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Config. | $^{(2s+1)}L_j$ | This Work | CAHS [19] | FMP [18] | WBEPMT [9] | QDT [9] | Experimental Results | RMBT [29 | Kazakov [30] | Kurucz [31] | Other Results |
| 4 | *4f* | $^2F_{3 \pm 1/2}$ | 4.50 | 4.52 | 4.51 | 4.51 | 4.51 | 7.0(4)[c], 4.9(2)[f], | 4.525(1) | 4.522 | 4.523 | 4.523[a] |
| | *5f* | $^2F_{3 \pm 1/2}$ | 8.71 | 8.739 | 8.722 | 8.71 | 8.72 | 9.7(7)[c], 9.0(8)[i] | 8.734(1) | 8.739 | 8.737 | 8.737[a] |
| | *6f* | $^2F_{3 \pm 1/2}$ | 14.92 | 14.93 | 14.91 | 14.9 | 14.9 | 9.3(10)[i] | 14.948(6) | 14.93 | 14.93 | 14.93[a] |
| | *7f* | $^2F_{3 \pm 1/2}$ | 23.55 | 23.49 | 23.45 | 23.4 | 23.5 | | 23.60(1) | 23.49 | | |
| | *8f* | $^2F_{3 \pm 1/2}$ | 34.97 | 34.77 | 34.72 | 34.7 | 34.3 | | 35.03(1) | 34.77 | | |
| | *9f* | $^2F_{3 \pm 1/2}$ | 49.59 | 49.48 | 49.13 | 49.1 | 49.1 | | 49.73(2) | 49.18 | 49.261 | |
| | *10f* | $^2F_{3 \pm 1/2}$ | 67.80 | 67.08 | | 66.9 | 66.9 | | | | 67.114 | |
| | *11f* | $^2F_{3 \pm 1/2}$ | 90.00 | 88.85 | | | | | | | | |
| | *12f* | $^2F_{3 \pm 1/2}$ | 116.58 | 114.9 | | | | | | | | |
| | *13f* | $^2F_{3 \pm 1/2}$ | 147.93 | | | | | | | | | |
| | *14f* | $^2F_{3 \pm 1/2}$ | 184.46 | | | | | | | | | |
| | *15f* | $^2F_{3 \pm 1/2}$ | 226.56 | | | | | | | | | |
| | *16f* | $^2F_{3 \pm 1/2}$ | 274.62 | | | | | | | | | |
| | *17f* | $^2F_{3 \pm 1/2}$ | 329.05 | | | | | | | | | |
| | *18f* | $^2F_{3 \pm 1/2}$ | 390.23 | | | | | | | | | |
| | *19f* | $^2F_{3 \pm 1/2}$ | 458.56 | | | | | | | | | |
| | *20f* | $^2F_{3 \pm 1/2}$ | 534.44 | | | | | | | | | |
| | *21f* | $^2F_{3 \pm 1/2}$ | 618.27 | | | | | | | | | |
| | *22f* | $^2F_{3 \pm 1/2}$ | 710.43 | | | | | | | | | |
| | *23f* | $^2F_{3 \pm 1/2}$ | 811.34 | | | | | | | | | |
| | *24f* | $^2F_{3 \pm 1/2}$ | 921.37 | | | | | | | | | |
| | *25f* | $^2F_{3 \pm 1/2}$ | 1040.94 | | | | | | | | | |
| | *26f* | $^2F_{3 \pm 1/2}$ | 1170.42 | | | | | | | | | |
| | *27f* | $^2F_{3 \pm 1/2}$ | 1310.23 | | | | | | | | | |
| | *28f* | $^2F_{3 \pm 1/2}$ | 1460.73 | | | | | | | | | |
| | *29f* | $^2F_{3 \pm 1/2}$ | 1622.35 | | | | | | | | | |
| | *30f* | $^2F_{3 \pm 1/2}$ | 1795.48 | | | | | | | | | |

[a] ref. [21], [b] ref. [23], [c] ref. [3], [d] ref. [22], [e] ref. [24], [f] ref. [25], [g] ref. [26], [i] ref. [27], [j] ref. [28].

TABLE 2. List of transition probabilities, oscillator strengths, and line strengths compared with NIST data

| Transition | Transition Probability ($s^{-1}$) | | | Oscillator Strength | | | Line Strength (a.u.) | | |
|---|---|---|---|---|---|---|---|---|---|
| | This work | NIST | % Error | This work | NIST | % Error | This work | NIST | % Error |
| 2S ---> 2P | 1.18E+08 | 1.13E+08 | 4.46630 | 0.17299 | 0.16596 | 4.23575 | 3.56656 | 3.42230 | 4.21527 |
| 2S --> 2P | 1.18E+08 | 1.13E+08 | 4.40154 | 0.34598 | 0.33198 | 4.21691 | 7.13312 | 6.84450 | 4.21679 |
| 2S ---> 3P | 1.69E+08 | 1.72E+08 | 1.79255 | 0.02713 | 0.02768 | 1.98565 | 0.18511 | 0.18890 | 2.00597 |
| 2S ---> 3P | 1.69E+08 | 1.72E+08 | 1.95232 | 0.05423 | 0.05540 | 2.10590 | 0.37003 | 0.37800 | 2.10925 |
| 2S ---> 4P | 9.63E+07 | 9.79E+07 | 1.59749 | 0.01022 | 0.01040 | 1.72409 | 0.05666 | 0.05770 | 1.80067 |
| 2S ---> 4P | 9.63E+07 | 9.76E+07 | 1.32908 | 0.02043 | 0.02070 | 1.28474 | 0.11328 | 0.11500 | 1.49511 |
| 2S ---> 5P | 5.41E+07 | 5.50E+07 | 1.68348 | 0.00486 | 0.00496 | 1.92708 | 0.02483 | 0.02530 | 1.85025 |
| 2S ---> 5P | 5.41E+07 | 5.50E+07 | 1.71442 | 0.00973 | 0.00991 | 1.85966 | 0.04965 | 0.05060 | 1.88212 |
| 2S ---> 6P | 3.26E+07 | 3.30E+07 | 1.22905 | 0.00270 | 0.00274 | 1.58319 | 0.01320 | 0.01340 | 1.48292 |
| 2S ---> 6P | 3.26E+07 | 3.30E+07 | 1.24155 | 0.00539 | 0.00547 | 1.41590 | 0.02640 | 0.02680 | 1.49562 |
| 2S ---> 7P | 2.10E+07 | 2.03E+07 | 3.32291 | 0.00165 | 0.00160 | 3.30564 | 0.00790 | 0.00764 | 3.36760 |
| 2S ---> 7P | 2.10E+07 | 2.03E+07 | 3.32879 | 0.00331 | 0.00320 | 3.31157 | 0.01580 | 0.01530 | 3.23843 |
| 2S ---> 8P | 1.42E+07 | 1.47E+07 | 3.18341 | 0.00109 | 0.00112 | 2.90421 | 0.00512 | 0.00529 | 3.28439 |
| 2S ---> 8P | 1.42E+07 | 1.47E+07 | 3.16361 | 0.00218 | 0.00225 | 3.31587 | 0.01023 | 0.01060 | 3.44697 |
| 2S ---> 9P | 1.01E+07 | 8.76E+06 | 15.03572 | 0.00075 | 0.00066 | 14.78235 | 0.00351 | 0.00306 | 14.74375 |
| 2S ---> 9P | 1.01E+07 | 8.75E+06 | 15.20381 | 0.00151 | 0.00131 | 15.16959 | 0.00702 | 0.00611 | 14.96831 |
| 2P ---> 3S | 1.36E+08 | 1.36E+08 | 0.27902 | 0.06408 | 0.06438 | 0.45934 | 0.74949 | 0.75290 | 0.45356 |
| 2P ---> 3S | 2.71E+08 | 2.72E+08 | 0.27902 | 0.06408 | 0.06438 | 0.45934 | 1.49897 | 1.50600 | 0.46678 |
| 2P---> 3D | 9.68E+08 | 9.22E+08 | 5.02624 | 1.29582 | 0.63200 | 105.03481 | 6.59918 | 6.29300 | 4.86542 |
| 2P---> 3D | 1.94E+08 | 1.84E+08 | 5.04903 | 0.13358 | 0.06321 | 111.32732 | 1.31984 | 1.25900 | 4.83211 |
| 2P---> 3D | 1.16E+09 | 1.11E+09 | 5.03087 | 0.59644 | 0.56890 | 4.84075 | 11.87833 | 11.33000 | 4.83967 |
| 2P ---> 4S | 4.74E+07 | 4.84E+07 | 2.07925 | 0.01016 | 0.01039 | 2.22786 | 0.08007 | 0.08191 | 2.24436 |
| 2P ---> 4S | 9.47E+07 | 9.67E+07 | 2.04887 | 0.01016 | 0.01039 | 2.22786 | 0.16014 | 0.16380 | 2.23243 |
| 2P---> 4D | 3.25E+08 | 3.14E+08 | 3.59168 | 0.12719 | 0.12300 | 3.40865 | 0.95721 | 0.92500 | 3.48255 |
| 2P---> 4D | 6.51E+07 | 6.27E+07 | 3.75689 | 0.01272 | 0.01230 | 3.40865 | 0.19144 | 0.18500 | 3.48255 |
| 2P---> 4D | 3.90E+08 | 3.75E+08 | 4.09202 | 0.11448 | 0.11000 | 4.06924 | 1.72302 | 1.66000 | 3.79649 |
| 2P---> 5S | 2.23E+07 | 2.26E+07 | 1.27329 | 0.00367 | 0.00372 | 1.38246 | 0.02532 | 0.02570 | 1.48573 |
| 2P ---> 5S | 4.46E+07 | 4.52E+07 | 1.27329 | 0.00367 | 0.00372 | 1.38246 | 0.05064 | 0.05140 | 1.48573 |
| 2P---> 5D | 1.51E+08 | 1.46E+08 | 3.71325 | 0.04779 | 0.04630 | 3.22637 | 0.32315 | 0.31300 | 3.24350 |
| 2P---> 5D | 3.03E+07 | 2.93E+07 | 3.35928 | 0.00478 | 0.00463 | 3.22637 | 0.06463 | 0.06260 | 3.24350 |
| 2P---> 5D | 1.82E+08 | 1.76E+08 | 3.24568 | 0.04302 | 0.04160 | 3.40367 | 0.58169 | 0.56300 | 3.32033 |

| Transition | Transition Probability ($s^{-1}$) | | | Oscillator Strength | | | Line Strength (a.u.) | | |
|---|---|---|---|---|---|---|---|---|---|
| | This work | NIST | % Error | This work | NIST | % Error | This work | NIST | % Error |
| 2P---> 6S | 1.23E+07 | 1.24E+07 | 0.73980 | 0.00178 | 0.00181 | 1.46383 | 0.01156 | 0.01170 | 1.23905 |
| 2P ---> 6S | 2.46E+07 | 2.48E+07 | 0.73980 | 0.00178 | 0.00180 | 0.91641 | 0.02311 | 0.02330 | 0.81518 |
| 2P---> 6D | 8.34E+07 | 8.08E+07 | 3.22919 | 0.02365 | 0.02290 | 3.25825 | 0.15152 | 0.14700 | 3.07639 |
| 2P---> 6D | 1.67E+07 | 1.62E+07 | 2.97430 | 0.00236 | 0.00229 | 3.25825 | 0.03030 | 0.02940 | 3.07639 |
| 2P---> 6D | 1.00E+08 | 9.71E+07 | 3.08035 | 0.02128 | 0.02070 | 2.80930 | 0.27274 | 0.26500 | 2.92081 |
| 2P---> 7S | 7.51E+06 | 7.60E+06 | 1.13924 | 0.00101 | 0.00103 | 1.52560 | 0.00634 | 0.00643 | 1.35632 |
| 2P---> 7S | 1.50E+07 | 1.53E+07 | 1.78539 | 0.00101 | 0.00103 | 1.52560 | 0.01269 | 0.01290 | 1.66219 |
| 2P---> 7D | 6.12E+07 | 5.95E+07 | 2.90241 | 0.01223 | 0.01190 | 2.81351 | 0.15201 | 0.14800 | 2.70823 |
| 2P---> 7D | 5.10E+07 | 4.95E+07 | 3.08070 | 0.01359 | 0.01320 | 2.99155 | 0.08445 | 0.08210 | 2.86604 |
| 2P---> 7D | 1.02E+07 | 9.89E+06 | 3.18492 | 0.00136 | 0.00132 | 2.99155 | 0.01689 | 0.01640 | 2.99148 |
| 2P---> 8D | 4.03E+07 | 3.91E+07 | 2.97413 | 0.00774 | 0.00753 | 2.73303 | 0.09424 | 0.09170 | 2.77284 |
| 2P---> 8D | 3.36E+07 | 3.26E+07 | 2.93129 | 0.00860 | 0.00837 | 2.70179 | 0.05236 | 0.05100 | 2.67050 |
| 2P---> 8D | 6.71E+06 | 6.52E+06 | 2.93129 | 0.00086 | 0.00084 | 2.70179 | 0.01047 | 0.01020 | 2.67050 |
| 2P---> 9D | 2.33E+07 | 2.28E+07 | 2.07489 | 0.00581 | 0.00571 | 1.68461 | 0.03490 | 0.03430 | 1.75580 |
| 2P---> 9D | 4.65E+06 | 4.57E+06 | 1.85153 | 0.00058 | 0.00057 | 1.68461 | 0.00698 | 0.00686 | 1.75580 |
| 2P---> 9D | 2.79E+07 | 2.74E+07 | 1.91212 | 0.00522 | 0.00513 | 1.84937 | 0.06282 | 0.06170 | 1.80819 |
| 3S---> 3P | 1.28E+07 | 1.26E+07 | 1.65085 | 0.28071 | 0.27670 | 1.44915 | 22.36679 | 22.05000 | 1.43671 |
| 3S---> 3P | 1.28E+07 | 1.26E+07 | 1.64285 | 0.56156 | 0.55350 | 1.45599 | 44.73425 | 44.09000 | 1.46121 |
| 3S---> 4P | 1.43E+07 | 1.41E+07 | 1.34336 | 0.02294 | 0.02270 | 1.07893 | 0.49486 | 0.48900 | 1.19796 |
| 3S---> 4P | 1.43E+07 | 1.41E+07 | 1.27587 | 0.04586 | 0.04540 | 1.00625 | 0.98898 | 0.97900 | 1.01923 |
| 3S---> 5P | 1.07E+07 | 1.06E+07 | 1.28939 | 0.00968 | 0.00959 | 0.94745 | 0.15646 | 0.15500 | 0.93981 |
| 3S---> 5P | 1.07E+07 | 1.06E+07 | 1.23747 | 0.01935 | 0.01910 | 1.31616 | 0.31274 | 0.30900 | 1.21136 |
| 3P---> 3D | 6.58E+04 | 6.58E+04 | 0.04534 | 0.08095 | 0.08113 | 0.22119 | 34.18412 | 34.26000 | 0.22149 |
| 3P---> 4S | 3.22E+07 | 3.23E+07 | 0.55490 | 0.13376 | 0.13470 | 0.69857 | 4.64281 | 4.67700 | 0.73099 |
| 3P---> 4D | 9.40E+07 | 9.09E+07 | 3.44070 | 0.53546 | 0.51900 | 3.17095 | 15.37810 | 14.90000 | 3.20872 |
| 3P---> 5S | 1.37E+07 | 1.39E+07 | 1.18300 | 0.02161 | 0.02190 | 1.31165 | 0.46142 | 0.46700 | 1.19407 |
| 3P---> 5D | 4.81E+07 | 4.65E+07 | 3.45453 | 0.13371 | 0.13000 | 2.85722 | 2.68293 | 2.60000 | 3.18956 |
| 3P ---> 6S | 7.32E+06 | 7.37E+06 | 0.66040 | 0.00798 | 0.00805 | 0.90771 | 0.14172 | 0.14300 | 0.89838 |
| 3P---> 6D | 2.73E+07 | 2.65E+07 | 2.86291 | 0.05595 | 0.05440 | 2.85220 | 0.96474 | 0.93800 | 2.85056 |
| 3P---> 7S | 4.39E+06 | 4.43E+06 | 0.85736 | 0.00396 | 0.00401 | 1.16488 | 0.06408 | 0.06480 | 1.11407 |
| 3P---> 7D | 1.69E+07 | 1.64E+07 | 2.98463 | 0.02946 | 0.02870 | 2.64609 | 0.46824 | 0.45600 | 2.68490 |
| 3P---> 8D | 1.12E+07 | 1.09E+07 | 2.61524 | 0.01767 | 0.01720 | 2.73903 | 0.26731 | 0.26000 | 2.81263 |

| Transition | Transition Probability ($s^{-1}$) | | | Oscillator Strength | | | Line Strength (a.u.) | | |
|---|---|---|---|---|---|---|---|---|---|
| | This work | NIST | % Error | This work | NIST | % Error | This work | NIST | % Error |
| 3P---> 3D | 1.31E+04 | 1.31E+04 | 0.06766 | 0.00808 | 0.00810 | 0.22299 | 6.83673 | 6.85200 | 0.22289 |
| 3P---> 3D | 7.87E+04 | 7.87E+04 | 0.03967 | 0.07279 | 0.07294 | 0.20465 | 61.53188 | 61.66000 | 0.20778 |
| 3P---> 4S | 6.43E+07 | 6.47E+07 | 0.52107 | 0.13379 | 0.13470 | 0.67532 | 9.28874 | 9.35400 | 0.69765 |
| 3P---> 4D | 1.88E+07 | 1.82E+07 | 3.32882 | 0.05356 | 0.05210 | 2.79398 | 3.07645 | 2.99000 | 2.89129 |
| 3P---> 4D | 1.13E+08 | 1.09E+08 | 3.51579 | 0.48198 | 0.46800 | 2.98689 | 27.68647 | 26.90000 | 2.92369 |
| 3P---> 5S | 2.75E+07 | 2.77E+07 | 0.82634 | 0.02162 | 0.02190 | 1.29939 | 0.92302 | 0.93400 | 1.17561 |
| 3P---> 5D | 9.62E+06 | 9.32E+06 | 3.22688 | 0.01337 | 0.01300 | 2.86369 | 0.53665 | 0.52100 | 3.00402 |
| 3P---> 5D | 5.77E+07 | 5.60E+07 | 3.07923 | 0.12035 | 0.11700 | 2.86275 | 4.82980 | 4.69000 | 2.98074 |
| 3P ---> 6S | 1.46E+07 | 1.47E+07 | 0.39090 | 0.00798 | 0.00805 | 0.89821 | 0.28347 | 0.28600 | 0.88372 |
| 3P---> 6D | 5.45E+06 | 5.30E+06 | 2.85441 | 0.00560 | 0.00545 | 2.66536 | 0.19296 | 0.18800 | 2.63879 |
| 3P---> 6D | 3.27E+07 | 3.18E+07 | 2.85441 | 0.05036 | 0.04900 | 2.77012 | 1.73665 | 1.69000 | 2.76026 |
| 3P---> 7S | 8.78E+06 | 8.89E+06 | 1.19304 | 0.00396 | 0.00402 | 1.40251 | 0.12817 | 0.13000 | 1.40543 |
| 3P---> 7D | 2.03E+07 | 1.97E+07 | 2.86863 | 0.02651 | 0.02580 | 2.76381 | 0.84286 | 0.82100 | 2.66316 |
| 3P---> 7D | 3.38E+06 | 3.29E+06 | 2.66163 | 0.00295 | 0.00287 | 2.64566 | 0.09365 | 0.09130 | 2.57678 |
| 3P---> 8D | 1.34E+07 | 1.30E+07 | 3.23238 | 0.01590 | 0.01550 | 2.60159 | 0.48116 | 0.46800 | 2.81244 |
| 3P---> 8D | 2.24E+06 | 2.17E+06 | 3.07720 | 0.00177 | 0.00172 | 2.73724 | 0.05346 | 0.05200 | 2.81540 |
| 3D---> 4P | 8.42E+06 | 8.76E+06 | 3.88555 | 0.01469 | 0.01530 | 3.95545 | 0.93455 | 0.97400 | 4.05022 |
| 3D---> 4P | 8.42E+05 | 8.77E+05 | 4.03629 | 0.00294 | 0.00307 | 4.31682 | 0.18681 | 0.19500 | 4.20095 |
| 3D---> 4F | 2.07E+08 | 2.07E+08 | 0.14122 | 1.01686 | 1.02000 | 0.30771 | 62.59556 | 62.50000 | 0.15289 |
| 3D---> 5P | 3.55E+06 | 3.74E+06 | 5.21302 | 0.00278 | 0.00293 | 5.28647 | 0.11820 | 0.12500 | 5.44109 |
| 3D---> 5P | 3.54E+05 | 3.74E+05 | 5.25275 | 0.00055 | 0.00059 | 5.49007 | 0.02363 | 0.02500 | 5.48466 |
| 3D---> 5F | 6.85E+07 | 6.82E+07 | 0.36725 | 0.15715 | 0.15700 | 0.09618 | 6.61805 | 6.61000 | 0.12179 |
| 3D---> 6F | 3.24E+07 | 3.22E+07 | 0.53169 | 0.05414 | 0.05400 | 0.26629 | 1.94619 | 1.94000 | 0.31928 |
| 3D---> 7F | 1.82E+07 | 1.81E+07 | 0.64236 | 0.02572 | 0.02570 | 0.09470 | 0.84963 | 0.84800 | 0.19206 |
| 3D---> 8F | 1.14E+07 | 1.14E+07 | 0.16914 | 0.01450 | 0.01450 | 0.02436 | 0.45506 | 0.45500 | 0.01312 |
| 3D---> 9F | 7.63E+06 | 7.29E+06 | 4.66193 | 0.00909 | 0.00870 | 4.47384 | 0.27572 | 0.26400 | 4.44091 |
| 3D---> 4P | 7.58E+06 | 7.87E+06 | 3.74730 | 0.01763 | 0.01830 | 3.67591 | 1.68155 | 1.75000 | 3.91134 |
| 3D---> 4F | 1.48E+07 | 1.47E+07 | 0.72076 | 0.04842 | 0.04830 | 0.25314 | 4.47125 | 4.46000 | 0.25223 |
| 3D---> 4F | 2.22E+08 | 2.21E+08 | 0.49412 | 0.96845 | 0.96600 | 0.25315 | 89.42446 | 89.20000 | 0.25163 |
| 3D---> 5P | 3.19E+06 | 3.37E+06 | 5.35752 | 0.00333 | 0.00352 | 5.42545 | 0.21269 | 0.22500 | 5.47212 |
| 3D---> 5F | 4.89E+06 | 4.87E+06 | 0.38989 | 0.00748 | 0.00747 | 0.17574 | 0.47271 | 0.47200 | 0.15040 |
| 3D---> 5F | 7.33E+07 | 7.31E+07 | 0.32401 | 0.14967 | 0.14900 | 0.44701 | 9.45440 | 9.44000 | 0.15251 |

| Transition | Transition Probability ($s^{-1}$) | | | Oscillator Strength | | | Line Strength (a.u.) | | |
|---|---|---|---|---|---|---|---|---|---|
| | This work | NIST | % Error | This work | NIST | % Error | This work | NIST | % Error |
| 3D---> 6F | 2.31E+06 | 2.31E+06 | 0.08872 | 0.00258 | 0.00258 | 0.07168 | 0.13901 | 0.13900 | 0.00656 |
| 3D---> 6F | 3.47E+07 | 3.46E+07 | 0.23588 | 0.05156 | 0.05160 | 0.06933 | 2.78025 | 2.78000 | 0.00882 |
| 3D---> 7F | 1.30E+06 | 1.30E+06 | 0.08110 | 0.00122 | 0.00122 | 0.40162 | 0.06069 | 0.06060 | 0.14053 |
| 3D---> 7F | 1.95E+07 | 1.94E+07 | 0.59698 | 0.02450 | 0.02440 | 0.40162 | 1.21370 | 1.21000 | 0.30606 |
| 3D---> 8F | 1.22E+07 | 1.22E+07 | 0.06407 | 0.01381 | 0.01380 | 0.08415 | 0.65003 | 0.64900 | 0.15927 |
| 3D---> 8F | 8.13E+05 | 8.11E+05 | 0.22672 | 0.00069 | 0.00069 | 0.05751 | 0.03250 | 0.03250 | 0.00833 |
| 3D---> 9F | 8.17E+06 | 7.81E+06 | 4.65556 | 0.00866 | 0.00828 | 4.53266 | 0.39385 | 0.37700 | 4.46854 |
| 3D---> 9F | 5.45E+05 | 5.19E+05 | 4.99841 | 0.00043 | 0.00041 | 4.79243 | 0.01969 | 0.01880 | 4.75300 |
| 4S---> 4P | 2.77E+06 | 2.75E+06 | 0.66270 | 0.38134 | 0.37900 | 0.61676 | 76.17664 | 75.80000 | 0.49689 |
| 4S---> 4P | 2.77E+06 | 2.76E+06 | 0.37181 | 0.76286 | 0.76100 | 0.24461 | 152.35308 | 152.00000 | 0.23229 |
| 4S---> 5P | 2.60E+06 | 2.55E+06 | 1.98133 | 0.02133 | 0.02090 | 2.06587 | 1.03984 | 1.02000 | 1.94543 |
| 4S---> 5P | 2.60E+06 | 2.53E+06 | 2.70066 | 0.04262 | 0.04160 | 2.46339 | 2.07773 | 2.03000 | 2.35129 |
| 4P---> 4D | 2.13E+04 | 2.12E+04 | 0.24055 | 0.14572 | 0.14500 | 0.49592 | 145.22058 | 145.00000 | 0.15212 |
| 4P ---> 5S | 1.01E+07 | 1.02E+07 | 0.97198 | 0.20560 | 0.20700 | 0.67529 | 15.78776 | 15.90000 | 0.70592 |
| 4P---> 5D | 1.86E+07 | 1.81E+07 | 2.54131 | 0.49915 | 0.48700 | 2.49480 | 31.15305 | 30.40000 | 2.47715 |
| 4P---> 6S | 4.87E+06 | 4.89E+06 | 0.40074 | 0.03329 | 0.03350 | 0.61285 | 1.48162 | 1.49000 | 0.56210 |
| 4P---> 6D | 1.15E+07 | 1.11E+07 | 3.25888 | 0.13535 | 0.13200 | 2.53671 | 5.59755 | 5.45000 | 2.70733 |
| 4P---> 7S | 2.82E+06 | 2.84E+06 | 0.73776 | 0.01238 | 0.01250 | 0.93892 | 0.44170 | 0.44500 | 0.74178 |
| 4P---> 7D | 7.28E+06 | 7.05E+06 | 3.21387 | 0.05934 | 0.05760 | 3.01335 | 2.03917 | 1.98000 | 2.98841 |
| 4P---> 8D | 4.87E+06 | 4.73E+06 | 2.96452 | 0.03225 | 0.03140 | 2.70910 | 0.99880 | 0.97200 | 2.75676 |
| 4P---> 4D | 4.23E+03 | 4.22E+03 | 0.34487 | 0.01455 | 0.01450 | 0.37193 | 29.04389 | 29.00000 | 0.15134 |
| 4P---> 4D | 2.54E+04 | 2.54E+04 | 0.13973 | 0.13104 | 0.13100 | 0.02692 | 261.39950 | 261.00000 | 0.15307 |
| 4P---> 5S | 2.02E+07 | 2.03E+07 | 0.47998 | 0.20565 | 0.20700 | 0.65235 | 31.58579 | 31.80000 | 0.67361 |
| 4P---> 5D | 3.71E+06 | 3.62E+06 | 2.55197 | 0.04993 | 0.04880 | 2.31112 | 6.23269 | 6.09000 | 2.34309 |
| 4P---> 5D | 2.23E+07 | 2.17E+07 | 2.64190 | 0.44932 | 0.43900 | 2.35082 | 56.08982 | 54.80000 | 2.35370 |
| 4P---> 6S | 9.74E+06 | 9.81E+06 | 0.70469 | 0.03330 | 0.03360 | 0.89717 | 2.96375 | 2.99000 | 0.87776 |
| 4P---> 6D | 2.29E+06 | 2.23E+06 | 2.79870 | 0.01354 | 0.01320 | 2.55000 | 1.11971 | 1.09000 | 2.72586 |
| 4P--> 6D | 1.38E+07 | 1.34E+07 | 2.64526 | 0.12183 | 0.11900 | 2.37764 | 10.07741 | 9.82000 | 2.62125 |
| 4P---> 7S | 5.64E+06 | 5.68E+06 | 0.73799 | 0.01238 | 0.01250 | 0.93046 | 0.88351 | 0.89100 | 0.84035 |
| 4P---> 7D | 8.73E+06 | 8.48E+06 | 2.97136 | 0.05341 | 0.05190 | 2.90440 | 3.67103 | 3.57000 | 2.83006 |
| 4P---> 7D | 1.46E+06 | 1.41E+06 | 3.21356 | 0.00593 | 0.00578 | 2.66527 | 0.40788 | 0.39700 | 2.74171 |
| 4P---> 8D | 5.84E+06 | 5.68E+06 | 2.89109 | 0.02903 | 0.02830 | 2.57142 | 1.79804 | 1.75000 | 2.74497 |

| Transition | Transition Probability ($s^{-1}$) | | | Oscillator Strength | | | Line Strength (a.u.) | | |
|---|---|---|---|---|---|---|---|---|---|
| | This work | NIST | % Error | This work | NIST | % Error | This work | NIST | % Error |
| 4P---> 8D | 9.74E+05 | 9.44E+05 | 3.18063 | 0.00323 | 0.00313 | 3.04307 | 0.19978 | 0.19400 | 2.97837 |
| 4D---> 5P | 4.40E+06 | 4.52E+06 | 2.55204 | 0.03614 | 0.03710 | 2.59979 | 4.98251 | 5.12000 | 2.68533 |
| 4D---> 5P | 4.40E+05 | 4.50E+05 | 2.15896 | 0.00722 | 0.00740 | 2.38527 | 0.99596 | 1.02000 | 2.35684 |
| 4D---> 5P | 3.96E+06 | 4.06E+06 | 2.39098 | 0.04335 | 0.04450 | 2.59078 | 8.96514 | 9.21000 | 2.65861 |
| 5S---> 5P | 8.58E+05 | 8.52E+05 | 0.69750 | 0.47943 | 0.47700 | 0.51015 | 192.89522 | 192.00000 | 0.46626 |
| 5S ---> 5P | 8.59E+05 | 8.53E+05 | 0.65754 | 0.95911 | 0.95500 | 0.43008 | 385.78580 | 384.00000 | 0.46505 |
| 5P---> 5D | 7.80E+03 | 7.82E+03 | 0.31887 | 0.20361 | 0.20500 | 0.67567 | 396.04773 | 398.00000 | 0.49052 |
| 5P---> 6S | 3.91E+06 | 3.93E+06 | 0.51046 | 0.27837 | 0.28100 | 0.93517 | 39.97715 | 40.30000 | 0.80111 |
| 5P---> 6D | 5.37E+06 | 5.27E+06 | 1.96922 | 0.49341 | 0.48500 | 1.73420 | 56.90054 | 55.90000 | 1.78987 |
| 5P ---> 7S | 2.04E+06 | 2.06E+06 | 0.73352 | 0.04499 | 0.04530 | 0.67437 | 3.59222 | 3.62000 | 0.76749 |
| 5P---> 7D | 3.69E+06 | 3.61E+06 | 2.35056 | 0.13814 | 0.13500 | 2.32383 | 10.16507 | 9.95000 | 2.16148 |
| 5P---> 8D | 2.53E+06 | 2.47E+06 | 2.27064 | 0.06193 | 0.06060 | 2.20119 | 3.69073 | 3.61000 | 2.23623 |
| 5P---> 5D | 1.55E+03 | 1.56E+03 | 0.44404 | 0.02034 | 0.02040 | 0.31598 | 79.20922 | 79.60000 | 0.49092 |
| 5P---> 5D | 9.33E+03 | 9.35E+03 | 0.23009 | 0.18309 | 0.18400 | 0.49608 | 712.89235 | 716.00000 | 0.43403 |
| 5P---> 6S | 7.82E+06 | 7.87E+06 | 0.63188 | 0.27844 | 0.28100 | 0.91161 | 79.98082 | 80.60000 | 0.76821 |
| 5P---> 6D | 1.07E+06 | 1.06E+06 | 1.40900 | 0.04936 | 0.04860 | 1.55716 | 11.38458 | 11.20000 | 1.64807 |
| 5P---> 6D | 6.45E+06 | 6.35E+06 | 1.56870 | 0.44421 | 0.43800 | 1.41804 | 102.46126 | 101.00000 | 1.44679 |
| 5P---> 7S | 4.09E+06 | 4.12E+06 | 0.73250 | 0.04500 | 0.04540 | 0.88180 | 7.18563 | 7.25000 | 0.88785 |
| 5P---> 7D | 4.43E+06 | 4.33E+06 | 2.41051 | 0.12435 | 0.12200 | 1.92798 | 18.30232 | 17.90000 | 2.24760 |
| 5P---> 7D | 7.39E+05 | 7.22E+05 | 2.35938 | 0.01382 | 0.01350 | 2.34248 | 2.03348 | 1.99000 | 2.18501 |
| 5P---> 8D | 3.03E+06 | 2.96E+06 | 2.41923 | 0.05575 | 0.05450 | 2.29566 | 6.64487 | 6.50000 | 2.22876 |
| 5P---> 8D | 5.05E+05 | 4.93E+05 | 2.48346 | 0.00619 | 0.00606 | 2.21451 | 0.73827 | 0.72200 | 2.25354 |

## Conclusion

An extensive study has been carried out by employing the single-electron approximation and quantum defect theory to calculate transition probabilities, oscillator strengths, and line strength of 6386 transitions of ionic states of beryllium ion. The salient findings of this research work are appended below.

1. The transition probabilities are compared with NIST data. While there are isolated cases with an error margin of 5%, the majority exhibit errors of less than 1%.
2. The 6386 transitions have been presented in the supplementary table. This table contains transition wavelength, energies, transition probabilities, oscillator strengths, line strengths, configurations, and terms of upper and lower levels. Out of 6386 transitions, only 149 are listed in the NIST database. Table 2 shows both calculated values and NIST values for 149 transitions.
3. The transition probabilities have been used to calculate the lifetimes of nS, nP, nD, and nF states. The lifetimes of 112 states are reported in this article for principal quantum number n < 31. Most of the lifetimes are presented for the first time. The lifetimes are also compared with the available published data.
4. The lifetimes are compared with the corresponding works of [3-9, 18, 19, 21-31]. The lifetimes for both sets computed in this work show excellent agreement with the results of CAHS [21], FMP [18], WBEPMT, QDOT [19], RMBT [29], and [30, 31].
5. The lifetimes of nS, nP, nD, and nF states exhibit a monotonically increasing trend, with an exception observed at the 3P state, where the lifetime is lesser than that of the 2P state.
6. A Python program was developed to calculate the radial integral. The same program was used to calculate the corresponding expectation values < r > to check the validity of the results.

## Compliance with Ethical Standards

The authors certify that they have complied with the Journal's ethical guidelines.

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