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ARTICLE

Form Factor of the Oriented Pyramidal Ice Crystals in the Wentzel-Kramers-Brillouin Approximation

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Abstract: In this work, the Wentzel-Kramers-Brillouin (WKB) approximation is applied to determine an analytical expression of the form factor of oriented pyramidal ice crystals. This study will focus on two special cases of the normal incident of light: flat incidence and edge-on incidence. This form factor is calculated using an adequate decomposition of the pyramid. Furthermore, the analytical expression of the extinction coefficient is derived for these two special cases. Finally, some numerical examples are analyzed to illustrate our results.

Keywords: Light scattering, Form factor, Wentzel-Kramers-Brillouin approximation, Pyramidal ice crystals, Extinction efficiency.

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1. Introduction

Light is one of the main messages used by humans to gather information about their environment after having interacted with an object. The light bears an imprint of this interaction that can be decoded to deduce certain properties of the object. The reading of this information can be done in the most classical way thanks to the eye which allows us to characterize the intensity, the direction or the average spectral distribution of the light. The information conveyed by the light is however still much richer than what the eye can decode; the polarization, the phase and the spectrum are magnitudes which are not measured by the eye and which are also rich in information.

The study of the scattering and absorption of light by small particles is of great interest in various scientific disciplines and many applications, such as medical technology, geophysics, metrology and radio astronomy [1-6]. In particular, in the field of photovoltaic energy conversion, this study is used to quantify the part of light converted into electrical energy [7-9]. Nevertheless, the investigation of the scattering of light by small particles continues to surprise us with new discoveries and exciting theoretical and experimental developments, such as optical trapping, abnormal light scattering and optical tweezers [10].

On another side, several studies carried out on the direct and indirect observation of the size and shape of ice crystals have again underlined the irregular nature of these particles [11]. Ice crystals in earth's atmosphere generally retain the shape of hexagonal columns or plates. However, the fluctuations in ambient temperature, pressure

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and wind speed can cause any degree of irregularity [12]. Recent analyses of holograms taken from airplanes in cirrus clouds show a wide variety of crystal structures [13]. According to the classification of ice crystals of Magano and Lee [14], these crystals are classified as the pyramid, the cup, the solid bullet, the hollow bullet, the solid column, the hollow column, the capped column and the bullet rosette [12]. Their role in the atmosphere is very important directly and indirectly. These particles modify the radiative balance of the atmosphere and a great uncertainty persists as to the value of their radiative forcing. It is therefore necessary to study them in more detail.

To study the scattering by a spherical particle, three possibilities exist according to the ratio of the particle size (α): Rayleigh's theory which studies light scattering by particles of very small size ($\alpha \ll 1$), the theory of geometrical optics which applies to large particles ($\alpha \gg 1$) and Mie theory which is used for intermediate-size particles. In principle, the interaction of the particle with incident electromagnetic radiation can be determined by solving the Maxwell equations with boundary conditions corresponding to the shape of the particle; there are exact solutions for certain shapes, such as spheres, ellipsoids and infinite cylinders [1, 15].

For non-spherical particles of very small sizes, the Rayleigh's theory remains valid. For particles of intermediate sizes and fairly simple shapes, there are theories for various shapes and for different sizes. For example, the T-matrix theory [16] allows a fairly fast calculation for flattened or elongated ellipsoid particles. For complex or arbitrary forms, theories begin to be elaborated [17], such as the finite-element methods [18] and the finite-difference domain method [19]. These techniques are very precise, but very expensive in terms of calculation time, which is their main disadvantage.

Under such circumstances, the use of approximation methods becomes preferable or even mandatory. The most widely used analytical approximations for practical situations are the Rayleigh-Gans (RG) approximation and anomalous diffraction (AD) of van de Hulst [1]. The Wentzel-Kramers-Brillouin (WKB) approximation [20] is a classical approximation, which takes the phase shift into account correctly, so it does not have restriction on the phase shift magnitude, contrary to other approximations. The WKB approximation has been successfully applied to spheres, spheroids and cylinders [20, 6]. Recently, this approach has been applied to model the scattering properties of hexagonal and prismatic particles [10, 21].

In this work, the WKB approximation is used to investigate the scattering of light by square pyramidal particles. This study is an extension of the work of Ibnchaikh (2016) [10] to other non-column particles. For this, the analytical expression of the form factor for square pyramid ice crystal was derived using the WKB method. Furthermore, the extinction coefficient is calculated in order to illustrate the results.

2. WKB Approximation

Consider a particle illuminated by a plane wave polarized in the direction $\overrightarrow{e_x}$ and propagating along the z-axis (Fig. 1).

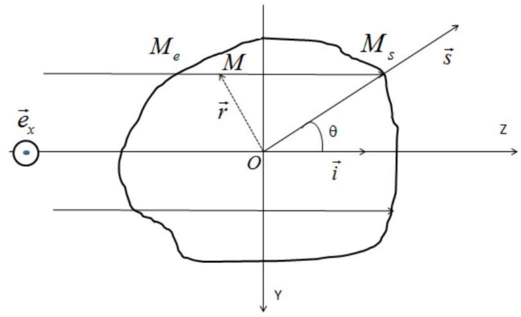

FIG. 1. Description of the scattering problem.

In the literature, the expression of the amplitude of light scattering in the WKB approximation, in a scalar form, is [20, 22]:

$$|f(\vec{s},\vec{i})| = \frac{k^2}{2\pi}\sin(\aleph)|(m-1)F(\theta,\varphi)| \quad (1)$$

where $\vec{s}$ and $\vec{i}$ are the unit vectors along the direction of scattering and propagation of light, respectively. $\aleph$ is the angle between the polarization vector $\overrightarrow{e_x}$ and the unit vector $\vec{s}$, θ is the scattering angle between $\vec{s}$ and $\vec{i}$, φ is the azimuth angle, k is the wave vector and m is the relative complex refractive index. The quantity $F(\theta,\varphi)$ is known as the form factor which represents the modification of the scattered irradiance due to the finite size of the particle and to its deviation from sphericity:

$$F(\theta,\varphi) = \iiint_v \exp\big(ik\vec{r}.(\vec{i}-\vec{s})\big)\exp(ikw)dv \quad (2)$$

where $\vec{r}$ is the position vector of any point within the scattering object, v is the volume of the studied particle and:

$$w = \int_{z_e}^{z} (m(z') - 1)dz' = (m-1)(z - z_e) \quad (3)$$

where w is the optical path for a homogenous particle which is introduced by the scattering object, z is the z-coordinate of the scatter element inside the particle and z_e is the z-coordinate of the initial position of penetration of the object.

In rectangular coordinates, the form factor can be expressed as follows:

$$F(\theta, \varphi) = \iiint_v \left. \begin{matrix} e^{-ikx \sin\theta \cos\varphi} e^{-iky \sin\theta \sin\varphi} \\ e^{ikz(m-\cos\theta)} e^{-ikz_e(m-1)} \end{matrix} \right\} dv \quad (4)$$

where x, y and z are the components of the position of the scattering element inside the object.

By integrating Eq. (4) over z, one obtains:

$$F(\theta, \varphi) = \frac{1}{ik(m-\cos\theta)} \cdot \iint e^{-ikx \sin\theta \cos\varphi} e^{-iky \sin\theta \sin\varphi} \, G(z_e, z_s) dxdy \quad (5)$$

with:

$$G(z_e, z_s) = e^{ik(m-\cos\theta)z_s} e^{-ik(m-1)z_e} - e^{ik(1-\cos\theta)z_e} \quad (6)$$

where z_e and z_s are the z-coordinates of the intersection of the light and the body lateral surfaces, as shown in Fig. 2.

To calculate the integral in Eq. (5) on the volume of the particle, it is necessary to determine the z-coordinates of the intersection of the light and the body lateral surfaces of the particle. For this, the pyramidal particle is cut in several square slices along its main axis (Fig. 2(c)).

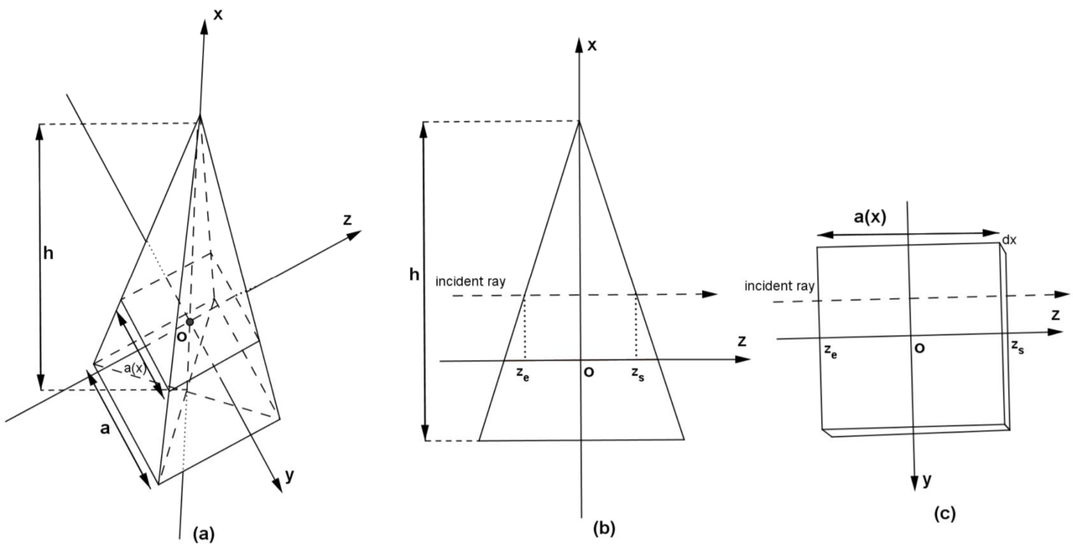

FIG. 2. Decomposition of the pyramid.

3. Results and Discussion

Consider a particle of pyramidal shape with equilateral faces and a square base oriented in the $y - z$ plane and of side length a. The origin of the Cartesian coordinate system orthonormal $R(O, X, Y, Z)$ coincides with the center of the particle (pyramid) and the principal axis of the pyramid is oriented along the x-axis.

The use of the WKB method for oblique incidence presents some difficulties. Thus, this approximation will be applied in this work to derive the form factor of a pyramidal particle for two special cases of normal incidence: flat incidence (Fig. 3(a)) and edge-on incidence (Fig. 3(b)). Therefore, the pyramid is cut into infinitely thin slices with thickness dx; these slices are perpendicular to the symmetric axis of the pyramid and are in the form of squares. The length of its rib is $a(x) = \frac{3a}{4} - \frac{ax}{h}$, with h being the height of the pyramid. So, for each slice, we can derive its contribution to the form factor.

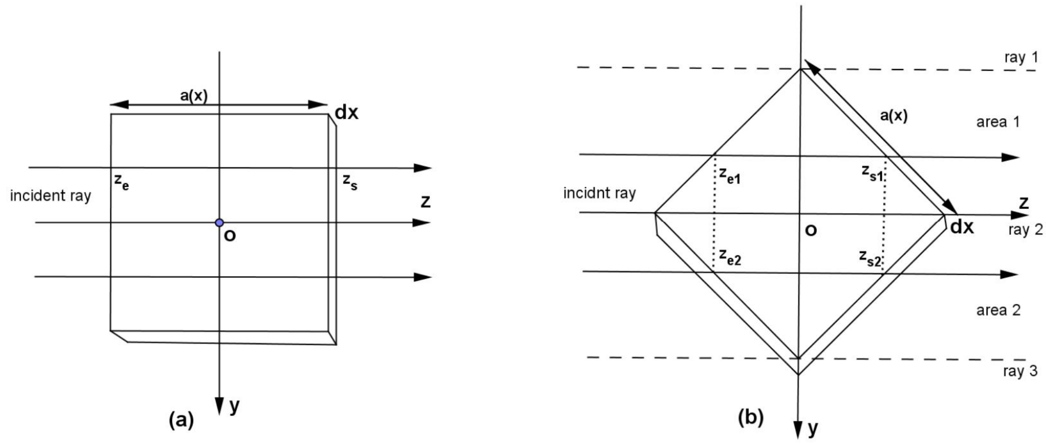

FIG. 3. Slices of the pyramid for flat incidence (a) and edge-on incidence (b).

The z-coordinates for the flat incidence are defined from Fig. 3(a):

$$z_s = -z_e = \frac{a(x)}{2}, -\frac{a(x)}{2} \leq y \leq \frac{a(x)}{2} \tag{7}$$

and the z-coordinates for the edge-on incidence are defined from Fig. 3(b):

$$\left.\begin{aligned} z_{s1} &= -z_{e1} = \frac{\sqrt{2}}{2}a(x) + y, -\frac{\sqrt{2}}{2}a(x) \leq y \leq 0 \\ z_{s2} &= -z_{e2} = \frac{\sqrt{2}}{2}a(x) - y, 0 \leq y \leq \frac{\sqrt{2}}{2}a(x) \end{aligned}\right\} \tag{8}$$

3.1 Flat Incidence

In this first case of incidence, Fig. 3(a) is used. Four particular cases are considered for which the analytical expression of the integral given by Eq. (5) is not defined for a number of values of θ and φ, as well as a general case. The form factor for the flat incidence is denoted by $F_1(\theta, \varphi)$.

a) *Case of* $\boldsymbol{\theta = 0}$

After some algebraic manipulations, one obtains the form factor:

$$F_1(0, \varphi) = \frac{a^2 h}{i2\rho}\left\{\frac{2}{i\rho}\left[\left(\frac{1-e^{i\rho}}{i\rho}\right) + e^{i\rho}\right] - 1\right\} \tag{9}$$

where $\rho = ka(m-1)$.

b) *Case of* $\boldsymbol{\theta = \pi}$

In this case, the expression of the form factor is given by:

$$F_1(\pi, \varphi) = \frac{h}{k^2(m+1)}\left\{\frac{1}{m}\left(e^{ikam}\left(1 - \frac{1}{ikam}\right) + \frac{1}{ikam}\right) + e^{-ika}\left(1 + \frac{1}{ika}\right) - \frac{1}{ika}\right\} \tag{10}$$

c) *Case of* $\boldsymbol{0 < \theta < \pi, \varphi = 0}$

By integration of Eq. (5) over y, the contribution of a slice $f^1(\theta, 0)$ to the form factor is given by the following expression:

$$f^1(\theta, 0)dx = \frac{dx}{ik(m-\cos\theta)}\left(e^{i\frac{3}{4}g}\left(\frac{3a}{4}I'_+ - \frac{a}{h}I'_-\right) - e^{i\frac{3}{4}g}\left(\frac{3a}{4}J'_+ - \frac{a}{h}J'_-\right)\right) \tag{11}$$

with:

$$I'_+ = e^{-ikx\sin\theta}e^{-i\frac{g}{h}x} \tag{12}$$

$$I'_- = xe^{-ikx\sin\theta}e^{-i\frac{g}{h}x} \tag{13}$$

$$J'_+ = e^{-ikx\sin\theta}e^{-i\frac{q}{h}x} \tag{14}$$

$$J'_- = xe^{-ikx\sin\theta}e^{-i\frac{q}{h}x}\,. \tag{15}$$

Therefore, the contribution from all slices to the form factor is expressed as:

$$F_1(\theta, 0) = \int_{-\frac{h}{4}}^{\frac{3}{4}h} f^1(\theta, 0)dx\,. \tag{16}$$

Integration over x gives:

$$F_1(\theta, 0) = \frac{ahe^{-i\frac{3}{4}kh\sin\theta}}{ik(m-\cos\theta)}\left\{\left(\frac{e^{iZ_g^+}}{iZ_g^+} - \frac{e^{iZ_q^+}}{iZ_q^+}\right) - \left(\frac{e^{iZ_g^+}-1}{(iZ_g^+)^2} - \frac{e^{iZ_q^+}-1}{(iZ_q^+)^2}\right)\right\} \tag{17}$$

with:

$$Z_g^+ = kh\sin\theta + g \tag{18}$$

$$Z_q^+ = kh\sin\theta + q \tag{19}$$

and the parameters used are: $\mu = \frac{ka}{2}(m - \cos\theta)$, $g = \frac{\rho}{2} + \mu$, $q = \frac{\rho}{2} - \mu$ and ρ as already defined in the case $\theta = 0$.

d) *Case of* $0 < \theta < \pi$ *and* $\varphi = \pi$

In this case, the same method is used to determine the expression of the form factor; it suffices to calculate the integral of Eq. (5) over y and x; so:

$$F_1(\theta,\pi) = \frac{ahe^{-i\frac{1}{4}kh\sin\theta}}{ik(m-\cos\theta)}\left\{\left(\frac{e^{iq}}{iZ_q^-} - \frac{e^{ig}}{iZ_g^-}\right) - \left(e^{iq}\frac{e^{iZ_q^-}-1}{(iZ_q^-)^2} - e^{ig}\frac{e^{iZ_g^-}-1}{(iZ_g^-)^2}\right)\right\} \quad (20)$$

with:

$$Z_g^- = kh\sin\theta - g \quad (21)$$

$$Z_q^- = kh\sin\theta - q\,. \quad (22)$$

e) *General Case*

In this general case, the values of θ and φ are different from those of the preceding cases. By integrating Eq. (5) over y, the expression of the contribution of a slice $f^1(\theta,\varphi)$ to the form factor becomes:

$$f^1(\theta,\varphi)dx = \frac{dx}{ik(m-\cos\theta)ik\sin\theta\sin\varphi}\left((I_+ - I_-) - (J_+ - J_-)\right) \quad (23)$$

with:

$$I_{\pm} = e^{-ikx\sin\theta\cos\varphi}e^{ikz_s(2m-1-\cos\theta)}e^{\pm iky_s\sin\theta\sin\varphi} \quad (24)$$

$$J_{\pm} = e^{-ikx\sin\theta\cos\varphi}e^{ikz_s(\cos\theta-1)}e^{\pm iky_s\sin\theta\sin\varphi}. \quad (25)$$

Thus, the contribution from all slices to the form factor is expressed as:

$$F_1(\theta,\varphi) = \int_{-\frac{h}{4}}^{\frac{3}{4}h} f^1(\theta,\varphi)dx\,. \quad (26)$$

By introducing the value of z_s defined in Eq. (7) and by integrating with respect to x, the previous integral takes the following form:

$$F_1(\theta,\varphi) = -\frac{\frac{ha^2}{4}}{ut}e^{-i\frac{3}{4}d}\left\{\left(\frac{e^{iA^+}-1}{iA^+} - \frac{e^{iA^-}-1}{iA^-}\right) - \left(\frac{e^{iB^+}-1}{iB^+} - \frac{e^{iB^-}-1}{iB^-}\right)\right\} \quad (27)$$

with:

$$A^{\pm} = d + g \pm t \quad (28)$$

$$B^{\pm} = d + q \pm t\,. \quad (29)$$

The parameters used are:

$$d = hk\sin\theta\cos\varphi \quad (30)$$

$$t = \frac{ka}{2}\sin\theta\sin\varphi. \quad (31)$$

3.2 Edge-on Incidence

In this second case of incidence, the slice of the pyramid is divided into two areas by ray 1, ray 2 and ray 3 (see Fig. 3(b)). The ray paths z_{ej} and z_{sj} for each area as well as the function $G(z_e, z_s)$ (see Eq. (6)) depend on the variables x and y. As in the case of flat incidence, there are four special cases and a general case to be treated.

a) *Case of* $\theta = 0$

In this case, the expression of the form factor is given by:

$$F_2(0,\varphi) = \frac{a^2h}{i\rho'}\left\{\frac{2}{i\rho'}\left(\frac{e^{i\rho'}-1}{i\rho'} - 1\right) - 1\right\} \quad (32)$$

where $F_2(\theta,\varphi)$ represents the form factor for the edge-on incidence, with $\rho' = \sqrt{2}ka(m-1)$.

b) *Case of* $\theta = \pi$

Similarly here, the expression of the form factor is easily calculated:

$$F_2(\pi,\varphi) = -\frac{h}{k^2(m+1)}\left\{\frac{1}{m}\left(\frac{e^{i\sqrt{2}kam}-1}{i\sqrt{2}kam} - 1\right) - \frac{e^{i\sqrt{2}ka}-1}{i\sqrt{2}ka} - 1\right\}. \quad (33)$$

c) *Case of* $0 < \theta < \pi$ *and* $\varphi = 0$

By the integration of Eq. (5) over y, the expression of the contribution of a slice $f^2(\theta,0)$ to the form factor is given by:

$$f^2(\theta,0)dx = \frac{dx}{ik(m-\cos\theta)}\left(f^{2-}(\theta,0) + f^{2+}(\theta,0)\right) \quad (34)$$

where $f^{2-}(\theta,0)$ and $f^{2+}(\theta,0)$ are respectively the contributions of area 1 and area 2 of the slice to the form factor, with:

$$f^{2\pm}(\theta,0) = \left(I''_{\pm} - J''_{\pm}\right) \quad (35)$$

and

$$I''_- = I''_+ = \frac{\sqrt{2}a}{i2g'}\left(e^{i\frac{3}{4}g'}I''_{1-} - I''_{2-}\right) \quad (36)$$

$$I''_{1-} = e^{-ikx\sin\theta}e^{-i\frac{g'}{h}x} \quad (37)$$

$$I''_{2-} = e^{-ikx\sin\theta} \quad (38)$$

$$J''_- = J'_+ = \frac{\sqrt{2}a}{i2q'}\left(e^{i\frac{3}{4}q'}J''_{1-} - J''_{2-}\right) \quad (39)$$

$$J''_{1-} = e^{-ikx\sin\theta} e^{-i\frac{q'}{h}x} \quad (40)$$

$$J''_{2-} = e^{-ikx\sin\theta}. \quad (41)$$

Then, the contribution from all slices to the form factor is expressed as:

$$F_2(\theta,0) = F_{2-}(\theta,0) + F_{2+}(\theta,0) \quad (42)$$

where $F_{2-}(\theta,0)$ and $F_{2+}(\theta,0)$ are the contributions of area 1 and area 2 to the form factor, respectively, with:

$$F_{2\pm}(\theta,0) = \frac{1}{ik(m-\cos\theta)} \int_{-\frac{h}{4}}^{\frac{3}{4}h} f^{2\pm}(\theta,0)dx \quad (43)$$

By performing the last integration, the expression of Eq. (42) becomes:

$$F_2(\theta,0) = \frac{\sqrt{2}ahe^{-i\frac{3}{4}kh\sin\theta}}{ik(m-\cos\theta)} \left\{ \frac{1}{ig'} \frac{e^{iZ^+_{g'}}-1}{iZ^+_{g'}} - \frac{1}{iq'} \frac{e^{iZ^+_{q'}}-1}{iZ^+_{q'}} + \frac{e^{ikh\sin\theta}-1}{ikh\sin\theta} \left(\frac{1}{iq'} - \frac{1}{ig'} \right) \right\} \quad (44)$$

where:

$$Z^+_{g'} = kh\sin\theta + g' \quad (45)$$

$$Z^+_{q'} = kh\sin\theta + q'. \quad (46)$$

The parameters used are: $\mu' = \frac{\sqrt{2}}{2} ka(m - \cos\theta)$, $g' = \frac{\rho'}{2} + \mu'$, $q' = \frac{\rho'}{2} - \mu'$ and the parameter ρ' as defined above.

d) *Case of* $0 < \theta < \pi$ *and* $\varphi = \pi$

The same method used above gives the expression of the form factor:

$$F_2(\theta,\pi) = \frac{\sqrt{2}ahe^{-i\frac{3}{4}kh\sin\theta}}{ik(m-\cos\theta)} \left\{ \frac{1}{ig'} \frac{1-e^{-iZ^-_{g'}}}{iZ^-_{g'}} - \frac{1}{iq} \frac{1-e^{-iZ^-_{q'}}}{iZ^-_{q'}} + \frac{1-e^{-ikh\sin\theta}}{ikh\sin\theta} \left(\frac{1}{iq'} - \frac{1}{ig'} \right) \right\} \quad (47)$$

with:

$$Z^-_{g'} = kh\sin\theta - g' \quad (48)$$

$$Z^-_{q'} = kh\sin\theta - q'. \quad (49)$$

e) *General Case*

The contribution from the slice to the form factor in this general case can be expressed in a simple form:

$$f^2(\theta,\varphi)dx = \frac{dx}{ik(m-\cos\theta)} \left(f^{2-}(\theta,\varphi) + f^{2+}(\theta,\varphi) \right) \quad (50)$$

with:

$$f^{2\pm}(\theta,\varphi) = \frac{e^{i\frac{3\sqrt{2}}{8}ka(2m-1-\cos\theta)}}{\mp ik(\sin\theta\sin\varphi \pm (2m-1-\cos\theta))} (I_{1\pm} - I_{2\pm}) - \frac{e^{i\frac{3\sqrt{2}}{8}ka(\cos\theta-1)}}{\mp ik(\sin\theta\sin\varphi \pm (\cos\theta-1))} (J_{1\pm} - J_{2\pm}) \quad (51)$$

$$I_{1\pm} = e^{-ikx\left(\sin\theta\cos\varphi + \frac{\sqrt{2}a}{2h}(2m-1-\cos\theta)\right)} \times e^{\mp i\frac{\sqrt{2}}{2}ka(x)(\sin\theta\sin\varphi \pm (2m-1-\cos\theta))} \quad (52)$$

$$I_{2\pm} = e^{-ikx\left(\sin\theta\cos\varphi + \frac{\sqrt{2}a}{2h}(2m-1-\cos\theta)\right)} \quad (53)$$

$$J_{1\pm} = e^{-ikx\left(\sin\theta\cos\varphi + \frac{\sqrt{2}a}{2h}(\cos\theta-1)\right)} \times e^{\mp i\frac{\sqrt{2}}{2}ka(x)(\sin\theta\sin\varphi \pm (\cos\theta-1))} \quad (54)$$

$$J_{2\pm} = e^{-ikx\left(\sin\theta\cos\varphi + \frac{\sqrt{2}a}{2h}(\cos\theta-1)\right)}. \quad (55)$$

The form factor can therefore be written as:

$$F_2(\theta,\varphi) = F_{2-}(\theta,\varphi) + F_{2+}(\theta,\varphi) \quad (56)$$

where:

$$F_{2\pm}(\theta,\varphi) = \frac{1}{ik(m-\cos\theta)} \int_{-\frac{h}{4}}^{\frac{3h}{4}} f^{2\pm}(\theta,\varphi)dx \quad (57)$$

After some algebraic manipulations, Eq. (57) becomes:

$$F_{2\pm}(\theta,\varphi) = -\frac{a^2}{2\mu'} \left\{ \frac{e^{i\frac{3}{4}g'}}{\mp(\sqrt{2}t \pm g')} \int_{-\frac{h}{4}}^{\frac{3h}{4}} (I_{1\pm} - I_{2\pm})dx - \frac{e^{i\frac{3}{4}q'}}{\mp(\sqrt{2}t \pm q')} \int_{-\frac{h}{4}}^{\frac{3h}{4}} (J_{1\pm} - J_{2\pm})dx \right\}. \quad (58)$$

Finally, by integrating Eq. (58) over the variable x, the expression of the form factor for each area is given by:

$$F_{2\pm}(\theta,\varphi) = -\frac{a^2h}{2\mu'} \left\{ \frac{e^{-i\frac{3}{4}d}}{\mp(\sqrt{2}t \pm g')} \left[\frac{e^{i(d\mp\sqrt{2}t)}-1}{i(d\mp\sqrt{2}t)} - \frac{e^{i(d+g')}-1}{i(d+g')} \right] - \frac{e^{-i\frac{3}{4}d}}{\mp(\sqrt{2}t \pm q')} \left[\frac{e^{i(d\mp\sqrt{2}t)}-1}{i(d\mp\sqrt{2}t)} - \frac{e^{i(d+q')}-1}{i(d+q')} \right] \right\} \quad (59)$$

where the parameters d and t have been defined in the general case of flat incidence.

Note that F_{2+} has a singularity at $\sqrt{2}t + q'$ when $\theta = \varphi = \frac{\pi}{2}$. In this case, the recalculation of F_{2+} gives the following result:

$$F_{2+}\left(\frac{\pi}{2},\frac{\pi}{2}\right)=\frac{h}{k^2 m}\left\{\left[\left(1+\frac{\sqrt{2}}{ika}\left(1+\frac{1}{2m}\right)\right)\left(1-e^{-i\frac{\sqrt{2}}{2}ka}\right)-1\right]+\frac{1-e^{i\frac{\sqrt{2}}{2}ka(2m-1)}}{i\sqrt{2}kam(2m-1)}\right\} \quad (60)$$

The analytical expressions of the form factor for a pyramidal particle as part of the WKB approximation are established; they have been obtained for both flat incident light and edge-on incidence. Then, the correctness of the analytical expressions of the form factor is checked. For this, the different double integrals which intervene here are calculated numerically using Gauss Legendre quadrature method. By taking 16×16 integration points for this quadrature, the results obtained with the analytical expressions of the form factor are reproduced numerically.

For illustration, Fig. 4 and Fig. 5 show the behavior of the normalized form factor ($|F(\theta,\varphi)/F(0,0)|^2$) as a function of the scattering angles θ and φ on a logarithmic scale for some values of ka and $\frac{h}{a}$. In these figures, the axis $\varphi=\pi$ is an axis of symmetry of the form factor; only the values of φ between 0 and π are considered.

The graphs representing the normalized form factor in the different cases show that when the scattering angle θ increases gradually from a certain value, the normalized form factor varies greatly with the azimuth angle φ. This is manifested by the appearance of a number of bumps and dips. Consequently, the azimuth angle φ plays an important role in the study of the scattering of light by non-spherical particles unlike the case of spherical particles; this is in agreement with the fact that the particle has non-spherical symmetry. On the other hand, the graphs also show the importance of the size parameter and the orientation (flat incidence and edge-on incidence) of the non-spherical particles. Finally, it is noted that the backscatter is nearly imperceptible compared to the forward scatter.

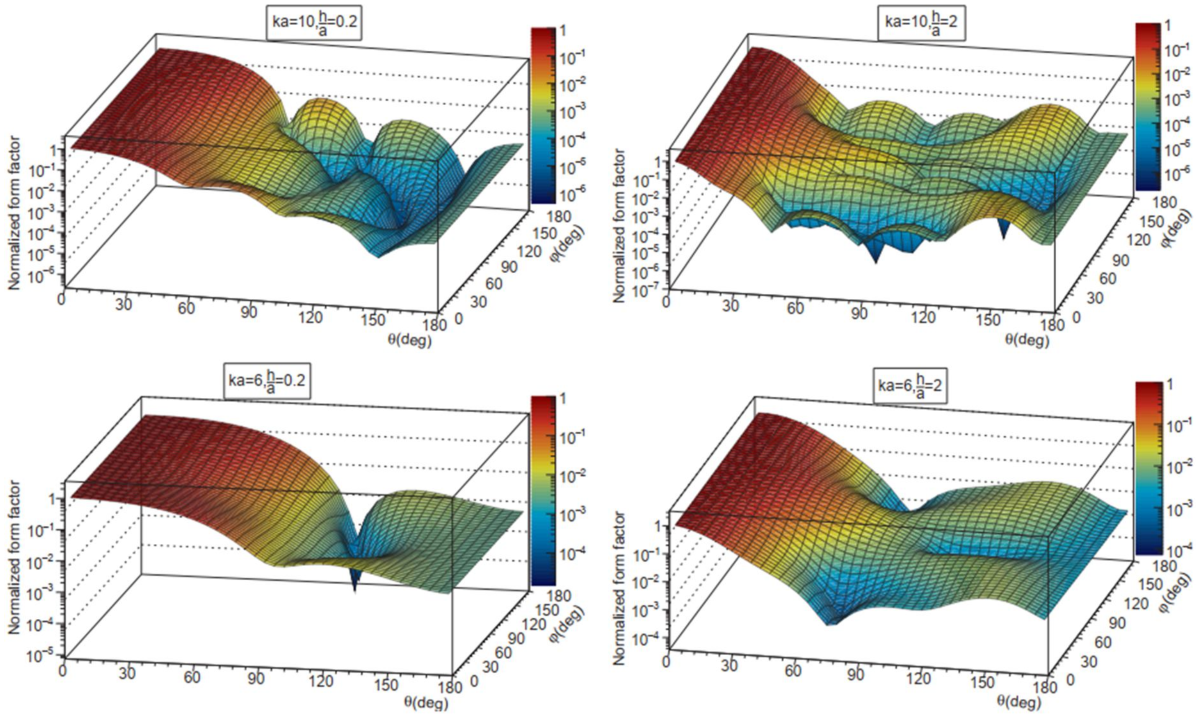

FIG. 4. Normalized form factor as a function of the scattering angles θ and φ for absorbing pyramidal particle for flat incident light at a complex refractive index $m=1.311+0.31\times10^{-8}i$ when $(ka=10, h/a=0.2)$, $(ka=10, h/a=2)$, $(ka=6, h/a=0.2)$ and $(ka=6, h/a=2)$.

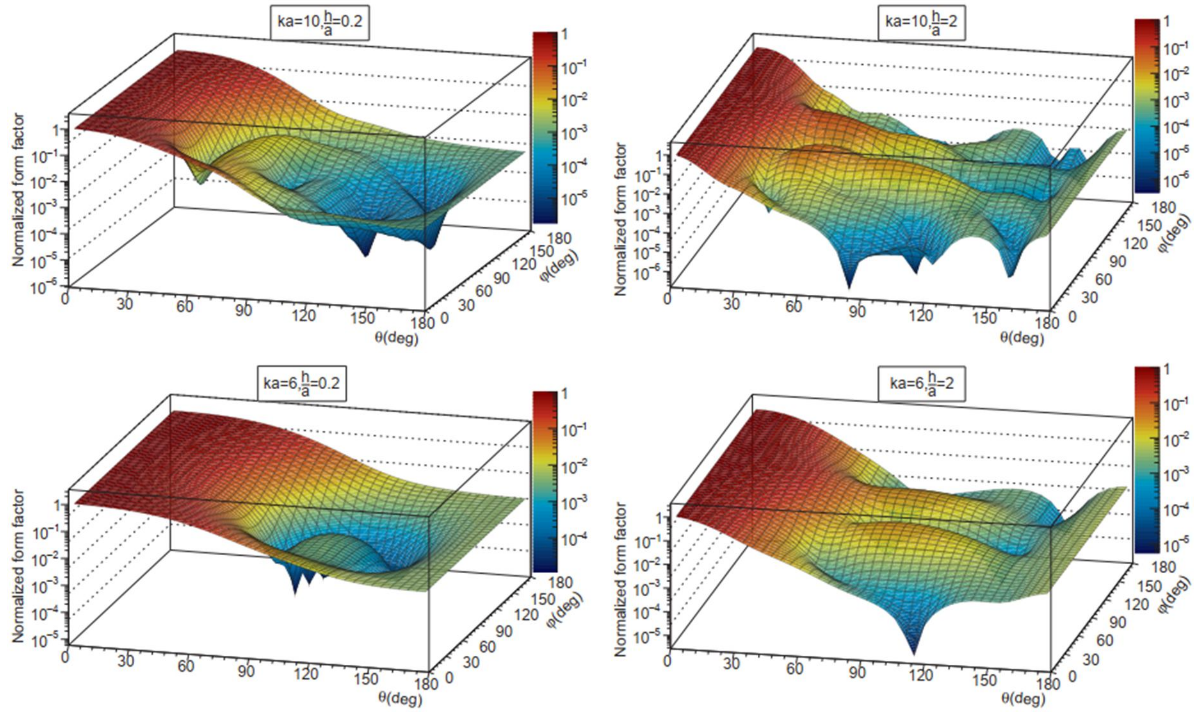

FIG. 5. Normalized form factor as a function of the scattering angles θ and φ for absorbing pyramidal particle for edge-on incidence at a complex refractive index $\mathrm{m} = 1.311 + 0.31 \times 10^{-8}\mathrm{i}$ when $(\mathrm{ka} = 10, \mathrm{h/a} = 0.2)$, $(\mathrm{ka} = 10, \mathrm{h/a} = 2)$, $(\mathrm{ka} = 6, \mathrm{h/a} = 0.2)$ and $(\mathrm{ka} = 6, \mathrm{h/a} = 2)$.

The form factors obtained here in Figs. 4 and 5, representing the modification of the scattered irradiance due to the finite size of the particle and to its deviation from sphericity, are studied for two particular cases: flat incidence (Fig. 4) and edge-on incidence (Fig. 5). In particular, the effects of the scattering angle θ and the azimuth angle φ on the form factor observed in this study can be used in the progress and development of numerous scientific applications, which are directly or indirectly linked to the phenomenon of light scattering by small particles, such as optical information, display and processing systems, telecommunications, photonics and optoelectronics [23]. Moreover, these results are used to deduce the optical properties of particles in other scientific disciplines, such as geophysics, astronomy, climatology and solar-energy technologies [17, 24, 25].

3.3 Extinction Efficiency

The extinction efficiency $\mathrm{Q_{ext}}$ is the extinction cross-section divided by the projection area of the particle. By using the optical theorem, the expression for the extinction efficiency is easily found from the forward form factor [1, 26]:

$$\mathrm{Q_{ext}} = \frac{2\mathrm{k}}{\mathrm{P}} \mathrm{I_m}((\mathrm{m}-1)\mathrm{F}(0,0)) \quad (61)$$

where P is the projected area of the particle on the plane perpendicular to the direction of the incident wave and the symbol $\mathrm{I_m}$ indicates the imaginary part.

The projected area of the pyramid in the first case (flat incidence) is given by: $\mathrm{P_1} = \frac{\mathrm{ah}}{2}$ and in the second case (edge-on incidence), it is given by: $\mathrm{P_2} = \frac{\mathrm{ah}}{\sqrt{2}}$. Thus, the extinction efficiencies of these two cases are given respectively by:

$$\mathrm{Q_{ext1}} = 2\mathrm{Re}\left(1 - \frac{2}{\mathrm{i}\rho}\left(\frac{(1-\mathrm{e}^{\mathrm{i}\rho})}{\mathrm{i}\rho} + \mathrm{e}^{\mathrm{i}\rho}\right)\right) \quad (62)$$

$$\mathrm{Q_{ext2}} = 2\mathrm{Re}\left(1 - \frac{2}{\mathrm{i}\rho'}\left(\frac{\left(\mathrm{e}^{\mathrm{i}\rho'}-1\right)}{\mathrm{i}\rho'} - 1\right)\right) \quad (63)$$

where the symbol Re designates the real part.

Since the parameters ρ and ρ' do not depend on the height h, the expressions of $\mathrm{Q_{ext}}$ above show that the extinction efficiency also does not depend on the height h.

For real refractive index, the extinction efficiencies become:

$$\mathrm{Q_{ext1}} = 2\left\{1 - 2\frac{\sin\rho}{\rho} + \frac{\sin(^{\rho}/_{2})^2}{(^{\rho}/_{2})^2}\right\} \quad (64)$$

$$\mathrm{Q_{ext2}} = 2\left\{1 - \frac{\sin(\frac{\rho'}{2})^2}{(\frac{\rho'}{2})^2}\right\} \quad (65)$$

Fig. 6 shows the variation of the extinction efficiency as a function of ka in two cases: flat incidence (Q_{ext1}) and edge-on incidence (Q_{ext2}). From this figure, it is noticed that the extinction efficiency oscillates around the value of 2 as the particle size further increases. For large values of ka, Q_{ext1} always oscillates around the value 2, while Q_{ext2} tends towards this same value 2. This extinction limit value corresponds to the extinction paradox described in the literature [1]. More recently, Berg in 2011 gave a more convincing explanation of this limit value 2 [27]. The particle intercepts a portion of the incident plane wave equal to its geometric section $\sigma_{geo} = P$. The resulting interaction vibrates the particle which re-emits a wave which is nothing other than the scattered wave. This scattered wave will, for its part, interfere with the incident wave in a destructive manner, again removing $\sigma_{geo} = P$ from the incident flux. We therefore find $\sigma_{ext} = 2\sigma_{geo}$ and finally $Q_{ext} = \frac{\sigma_{ext}}{P} = 2$.

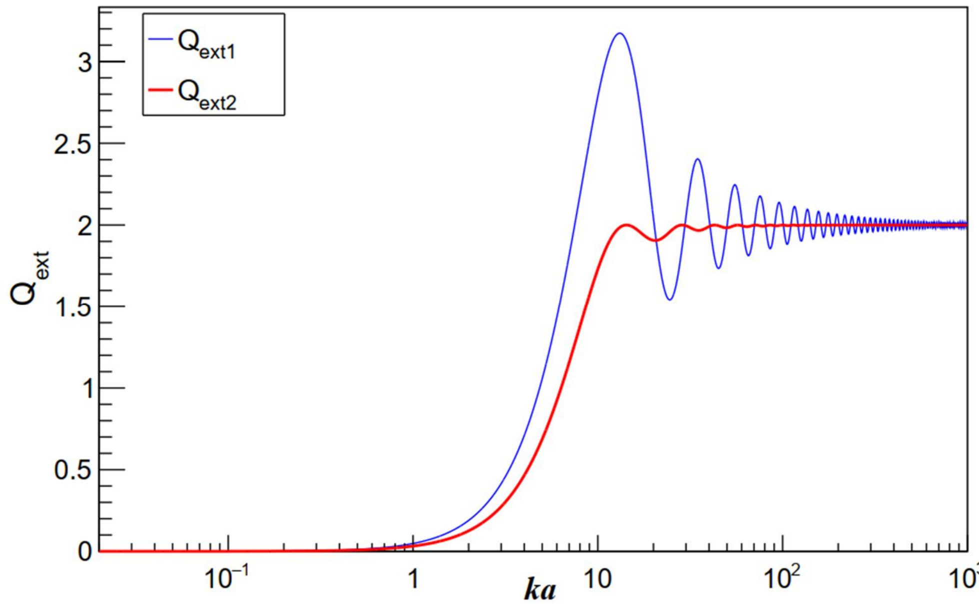

FIG. 6. Variation of the extinction coefficient as a function of the size parameter ka for a complex refractive index $m = 1.311 + 0.31 \times 10^{-8}i$.

4. Conclusion

In this work, the WKB method is used to study two particular cases of the scattering of light by a nonspherical pyramidal particle for normal incidence (flat incidence and edge-on incidence). The expression of the light scattering form factor of this particle is determined analytically for the two cases; it depends on the relative refractive index, the size parameter and the geometry of the particle. Unlike the case of spherical particles, the form factor is also dependent on the scattering angle θ and the azimuth angle φ. It is also seen that the particles that do not have spherical symmetry make the calculation of the form factor of light scattered by a pyramidal particle much more complicated.

Finally, the extinction efficiency for a pyramidal particle is calculated in the two cases of normal incidence according to the parameter size of the particle. The value of this coefficient tends to 2 when the particle size becomes infinite, which is in perfect agreement with other works in the literature.

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